THE PLURIPOTENCE OF BACKCALCULATED MODULI

A

THESIS

Presented to the Faculty

Of the University of Alaska Fairbanks

In Partial Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

By

Judith Ann Atkinson, B.S., M.S.

Fairbanks, Alaska

December 2002

UMI Number: 3071424

THE PLURIPOTENCE OF BACKCALCULATED MODULI

By

Judith Ann Atkinson

RECOMMENDED:

Advisory Committee Chair

Department Head

APPROVED:

Dean, College of Science, Engineering, and Mathematics

Dean of the Graduate School

9-24-02

Date

Abstract

The two main objectives of this research were first to investigate existing backcalculation techniques and the theories behind them for pluripotence (mathematical uniqueness of solution). Second to develop a backcalculation technique that addresses the pluripotence issue and attempts to overcome it.

The first objective was accomplished by demonstrating that most existing backcalculation programs do have pluripotence issues of some sort. Each program was analyzed and an attempt was made to determine how pluripotence is evaded. To accomplish the second objective the computer program KISS was developed based on Composite Plate theory. The program was verified by analyzing the deflection equations used for Composite Plate theory and comparing them to some popular deflection equations used in backcalculation today. The Composite Plate theory equations were also validated using finite element analysis.

Two types of sensitivity analysis were conducted. First the Composite Plate theory deflection equation was analyzed for sensitivity to various parameters. It was found that the deflections are most sensitive to layer thickness values and least sensitive to Poisson's ratio values. Secondly, the KISS program was analyzed for sensitivity. Plate size was investigated and it was determined that the plate size has a significant effect on backcalculation results. Variations in applied stress, seed values for $\overline{D}$ and k and AC assumed strength (strong, average or weak) were also included in the sensitivity analysis.

The KISS program was then compared with other popular backcalculation programs using data from the LTPP database. Three types of comparisons were made. First a Program-to-Program comparison was made in which the KISS program was compared to each of the other programs on a one-to-one basis. Secondly a Case-by-Case comparison was made in which the results from all programs were compared for each pavement system from the LTPP database. Thirdly, theoretical deflections from three different sources (Composite Plate theory, Elastic Layer theory, and finite element method) were generated for eight different theoretical pavement systems. In this method, the original moduli are known and were used as a basis for comparison.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

First and foremost I would like to thank Dr. Lutfi Raad for serving as chair of my committee over these nine years that it took to do this work. He did not give up on me even through some difficult times and I am truly grateful. Also I would like to acknowledge Dr. Jonah Lee of the Mechanical Engineering Department who has also served on my committee for nine years. I would like the thank Dr. J. Leroy Hulsey of the Civil Engineering Department for sharing his computer programming wizardry, help with the SAP2000 program and for lots of advice and encouragement over the course of this work. I would like to acknowledge Dr. Gary Gislason who served on my committee for seven years until his retirement and a special thanks to Dr. Ed Bueler of the Mathematics Department who kindly stepped in after Dr. Gislason's retirement.

I owe a debt of gratitude to those who provided technical assistance from afar when I had difficulties with the various computer programs. Linda M. Pierce, P.E. of Washington State Department of Transportation provided technical assistance with the EVERCALC computer program. Dave Bush of Dynatest fielded questions regarding the ELMOD program. Dr. Lynne H. Irwin provided invaluable assistance in understanding the innerworkings of the MODCOMP3 program.

I would like to thank Rosalind J. Kan, P.E., Janet L. Brown, P.E., and Bill Townsend, P.E. all of Alaska Department of Transportation and Public Facilities who have encouraged me and allowed me time to work on my research.

I have more friends and family than I could even begin to list who have encouraged me through the years. I owe a large debt of gratitude to my parents Joe and Joanna Atkinson who have been a great source of support and encouragement (and baby sitting) in my academic endeavors. Last but not least I would like to thank my children Alyna and Vallyn Atkinson who have served as my inspiration.

Chapter 1
Introduction

1.1 Problem Definition

In analysis of pavements, it is extremely valuable to know the properties of the materials within the pavement system. Non-destructive field testing techniques have been developed to estimate the moduli of multilayer pavement structures from surface deflection measurements. Loading equipment that simulates a moving wheel load, such as the Dynaflect and the Falling Weight Deflectometer (FWD), is generally used to apply an impact load to the pavement. The surface deflections associated with the applied load are then used to backcalculate the underlying layer moduli.

The backcalculated results can be used to provide material properties for the design of pavement overlays, and in developing rehabilitation recommendations and optimum maintenance strategies. The uniqueness (or non-uniqueness) of the moduli resulting from these backcalculations is the subject of this research.

In the past, non-uniqueness has been defined as meaning that if different initial seed moduli are used, then different backcalculated moduli will result (Lee, et al, ref 160). The intent of this paper is to demonstrate that there is more to solving the issue of uniqueness than finding the right seed values to start a backcalculation procedure.

First, a more in-depth definition of solution uniqueness is in order. For the purposes of this paper, uniqueness will be defined in a mathematical sense. The problem lies in the possibility that the equations used to determine (backcalculate) the moduli values may have more than one solution (possibly infinitely many!). I will term equations with more than one possible solution as pluripotent (i.e. having more than one potential solution).

1.2 Objectives

The objectives of this research are twofold. The first objective is to investigate existing backcalculation techniques and the theories behind them for pluripotence. The second objective is to develop a backcalculation technique that addresses the pluripotence issue and attempts to overcome it.

1.3 Scope of Work

This paper includes an exhaustive literature review of popular backcalculation theories and an investigation into the pluripotence of the equations involved. A new theoretical approach to backcalculation is developed and put into practice in the form of a computer program. This new technique is scrutinized with a sensitivity analysis, verification of the equations used, and comparison with several popular backcalculation programs.

1.4 Thesis Outline

The research in this paper is organized as follows:

Chapter 1: This chapter gives a brief introduction and description of this research.

Chapter 2: The equations used by various popular backcalculation programs are examined to determine if they are pluripotent. Several popular backcalculation programs are examined to determine how they get around the pluripotence of their equations. That is, how they "narrow it down" to one solution of potentially many.

Chapter 3: The equations for a composite plate resting on a Winkler foundation are used to develop a new backcalculation technique. A Program, using this technique, is developed for asphalt over a stabilized base.

Chapter 4: The deflection equations for Composite Plate Theory are verified by comparison to other theoretical deflections and to finite element generated deflections.

Chapter 5: The sensitivity of both the Composite Plate Theory deflection equation and the backcalculation program KISS, to changes in various parameters, are investigated.

Chapter 6: The new backcalculation technique is compared with other available backcalculation programs using actual field data and theoretical deflections.

Chapter 7: The information gathered in this research is summarized and conclusions are made. Suggestions are made for future research.

Chapter 2
Literature Review

2.1 Introduction

In this Chapter, the theories behind several popular backcalculation techniques are reviewed. The equations involved are examined for pluripotence. Several popular backcalculation programs are examined in detail to see how they get around the pluripotence of their equations.

The three major theories that will be reviewed are Elastic Layer Theory, the Method of Equivalent Thickness (MET), and Dimensional Analysis. Following the theoretical discussion, several popular backcalculation programs will be discussed in detail. The following is a list of programs to be included in the discussion: MODCOMP3, EVERCALC, MODULUS, CHEVDEF, BISDEF, ELSDEF, WESDEF, BOUSDEF, ELMOD, and ILLIBACK.

Finally a discussion of the history of research on the uniqueness of solutions will be included. An extensive, although not exhaustive listing of references on topics related to backcalculation is given in Appendix 2A. This appendix includes references on new and improved backcalculation methods, factors affecting backcalculation, non-destructive testing methods, articles comparing backcalculated results to each other as well as lab results, and finally articles on applications of backcalculated results.

2.2 Elastic Layer Theory

2.2.1 Background

Programs that assume linear elastic layers in the pavement system typically use equations that were developed by Burmister (1943, 1945, 1956, 1957). Burmister developed his equations based on the Theory of Elasticity derived by Love (1923). Equation 2-1 below is Burmister's equation for the deflection due to an arbitrary loading on the surface of the top layer in a two-layer system (pavement over a subgrade). It will be demonstrated that this equation is not pluripotent in a mathematical sense, that is, mathematically the equation appears to have a unique solution. However, there are some problems with the equation which cause pluripotence of a different kind. This will be explained in detail below.

$$W(r) = \frac{1.5 \cdot P}{2 \cdot \pi \cdot E_1} \int_0^{\infty} \left[\frac{e^{2mh} + 4Nmh - N^2 e^{-2mh}}{e^{2mh} - 2N(1 + 2m^2h^2) + N^2 e^{-2mh}} \right] \cdot J_0(mr)dm \qquad (2\text{-}1)$$

where,

$W(r)$ = Deflection at a distance r from the center of the load [in.]

P = an equivalent concentrated load at the surface of layer 1 [lb]

E_1 = modulus of layer 1 [psi]

h = thickness of layer 1 [in.]

r = distance from the center of the load [in.]

N = strength coefficient = $\dfrac{E_1 - E_{sg}}{E_1 + E_{sg}}$

E_{sg} = modulus of the subgrade [psi]

$J_0(mr)$ is a Bessel function of the first kind of order zero.

$$= \sum_{n=even}^{\infty} (-1)^{n/2} \frac{(mr)^n}{2^n\left(\frac{n}{2}!\right)^2} \text{ [unitless]}$$

m = variable of integration [1/in]

2.2.2 Theoretical Considerations

In the past, a mathematical proof of the uniqueness of solution (non-pluripotence) for Burmister's equation has not been obtained (refer to Mings, 1993*). However, non-pluripotence has been graphically indicated. Here the author intends to demonstrate non-pluripotence using another graphical approach. While this is not a mathematical proof, it shows clear evidence to support the idea of non-pluripotence for Burmister's equation.

It is evident from observing Burmister's equation, that if E_1 is changed, N would also have to change to compensate for the change in E_1 in order to keep the same deflection value. Upon further observation, it can be seen that if E_1 remains unchanged then N cannot change, if it did, a different deflection would result. This tells us that for a given N, there is only one (E_1, E_{sg}) pair that will generate a specified deflection value. This holds true for any given distance (r) from the center of the load.

On the other hand, if N is changed, the following tables and graph will demonstrate that while there are an infinite number of moduli pairs that can generate any

* The author's last name has been changed to Atkinson since the publication of the referenced work.

given deflection, there is only one pair that can generate the entire deflection basin. That is to say that there is only one N (and hence only one E_l, E_{sg} pair) that will generate a specified deflection basin.

The first step in analyzing this equation was to determine an appropriate approximation for the infinite integral. It was determined using the EXCELL computer software that using Simpson's rule with 60 steps and integrating to approximately the twelfth root of the Bessel function provided sufficient convergence for the purposes of this paper. When integration was carried out any further, problems developed due to round off error in the program.

As a base case, (set 0) the values of E_l = 1,000,000 psi and E_{sg} = 500,000 psi were used. Table 2-1 gives the deflection basin calculated using Burmister's equation and the computer spreadsheet program EXCELL. For all of the following cases presented, a load of P = 9000 lb was used.

Table 2-1: Initial Deflection Values

r (in.)	W(r) (in.)
0	1.38E-02
6	6.92E-04
12	3.57E-04
24	1.68E-04
36	1.10E-04
54	7.35E-05
72	5.48E-05

r = distance from the center of the load.
W(r) = the deflection at a distance
r from the center of the load.

For any specific W(r), I will use W(12) for example, there are an infinite number of E_l, E_{sg} pairs which will generate that deflection. Table 2-2 gives a list of some of the E_l, E_{sg} pairs which will generate the same W(12) as our base set (set 0). Note that for any value of N between 0 and 1, you can determine an E_l, E_{sg} pair which will generate W(12). As N gets close to 0 or 1, some of the moduli values become unrealistic, but still, there are an infinite number of choices for N between 0 and 1.

Table 2-2: Moduli Pairs That Will Generate The Same W(12) as Set 0.

	N	E1 (psi)	Esg (psi)
set 0	0.3333	1000000	500000
set 1	0.1	550391	450320
set 2	0.2	709475	472983
set 3	0.3	916542	493523
set 4	0.4	1195814	512492
set 5	0.5	1590538	530179
set 6	0.6	2185966	546491
set 7	0.7	3176191	560504
set 8	0.8	5119301	568811
set 9	0.9	10579108	556795

However, when looking at the entire deflection basin generated by these moduli pairs, it is seen that the rest of the deflection values differ from our base set (given in Table 2-1).

Table 2-3: Deflection Bowls Generated by Moduli Pairs Given in Table 2-2.

R	0	6	12	24	36	54	72
set 0	1.38E-02	6.91E-04	3.57E-04	1.68E-04	1.11E-04	7.34E-05	5.47E-05
set 1	2.43E-02	7.12E-04	3.57E-04	1.75E-04	1.16E-04	7.72E-05	5.77E-05
set 2	1.91E-02	7.05E-04	3.57E-04	1.71E-04	1.13E-04	7.54E-05	5.63E-05
set 3	1.50E-02	7.43E-04	3.57E-04	1.69E-04	1.11E-04	7.39E-05	5.51E-05
set 4	1.17E-02	6.81E-04	3.57E-04	1.66E-04	1.09E-04	7.25E-05	5.40E-05
set 5	8.99E-03	6.63E-04	3.57E-04	1.64E-04	1.07E-04	7.12E-05	5.31E-05
set 6	6.74E-03	6.40E-04	3.57E-04	1.63E-04	1.06E-04	7.00E-05	5.22E-05
set 7	4.85E-03	6.09E-04	3.57E-04	1.63E-04	1.04E-04	6.91E-05	5.16E-05
set 8	3.23E-03	5.68E-04	3.57E-04	1.67E-04	1.05E-04	6.88E-05	5.14E-05
set 9	1.82E-03	5.02E-04	3.57E-04	1.82E-04	1.11E-04	7.10E-05	5.31E-05

As can be seen from Table 2-3, the values of W(12) are equal, but the rest of the deflections are not. This is further demonstrated graphically in Figure 2-1.

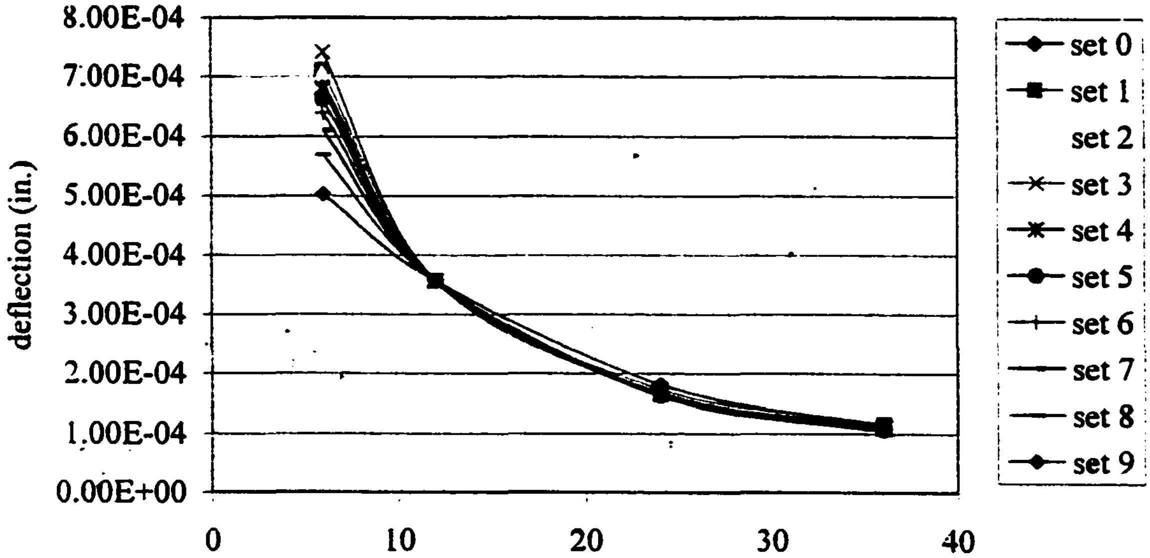


Figure 2-1: r vs Deflection

Figure 2-1 only shows deflections at r = 6, 12, 24, and 36 inches. This was done for clarity of the plot. As mentioned above, this is not a mathematical proof, but clearly demonstrates that there is only one pair of moduli that will give a specified deflection basin. The Figure and Table 2-3 demonstrate that for the other sets of E1, E_{sg} pairs that give the same W(12) none of the other deflections match so the resulting deflection basins are all different but intersect at W(12).

Having demonstrated the non-pluripotence (uniqueness) of Burmister's method, some explanation of the wide variety of solutions that can be achieved from different programs that use Burmister's equation will now be offered. As observed from Equation 2-1, Burmister's equation involves an infinite integral. Also note that the Bessel function is actually an infinite sum. Both the infinite integral and Bessel function cannot be evaluated outright but must be approximated. Each of the methods that use Burmister's equation have their own methods of approximation. Herein lies the difficulty of this method. In approximating the infinite integral and the infinite sum, errors are introduced.

2.3 Method of Equivalent Thickness

2.3.1 Background

The method of equivalent thickness assumes that any two layers with similar structural stiffness will distribute loading in the same way. This method was first developed by N. Odemark in 1949 (Ullidtz, 1987). In this method, all layers in a multilayered structure can be converted to one layer with equivalent stiffness (Zhou et. al., 1990). Programs that apply the Method of Equivalent Thickness (MET) employ Boussinesq theory to calculate deflections (Ullidtz, 1987, 1998). The thickness for an equivalent single layer of the same modulus as the subgrade is calculated for the upper n-1 layers (the nth layer is the subgrade). The following equation is used (Zhou et al., 1990):

$$H_{eq} = \sum_{i=1}^{n-1} h_{ei} = \sum_{i=1}^{n-1} f \cdot_i h_i \cdot \left[\frac{E_i}{E_n} \frac{(1-\mu_n^2)}{(1-\mu_i^2)} \right]^{1/3} \tag{2-2}$$

Where

E_i represents the modulus of the ith later,

E_n represents the modulus of the subgrade (nth later),

f_i = is a correction factor equal to 0.8 except for the first layer, where it is 0.9 for a one-layer (pavement on subgrade) and 1.0 for a multi-layer structure,

and μ and h represent the Poisson's ratio and thickness (respectively) for the layer indicated by the subscript.

2.3.2 Theoretical Considerations

Boussinesq theory involves assuming a single semi-infinite layer. In the backcalculation process using MET, the calculated deflections are generally compared to the measured deflections and the layer moduli adjusted accordingly (Ullidtz, 1987). The adjusted layer moduli are used to calculate a new equivalent thickness and deflections are compared again. This continues until an appropriate convergence criterion is met. Figure 2-2 demonstrates that for as few as two layers there are many combinations of moduli (E_1, E_2) that will give the same equivalent thickness, H_{eq}. As the number of layers in a system is increased, the number of combinations of moduli values that will generate any given

H_{eq} will only increase. Thus the equations used in the method of equivalent thickness are pluripotent. Figure 2-2 shows the solution set of (E_1, E_2) pairs that will generate a specific value of H_{eq}. The solution set in Figure 2-2 was generated by starting with an initial E_1 of 1,000,000 psi and E_2 of 50,000 psi, with an E_{sg} (subgrade) of 10,000 psi this generates and H_{eq} of 55.4062. All of the (E_1, E_2) pairs on the plot, in Figure 2-2, generate the same H_{eq} for a subgrade of E_{sg} = 10,000 psi. For a different value of H_{eq} a different solution set is obtained. However, the point here is that each solution set contains an infinite number of potential solutions.

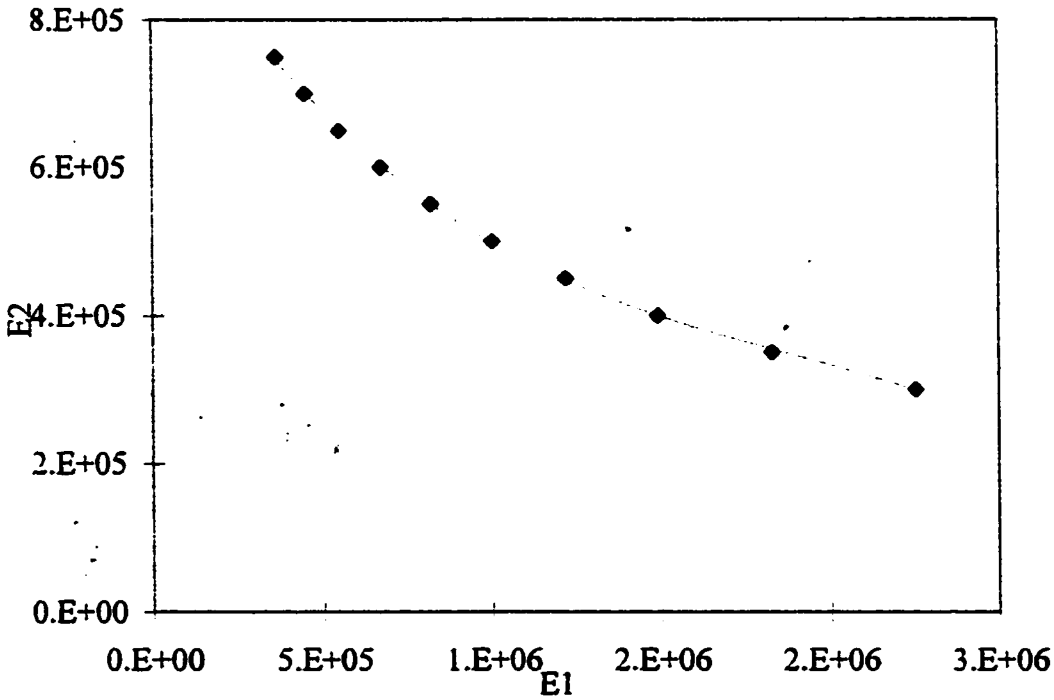


Figure 2-2: Pluripotentiality of Heq

2.4 Dimensional Analysis approach

2.4.1 Background

The Dimensional Analysis approach was developed by Ioannides for application with rigid pavements (1988, 1990). The equations for dimensionless deflections were derived based on the original works of Westergaard (1926, 1939), Hogg(1938), and Losberg(1960). The parameter AREA proposed by Hoffman (1980) was developed for the entire deflection basin and is calculated as follows :

$$\text{AREA} = \frac{\Lambda}{2D_0}\left[D_0 + 2\left(D_1 + D_2 + \cdots + D_{n-1}\right) + D_n\right] \tag{2-3}$$

Where,

AREA = a parameter characterizing the deflection basin [in.]

D_i = measured deflection at i^{th} sensor [in.]

Λ = spacing between the sensors. [in.]

Relationships between AREA and radius of relative stiffness, l, were developed. For a detailed description of these relationships the reader is referred to Hall and Mohseni (1991). These relationships are used to obtain l, which is then used to calculate nondimensional deflections based on Westergaard's work (Ioannides, 1994). The calculated deflections are in turn used to determine D (slab stiffness), which is then used to backcalculate layer modulus. It should be noted here that this method is only used on systems with one layer over a subgrade (a one-layer system) which means that D is a function of E_1 only.

2.4.2 Theoretical Considerations

The relationships between AREA and l were developed graphically and are one-to-one (see Figure 2-3). This means that from a given AREA value, the l value obtained is unique. Thus the calculated deflection, layer stiffness, and layer modulus are unique as well for a given AREA value. However, the AREA parameter is not unique in that it can be generated by more than one deflection basin. As a simplified example, suppose one has three measured deflections, say $D_0 = 0.01$, $D_1 = 0.004$, $D_2 = 0.001$, (assume $\Lambda = 6$ in.). This generates an AREA of 5.7. On the other hand, if the measured deflections were $D_0 = 0.01$, $D_1 = 0.0035$, $D_2 = 0.002$, (again assume $\Lambda = 6$ in.), again an AREA of 5.7 would be obtained. Thus two different deflection basins would generate the same AREA and hence the same radius of relative stiffness, l. While this is a pluripotence problem of a different kind than discussed previously, it still merits consideration.

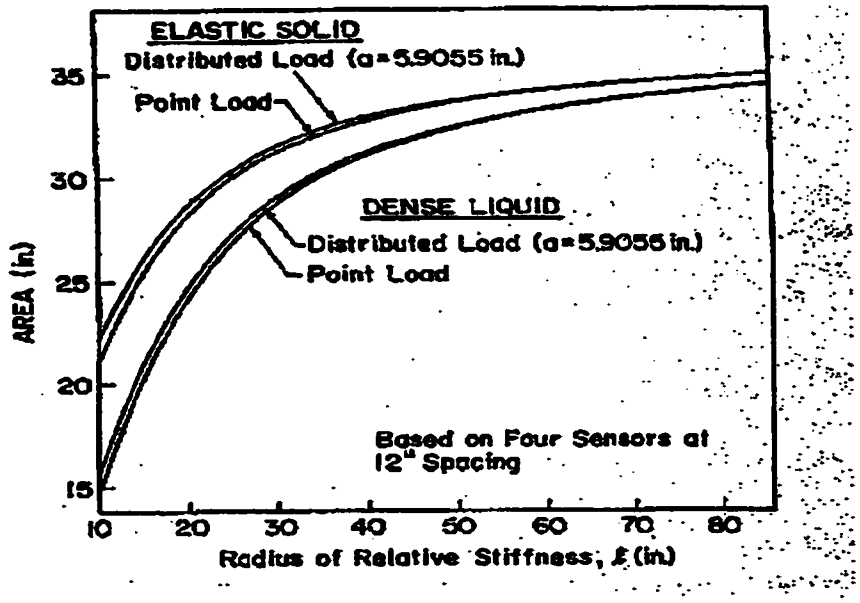


**Figure 2-3: AREA vs Radius of Relative Stiffness (Ioannides, 1988)
(Reproduced with permission)**

2.5 Applicability

In this section, several available backcalculation programs, which utilize some of the theories discussed above, will be outlined. Each program is summarized in a step-by-step procedure (without divulging too many trade secrets, as they weren't available) and a discussion of how a solution is obtained.

2.5.1 MODCOMP3

MODCOMP3 was developed by Irwin (1994). The original MODCOMP1 program was developed as a linear elastic program that has since been upgraded to include nonlinear layers. MODCOMP3 has the capacity to treat any combination of layers as being either linearly elastic or nonlinearly stress dependent. The program contains a total of eight nonlinear models of the form log-log constitutive model: $E = k_1 S^{k2}$ or the semi log form: E = k1*exp[S*k2]. The parameter S is a stress-strain parameter that depends on which of the eight models is chosen by the user, a detailed description of these models can be found in Irwin, 1994. The parameters k_1 and k_2 may be known or unknown. If unknown, deflection data from at least three load levels must be provided for the program to determine their values.

This program uses a modified version of the Chevron elastic layer computer program (CHEVLAY2) to calculated deflections (Dynatest et. al., 1993). Thus the program is based on Linear Elastic Layer theory. The program can handle up to twelve layers however it is recommended that no more than five of these have unknown moduli.

Dr. Irwin states that beyond five unknown moduli, there is a possibility that the solutions may not be unique, that is the specified fit might be accomplished by more than one set of layer moduli (1994). Further, with too many unknowns the program may not converge to a solution at all.

The following is a step-by-step description of the backcalculation procedure followed by MODCOMP3.

i) The program starts with a set of user supplied seed moduli from which deflections are calculated using CHEVLAY2.

ii) Each layer of unknown moduli is associated with a single deflection in one of two ways:

a) By default, MODCOMP3 uses a 34° line to estimate the line below which 95% of the deflection is caused, as demonstrated in Figure 2-4.

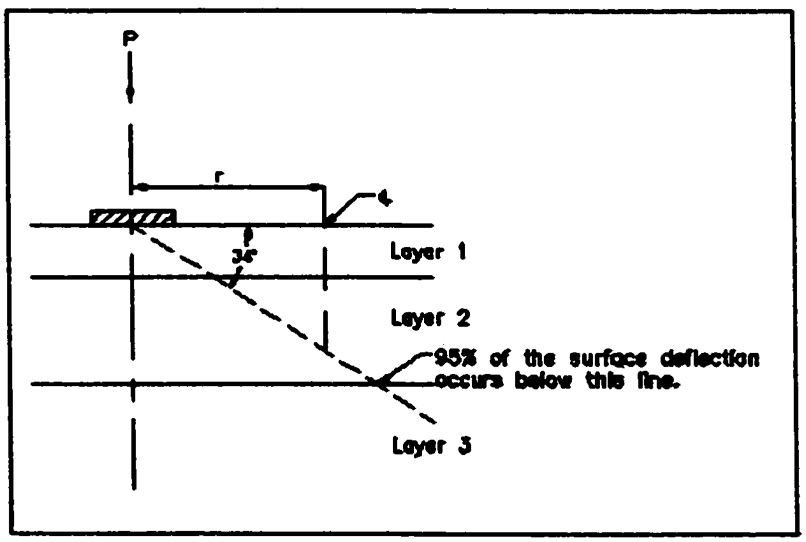


Figure 2-4: Pavement System Showing Line of 95% Deflection

NOTE: 1) If there is no deflection for a given layer (due to widespread sensors, or a thin layer) MODCOMP3 uses a curve fitting spline to interpolate a "measured" deflection to associate with the layer.

2) For this procedure, it is required that unknown layers should be no deeper than the radial distance from the center of the load to the outermost deflection sensor. Otherwise the layer would not intersect the 34° line within the range of the deflection sensors and could not be associated with a measured (or interpolated) deflection.

b) Alternatively, The user may specify the association.

iii) For the first iteration, the seed modulus for a given layer and its max or min value are used to generate an equation of the form $\log(E) = A \log(d) + B$ (refer to Figure 2-5). Linear interpolation between the log of the modulus and log of deflection is used (Lee, 1988). The seed value is input by the user and the minimum and maximum moduli may be input by the user, or the program will determine the values by multiplying 1/10 and 10 times the seed moduli.

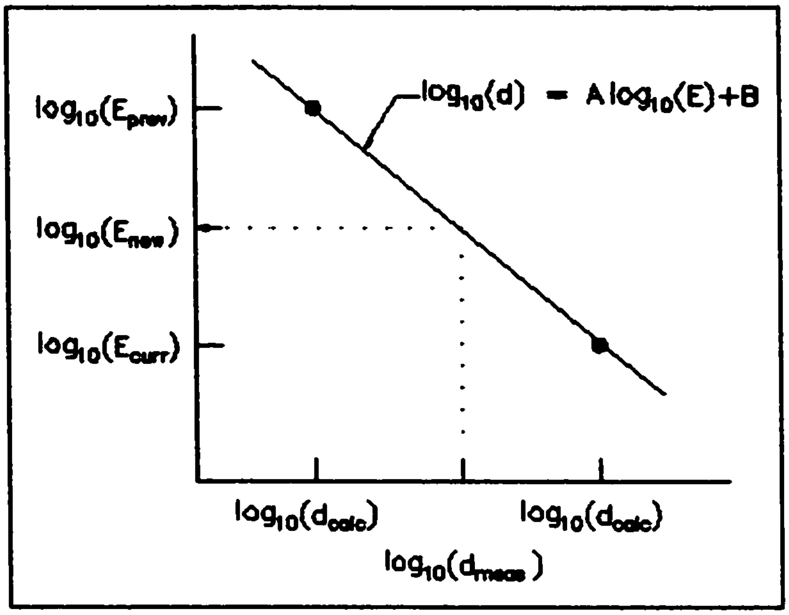


Figure 2-5: Interpolation of Modulus Using Calculated and Measured Deflections

NOTE: E_{prev} = modulus from previous iteration

E_{curr} = current modulus value

E_{new} = interpolated new modulus value for next iteration

d_{calc} = calculated deflection

d_{meas} = measured deflection

iv) The measured deflection is plugged into this equation to get an E value for the next iteration.

v) This new E is used in conjunction with the previous E to generate a new equation and a new E.

vi) This process is continued until convergence of RMS%(d_m,d_c) or max iterations is reached.

$$RMS\% = \left(\sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{d_{ci} - d_{mi}}{d_{mi}} \right)^2 } \right) (100)$$

Where

RMS% = root-mean-square percent error

d_{mi} = measured deflection at i^{th} location

d_{ci} = calculated deflection at i^{th} location

n = number of deflections measured.

vii) When deflection data for 3 or more load levels is available, regression is used to produce stress sensitivity coefficients (k_1 and k_2) for stress sensitive layers.

2.5.2 EVERCALC

This program was developed by the University of Washington for the Washington Department of Transportation (WSDOT) (Dynatest et. al., 1993). It is for use with flexible pavements with FWD. The program can handle up to 7 sensors and 8 different loads (EVERCALC user's guide, Feb. 1995). This program is based on linear elastic layer theory and can handle up to 5 layers. It should be noted here that the original EVERCALC program developed by Lee (1988) used the Chevron Elastic Layer program to calculate deflections. The program has since been updated for use by the Washington State Department of Transportation (WSDOT) and uses the program WESLEA, which is also based on elastic layer theory, to calculated deflections (EVERCALC User's Guide, Feb. 1995). The program can handle nonlinear material. Stress sensitivity coefficients can be determined when basins from at least two different load levels are given. The program adjusts AC moduli to WSDOT lab standard conditions (77°F, 100 millisec load duration). The program also contains the option of estimating the depth to a stiff layer using the scheme reported by Rhode and Scullion for the MODULUS program (EVERCALC User's Guide, Feb. 1995).

The following describes the basic procedure followed by the program:

i) The program estimates initial seed moduli using internal regressions which are algorithms developed using regression between pavement layer moduli, load, and various deflection basin parameters. Alternatively, the user may input seed moduli.

ii) The surface deflections are calculated with the seed moduli and the calculated deflections are compared with the measured deflections. The program uses moduli convergence [|(Ei(k+1)-Ei(k)|/Ei(k)]*100 and RMS% error between deflections to measure convergence. Also a maximum number of iterations is used to avoid getting into an endless loop.

iii) If the comparison shows deflection differences less than the allowable tolerance, the moduli used for the deflection calculation are the final solutions. Otherwise, the moduli are adjusted based on deflection difference. The modulus adjustment process utilizes the seed moduli, prior iteration moduli and constant correction factors. The deflection difference which provides the basis for the adjustment is determined as follows (Lee, 1988):

$$DD = \sum_{j=1}^{ND} (Dm_j - D_j) \tag{2-4}$$

Where: DD = deflection difference between measured and calculated deflections

D_{mj} = deflection measured at the j^{th} offset,

D_j = deflection calculated at the j^{th} offset, and

ND = number of deflection measurements.

If DD is positive, that is, the calculated deflections are smaller than the measured ones, the modulus adjustments are as follows:

$E_i^a = E_i^o \times 0.5$ for the first iteration and

$E_i^a = (E_i^o \times 3 + E_i^s/10)/4$ for the second and later iterations

If DD is negative, that is the calculated deflections are larger than the measured ones, the adjustments are as follows:

$E_i^a = E_i^o \times 1.5$ for the first iteration and

$E_i^a = (E_i^o \times 3 + E_i^s \times 10)/4$ for the second and later iterations

Where: E_i^a = adjusted modulus of the i^{th} layer,

E_i^o = original (or prior iteration) modulus of the i^{th} layer and

E_i^s = seed moduli of the ith layer.

This modulus adjustment process was developed based on a trial and error process. This optimization convergence technique requires at least as many deflection measurements as the number of pavement layers whose moduli are to be estimated.

iv) The surface deflections are calculated using the adjusted modulus for each layer and the sensitivity of the deflections to the modulus change is analyzed based on the relationship between deflection and a logarithm of the moduli values. The basic assumptions for the process are that the deflections depend on the unknown modulus and that the deflections have a linear relationship with the logarithm of the modulus (Lee, 1988). EVERCALC differs from MODCOMP in that instead of associating one deflection with one layer's modulus, each deflection is assumed to depend on the moduli of all layers. The equations used are of the form:

$$D_j = D_j^o + \sum_{i=1}^{N} S_{ji}(\log_{10} E_i - \log_{10} E_i^o)$$

Where

D_j = the deflection at radial offset j

D_j^o = calculated deflection for the original E_i values.

S_{ij} = the sensitivity of the deflection D_j (this is the slope in the equation of the form $D_j=A_{ij}+S_{ij}\log_{10}(E_i)$ which is determined for each Dj and Ei combination.)

E_i = latest modulus of layer i.

E_i^o = original modulus of layer i

N = total number of layers in the system.

This analysis is described in detail in Lee's PhD thesis, 1988.

v) The optimal set of the moduli is determined using a modified augmented Gauss-Newton algorithm, which is based on the optimization convergence method used in the program BISDEF (BISDEF is discussed in Section 2.5.4.2 of this paper). This optimization technique uses the sensitivity of the deflection basin to the change in layer moduli to determine a set of optimum modulii which minimizes the difference between the measured and calculated deflections (Lee, 1988).

Steps iii through v are repeated until the iteration reaches the maximum iteration specified in the input data or the error tolerance is met.

2.5.3 MODULUS

Several versions of this program have been developed (Uzan et al., 1988, 1989; Rhode and Scullion, 1990; Scullion et al, 1990; Rada et al., 1994). MODULUS 4.2 was written in conjunction with SHRP's long term pavement research efforts. Of the many programs evaluated, MODULUS was the program of choice for the LTPP research project (Rada et al., 1994). This program was also selected and adapted for use by the Texas Department of Highways and Public Transportation (Uzan et al., 1988).

The computer program MODULUS 4.2 can be used on a 2, 3, or 4 layer pavement system. The program first performs a series of layered-elastic program runs to construct a database of deflection basins based on user supplied moduli ranges. It then utilizes a pattern search procedure to match each observed deflection basin in the field data file to one of the calculated deflection basins in the predetermined data base. Interpolation is used if a measured deflection basin falls between two calculated basins (Dynatest et. al., 1993). The MODULUS program procedure is outlined below.

i) The program uses a linear elastic program to generate a database of deflection basins. Different versions of MODULUS use different elastic layer programs to calculate the deflection basins, some use BISAR, some WES5 for example.

ii) Modular ratios (E^*/ E_{sg}) are used to calculate deflections, where E^* is the modulus for a given layer.

iii) The modular ratios used to generate the database of deflection basins are determined from user input ranges of layer moduli.

v) For each set of modular ratios, a deflection basin is generated and a seed for E_{sg} is determined. A detailed explanation of how this is accomplished can be found in Scullion et. al., 1990.

vi) The squared error associated with each set of modular ratios $\left(\frac{E_1}{E_{sg}},\ldots\frac{E_k}{E_{sg}},\ldots,\frac{E_n}{E_{sg}}\right)$ is determined using the measured deflections and deflections calculated using each set of modular ratios.

vii) The value of E_{sg} associated with the set of ratios with the least squared error is used as a seed for Esg. Then seed values for the moduli of the remaining layers are calculated.

viii) These seed values are used as a starting point for a pattern search routine. The pattern search routine is based on the Hooke and Jeeves optimization algorithm and is used to minimize the error between the measured and calculated deflections (Scullion et. al., 1990). To avoid having to make multiple calls of the deflection calculation program, a 3-point Lagrange interpolation technique is used to determine the calculated deflections from the deflection basin database.

viii) The program performs a convexity test to see if the routine converged to a local or global minimum. If local, the program concludes that the limits of search were too narrow.

ix) The size of the error between the measured and calculated deflection basins is used to determine how much and in which direction to change the moduli values for the next iteration.

x) Once the minimum error is found, the program converts the associated moduli ratios into modulus values.

2.5.4 *DEF approach

These programs include CHEVDEF, BISDEF, ELSDEF, WESDEF, and BOUSDEF. These are a series of programs developed at the U.S. Army corps of Engineers Waterways Experiment Station (WES). In addition to the similarity of their names, these programs all use very similar iterative approaches to backcalculation. That is the moduli values of each layer are changed iteratively until a satisfactory match between the theoretical and measured deflections is obtained, a successive linear least squares approach is used (Sivaneswaran, et. al., 1991). The iterative process involves development of a set of equations which define the slope and intercept for each deflection and unknown modulus, the process assumes a linear relationship between the log of the deflection and the log of the unknown modulus as described below (Dynatest et. al., 1993). All of these programs are based on Elastic Layer Theory.

2.5.4.1 CHEVDEF

CHEVDEF was developed in 1980 by Bush as part of a project to develop nondestructive methodologies for light aircraft pavements conducted by the Waterways Experiment Station of the U.S. Army Corps of Engineers. According to Elastic Layer Theory, deflection is assumed to be a function of the layer moduli values (refer to section 2.2 of this paper).

Bush (1980) determined that the summation of the strains in the bottom layer, assuming the layer to be semi infinite, using the layered elastic model tends to give larger deflections than the measured values. To compensate for this, a rigid layer is placed at a depth of 20 ft. The program proceeds as follows: (Description based on Bush et. al., 1985; Bush, 1980; and Lee, 1988)

i) The program begins with a set of user defined seed moduli, a range of modulus values, and a set of measured deflections.

ii) The deflection basin (Δ_j^o for j = 1 to ND[number of deflections]) is calculated for the seed moduli via the Linear Elastic Layer program CHEVRON. These are considered the baseline moduli and deflection values for the first iteration.

iii) The squared error is calculated between measured and calculated deflections as follows:

$$\mathrm{ERROR}^2 = \sum_{j=1}^{ND}\left[MD_j - \Delta_j^o\right]^2 \qquad (2\text{-}5)$$

Where,

ND = number of measured deflections

MD_j = the value of the measured deflection at location j (j=1 to ND)

Δ_j^o = the deflection calculated using the current moduli (seed for first iteration)

iv) If the squared error is not within tolerance, a relationship of the form $\Delta_j = A_{ij} + S_{ij}\log_{10}E_i$ for each deflection (j = 1 to ND) and each unknown moduli (i = 1 to NL[number of layers]). This is precisely the method used in the EVERCALC program. Total deflection is written as a function of all the moduli values as $\Delta_j = \Delta_j^o + \sum_{i=1}^{NL} S_{ji} \log_{10} E_i$ where the summation represents the changes in Δ_j^o due to changes in the E_i's. (NOTE: the A's and S's are determined using the most current and previous values of E's and Δ's as demonstrated in Figure 2-6. If it is the first iteration, seed and max or min moduli are used.)

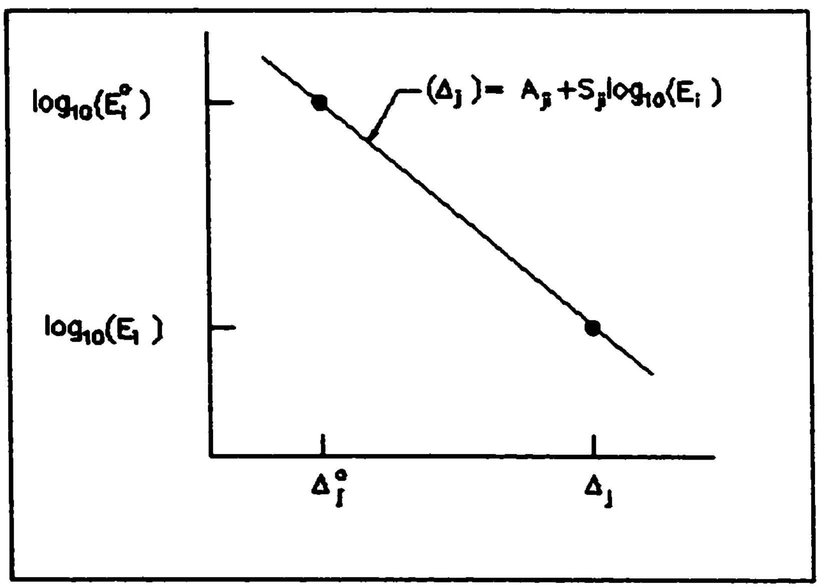


Figure 2-6: Determination of A_{ji} and S_{ji}

NOTE: MD_j = measured deflection at offset j

A and S values represent the slopes and intercepts

E_i^o = modulus value of the ith layer for the previous round

E_i = current modulus value for the ith layer

Δ_j^o = the deflection calculated using the previous moduli

Δ_j = the deflection calculated using the current moduli

By the nature of these equations, Δ_j^o can be written in terms of any of the unknown E's, (E_{NL} (bottom layer) in particularly) so $\Delta_j^o = A_{ij} + S_{ij}\log_{10}E_{NL}^o$ and hence the deflection equation is $\Delta_j = A_{jNL} + S_{jNL}\log_{10}E_{NL}^o + \sum_{i-1}^{NL} S_{ji}\log_{10}E_i$.

v) The equation for the squared error is now developed as

$$\text{ERROR}_j^2 = \left\{ MD_j - \left[A_{jNL} + S_{jNL}\log_{10}E_i^o + \sum_{i=1}^{NL} S_{ji}(\log_{10}E_i - \log_{10}E_i^o \right]\right\}^2$$

So the total squared error then is

$$\sum_{j=1}^{ND} \text{ERROR}_j^2 = \sum_{j=1}^{ND} \left\{ MD_j - \left[A_{jNL} + S_{jNL}\log_{10}E_i^o + \sum_{i=1}^{NL} S_{ji}(\log_{10}E_i - \log_{10}E_i^o \right]\right\}^2$$

where

MD_j = measured deflection at offset j

A and S values represent the slopes and intercepts determined in step iv above

E_i^o = modulus value of the ith layer for the previous round

E_i = current modulus value for the ith layer

A weight factor (W_j) may be added in if desired. The effect is to cause the small outer deflections away from the load to contribute to the total error as much as those near the load. The error is checked, if within tolerance the program is done.

vi) If the error is not within tolerance, new E's are determined as follows: By taking the partial derivatives, of the above error function, with respect to each unknown E_{ki}, a set of equations is obtained of the form:

$$0 = \sum_{j=1}^{ND} S_{jk} W_j \left\{ MD_j - \left[A_{jNL} + S_{jNL} \log_{10} E_i^o + \sum_{i=1}^{NL} S_{ji} (\log_{10} E_i - \log_{10} E_i^o \right] \right\}^2$$

This set of equations is converted into a matrix format [B]{E} = {C} that can be solved simultaneously giving an adjusted set of moduli for the minimum error between the measured and computed deflection basins.

vii) The error for this new set is checked, if not within tolerance the program returns to step vi and cycles through steps vi to vii until error is within tolerance or the maximum number of iterations is reached.

2.5.4.2 BISDEF/ELSDEF

BISDEF was developed for the Texas Transportation Institute by modifying the CHEVDEF program developed by the Corpse of Engineers, Waterways Experiment Station (WES) (Uzan et. al., 1988). The most significant modification was the replacement of the CHEVRON layered elastic program with the BISAR layered elastic program. The advantage of BISAR is that it can handle multiple loads and variable interface conditions (CHEVRON handles neither condition) (Bush and Alexander, 1985). ELSDEF is identical to BISDEF, with the exception that ELSYM5 is used for the forward calculations rather than BISAR (Kim, 1993). The procedure for both programs is outlined below:

i) Inputs to the program include an allowable range of moduli.

ii) The deflection measured at an offset of 183 cm (72 in) is used to estimate the initial subgrade modulus as

$$E_{sg} = 59304.82(D72)^{-0.98737}$$

Where D72 is the deflection in mils measured at a distance of 183 cm (72 in) from the applied load. This equation is for an applied load of 111 206 N (25,000 lb). A range for the subgrade modulus is then established as the predicted value plus or minus 34.5 Mpa (5000 psi) (Alexander, et. al., 1989).

iii) A linear relationship between the log of the modulus versus the deflection for each unknown modulus and each deflection as described in the procedure for CHEVDEF is determined, initially by using the input range values and later by using the latest modulus value.

iv) This relationship is then used iteratively to find moduli that minimize errors by plugging in the measured deflection and computing an adjusted modulus value, recalculating deflection and developing another relationship between log of deflection and log of modulus until error between calculated and measured deflections is sufficiently small or maximum number of iterations is reached.

v) Small deflections away from the load are weighted so that they contribute equally to those near the load in the solution process.

vi) The number of layers having an unknown modulus cannot exceed the number of measured deflections.

vii) A rigid layer of infinite thickness having a modulus of elasticity of 7000 MPa (1,000,000 psi) and a Poisson's ratio of 0.5 is assumed at a depth of 20 ft below the subgrade layer. This may be changed by the user if a stiff layer is known to be at a different depth (Alexander, et.al., 1989).

2.5.4.3 WESDEF

WESDEF is almost identical to BISDEF except for replacing BISAR with WESLEA (Kim, 1993). CHEVRON (used by CHEVDEF) does not handle variable interface conditions, BISAR (used by BISDEF) assumes a linear change between full and no friction whereas WESLEA uses a model which assumes that the interface friction satisfies Coulomb's law (Van Cauwelaert et. al., 1989). WESLEA also differs from CHEVRON and BISAR in that it uses a more advanced technique to approximate the necessary integration for calculating the deflections (Van Cauwelaert et. al., 1989). No procedural description is offered here, the reader is referred to the BISDEF procedure.

2.5.4.4 BOUSDEF

BOUSDEF, developed at Oregon State University, is based on the method of equivalent thickness and Boussinesq theory (Zhou et. al., 1989). The reader is referred to Section 2.3 of this paper for a description of the Method of Equivalent Thickness (MET). BOUSDEF is an iterative procedure similar to BISDEF, with the exception that the deflections are calculated by an equivalent thickness subroutine rather than an elastic layer subroutine (Hall and Mohseni, 1991).

BOUSDEF was developed for flexible pavements and assumes that the pavements consist of a bituminous surface/base, and a coarse grained aggregate base/subbase on the top of a fine grained subgrade (BOUSDEF User's Guide). There are some limitations involved in applying the method of equivalent thickness, according to Zhou et. al. (1990), the pavement layer moduli should decrease with depth, preferably by a factor of at least two between consecutive layers. Further, the equivalent thickness of a layer should be larger than the radius of the loaded area. BOUSDEF considers the base and subgrade to be nonlinear stress dependent layers. Therefore, deflections for more than one load level are required.

The following is a description of the process that the program follows:

i) The seed moduli and layer thicknesses are used to calculate baseline deflections by first calculating the equivalent thickness and then calculating deflections using Boussinesq theory.

ii) The sum of the error from percent difference between measured and calculated deflections is calculated and checked if it is within tolerance.

iii) If not within tolerance, an alternate E is computed for each layer. This is done by the same method described for CHEVDEF in section 2.5.4.1, using an equation of the form $d_j = A_{ij} + S_{ij} Log(E_i)$

iv) This process repeats until the sum of the differences between measured and calculated deflections is less than the tolerance or the maximum number of iterations has been reached.

v) The entire procedure is repeated for the data from each load level input by the user.

vi) The moduli determined from each set of data (for the different load levels) are used to calculate normal stresses induced by the load. These stress values and moduli are then regressed to find coefficients k_1 and k_2 for both base and subgrade layers.

2.5.5 ELMOD

ELMOD was developed by Ullidtz. ELMOD is an acronym for Evaluation of Layer Moduli and Overlay Design. Like BOUSDEF, the program is based on the method of equivalent thicknesses (Dynatest et. al., 1993). However, this program first makes use of the outer deflections, assuming that they are almost completely controlled by the subgrade modulus (Ullidtz et. al., 1987). The Program is supplied by Dynatest as part of the package offered to customers who purchase a Dynatest FWD (Kim and Nokes, 1993). The following description is based on Ullidtz et. al., 1987, and "ELMOD4 Training Manual".

i) First, the change in deflection with distance from the center of the load is used to determine if there is a stiff layer below.

a) If so, depth to stiff layer is calculated the stiff layer is assumed to be infinitely stiff.

b) If not, outer deflections are used to estimate C and n for subgrade moduli equation: $Em = C(\sigma_1/\sigma')^n$

Where σ_1 is the major principal dynamic stress,

σ' is the reference stress and

C and n are constants, n is negative.

ii) Moduli of surface and base (if present) are determined through an iterative process. ELMOD offers two options for this.

a) The first is used for systems with 3 or fewer layers (this includes the subgrade as a layer). This option is called the "curve fitting" method and modifies the f values (refer to equation 2-2 in Section 2.3 of this Chapter) to obtain a best approximation of a "WES5" deflection basin. WES5 is a linear elastic deflection calculation program.

b) The second option is called the "radius of curvature" method. This method does not attempt to "fit" deflection points within the basin, but uses the radius of curvature along with the non –linear subgrade properties (C and n) to determine moduli of the system layers.

iii) The subgrade modulus at the centerline is adjusted according to the stress level. The outer deflections are then checked and a new iteration carried out, if necessary.

2.5.6 ILLIBACK

ILLIBACK is a program developed by A.M. Ioannides, 1988. The ILLIBACK procedure was developed for a one-layer system consisting of a rigid pavement slab resting on an elastic solid (ES) or a dense liquid (DL) foundation (Ioannides, 1988). The program is based on closed-form solution of plate theory equations (Hall and Mohseni, 1991). The concept of dimensional analysis described in section 2.4 of this paper is utilized. Dimensional analysis in combination with Westergaard's equations are used to backcalculate a modulus for the slab and either E_s (for elastic solid foundation) or k (for dense liquid foundation) as described below.

i) The parameter AREA (see equation 2-3) is calculated using the measured deflections from the field.

ii) Relationships between AREA and radius of relative stiffness, l, are used to obtain l. These relationships are discussed in Section 2.4 of this paper.

iii) The value of l is in turn used to calculate the deflections using Westergaard's equation.

iv) Equations for dimensionless deflections, d_i, as a function of a/l only (where a is the radius of the circular load) were derived and used to calculate d_i. (Note that this need only be done for one chosen value of i, not for all deflections). It should be noted here that these equations are very complicated involving several Kelvin/Bessel functions so as with Elastic Layer Theory the problem of approximating several infinite integrals is involved.

v) The corresponding measured deflection D_i and load P are plugged into the equation $d_i = \frac{D_i D}{P l^2}$ and D (slab flexural stiffness) is backcalculated.

vi) From D, the surface modulus E is determined using the equation $D = \frac{Eh^3}{12(1-\mu^2)}$.

vii) Once E for the surface layer is determined, the equation for l can be used to determine k (if a dense liquid foundation was assumed) or E_S (if an elastic solid foundation was assumed).

2.6 Pluripotence

All of the theoretical methods described in Section 2.2 through 2.4 have pluripotence problems of some form. Linear Elastic Layer Theory has the problem of approximating two infinite integrals. The Method of Equivalent Thickness equations have been shown pluripotent, that is there are infinitely many combinations of moduli values which will give H_{eq}. Dimensional Analysis is non-pluripotent to the extent that once the stiffness (D) is determined, there is only one E which can satisfy $D = (Eh^3/(12(1-\mu^2)))$. However, the AREA parameter used to generate D is pluripotent as discussed in Section 2.4.2.

In addition, all of the specific programs, discussed in Section 2.5, which use these theories introduce more equations which further complicate the issue. Each program has it's own method of "narrowing it down" to one solution. These methods are the topic of this section. The programs discussed above will be grouped according to how they obtain a single solution and discussed.

2.6.1 MODCOMP3

The key to obtaining a single solution for the MODCOMP program is by associating each deflection with a single unknown modulus. The association of a single deflection to each unknown layer results in a set of n equations and n unknowns. These equations are linearly independent and hence they have a unique solution. The only drawback is that the assignment of deflections to layers assumes that the specified deflection depends solely on that layer. While it has been demonstrated that the further out a deflection is, the less it depends on the upper layers, it is unlikely that any deflection depends entirely on a single layer. The method is also dependent on where you start your search (seed values) and what limiting values are used (max and min moduli). If the seed and max and min values are changed, you will not necessarily end up with the same final solution because the search will progress differently, different slopes and intercepts will be obtained.

2.6.2 EVERCALC and the *DEF Programs

These programs are lumped together because they all have similar approaches to obtaining a single solution. By taking the partial derivatives of the error equation (described in Section 2.5) with respect to each of the unknown moduli, a system of n linearly independent equations with n unknowns is developed. Which results in a unique solution for the layer moduli.

2.6.3 MODULUS

The MODULUS program is able to obtain a single solution because of the Hooke and Jeeves pattern search routine that is used to match up the measured deflection basins with the basins in the data base of calculated basins. This pattern search routine is guaranteed (mathematically speaking) to converge to a solution. So you will always get "a" solution, even if it is not "the" solution. Further, the moduli ratios assumed also direct the final results to a single solution. Once the ratios are chosen and a subgrade modulus is determined, the rest of the moduli values are set.

2.6.4 ELMOD

There were not enough details available about how exactly the ELMOD program works to determine how a single solution is obtained. It is expected, however, that the key lies in the "curve fitting" or "radius of curvature" methods that are used.

2.6.5 ILLIBACK

As pointed out previously the relationship generated between AREA and radius of relative stiffness, l, is one-to-one. Therefore any given AREA will result in a single l value. However, as pointed out in the program description the equation used to determine the dimensionless deflection d_j is very complicated involving several Kelvin/Bessel functions. So as with Elastic layer theory there is the problem of having to estimate several infinite sums. However, once d_j is approximated, a unique value of stiffnes (D), and hence modulus (E), is obtained. However, the E is only unique because we are dealing with a slab-on-grade (one-layer) system. If more layers were involved, D would be a function of the E's of all the layers above the subgrade and would be pluripotent as is the case for Composite Plate Theory.

2.7 Research on Uniqueness

Appendix 2A gives an extensive, although not exhaustive, listing of papers relating to the subject of backcalculation. In viewing this appendix it can be seen that there are many important issues that have been researched regarding backcalculation. However, it can also be seen that there are very few papers that discuss the issue of pluripotence (or uniqueness of solutions).

In fact, only one paper has been found which directly addresses this issue. This was a paper by Stolle and Hein (1989). This paper discusses the inaccuracies introduced by certain assumptions such as layer thickness, seed moduli, and the sensitivity of the backcalculated results to these assumed values. The paper also addresses the problems induced by the various convergence criteria used in backcalculation programs. This is a ground-breaking paper in that it directly discusses the nonuniqueness issue, however the uniqueness problems pointed out in this paper are due to the sensitivity of the equations to their inputs. The mathematical pluripotence of the equations is not addressed.

In a paper by the ASTM Subcommittee working toward a standard guideline for backcalculation (May and Von Quintus, 1994) several factors effecting backcalculation were discussed. It was stated that uniqueness of solution is an underlying assumption but that in reality variations in input parameters can cause several combinations of moduli to "match" the deflection basin reasonable well. Again mathematical uniqueness was not addressed, in fact it was assumed.

In a paper by Lee, Mahoney, and Jackson, (1988) nonuniqueness of solutions was defined simply to mean that if different initial seed moduli are used, then different backcalculated moduli result in the final solution. This has been a misconception for many researchers in backcalculation. Hopefully this paper will make it clear that there is more to the nonuniqueness issue than choice of seed moduli.

The following is a quote from an excellent paper by Lytton (1989). "It is advisable not to assert that the set of moduli derived from any search is the only set of moduli possible without having mathematical rather than empirical proof of the point."

The limited amount of work that has been done on this subject is pretty incredible when you consider that the pluripotence or non-pluripotence of the equation used in backcalculation is at the very heart of the matter when it comes to deciding if you can really depend on your final results. All of the other issues are very important, it can be seen that a lot of time and money have gone into researching these issues while very little has been spent on investigating pluripotence. It is hoped that this paper will awaken the world of backcalculation to the importance of pluripotence and that more in-depth mathematical research into the issue will be inspired.

Chapter 3
Composite Plate Theory Approach to Backcalculation

3.1 Introduction

Composite plate theory has been investigated as a potentially new backcalculation model. This involves modeling the pavement as a composite plate (a single plate made up of several materials) resting on an elastic (Winkler) foundation. The equations used in this theory have been mathematically proven to be quasi-pluripotent (Mings, 1993). Quasi-pluripotent means that the solutions are unique up to a point (some of the backcalculated parameters are unique, but some are not). Methodologies to overcome this dilemma have been incorporated into a new backcalculation program called KISS. This program utilizes a multi-directional search algorithm developed by Torczon (1991). As an aside, the name for this program was chosen as a reminder that Keeping It Simple is a very important factor in obtaining a successful backcalculation methodology. As will be seen, this is often much easier said than done.

3.2 Theory and Equations

The theory and equations behind the composite plate model have been completely developed by Mings (1993). The necessary assumptions and equations will be presented here without derivation. The following is a list of the basic assumptions:

(1) The composite plate is assumed to be simply supported over an elastic (Winkler) foundation.

(2) The neutral plane of the plate remains unstrained during bending. (Note: In a single material plate, the neutral plane is the middle plane; this is not the case for composite plates.)

(3) Normals to the neutral surface before deformation remain normal to the same surface after deformation. (This implies that displacement caused by transverse shear strain is negligible.)

(4) Normal stresses transverse to the plate are negligible.

(5) Each material in the composite is homogeneous, isotropic, continuous, and linearly elastic. (This permits the use of the stress-strain relationships in terms of two elastic constants, these being E and μ.)

(6) Poisson's ratio is assumed to be equal for all materials in the composite. Without this assumption, equilibrium cannot be satisfied. This is not an unrealistic assumption because changes in Poisson's ratio do not significantly affect calculations of deflection.

The equations for stresses, strains, and displacements for the theory of a composite plate on an elastic (Winkler) foundation are listed below.

The equation for deflection in inches is given as:

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\frac{A_{mn}}{\pi^4\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2\overline{D}+k}\right\}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \tag{3-1}$$

where

A_{mn} = the load distribution expressed as a double Fourier sine series. That is,

$$= \frac{4}{ab}\int_0^a\int_0^b P(x,y)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)dydx$$

P(x,y) is the known load distribution [lb]

m,n = the dummy variables over which summation is carried out [unitless]

a,b = the plate dimensions. [in.]

$\overline{D} = \frac{1}{3(1-\mu^2)}\sum_{i=1}^{\bar{n}} E_i\left(\zeta^3\right)_{h_i}$ represents the composite plate stiffness. [lb-in.] (3-2)

E_i = the modulus of the i^{th} layer. [lb/in^2]

$\bar{n}$ = number of materials in the composite plate [unitless]

ζ = distance to the neutral plane from the middle of the i^{th} layer [in.]

μ = Poisson's ratio [unitless]

k = modulus of subgrade reaction [lb/in^3]

For the purposes of this backcalculation program, a rectangular load of dimensions c and d of an equally distributed pressure P_o centered at (ξ_o,η_o) on a rectangular plate of dimensions a and b is assumed. Performing the double integration gives the following:

$$A_{mn} = \frac{16P_o}{mn\pi^2}\sin\left(\frac{m\pi\xi_o}{a}\right)\sin\left(\frac{m\pi c}{2a}\right)\sin\left(\frac{n\pi\eta_o}{b}\right)\sin\left(\frac{n\pi d}{2b}\right)$$

Stress and strain equations can be derived from the deflection equation as follows:

Strains:

$$\varepsilon_x = -\zeta \frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = -\zeta \frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = -2\zeta \frac{\partial^2 w}{\partial x \partial y}$$

where,

w = deflection

ζ = the distance of the point of interest from the neutral plane of the plate.

Stresses:

$$\sigma_x = -\zeta \frac{E_i}{(1-\mu^2)} \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)$$

$$\sigma_y = -\zeta \frac{E_i}{(1-\mu^2)} \left(\mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$\tau_{xy} = -\zeta \frac{E_i}{(1+\mu)} \left(\frac{\partial^2 w}{\partial x \partial y} \right)$$

Taking the required derivatives gives the following:

$$\frac{\partial^2 w}{\partial x^2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} -\left(\frac{m\pi}{a}\right)^2 \left\{ \frac{A_{mn}}{\pi^4\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 \overline{D} + k} \right\} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$\frac{\partial^2 w}{\partial y^2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} -\left(\frac{n\pi}{b}\right)^2 \left\{ \frac{A_{mn}}{\pi^4\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 \overline{D} + k} \right\} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$\frac{\partial^2 w}{\partial x \partial y} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) \left\{ \frac{A_{mn}}{\pi^4\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 \overline{D} + k} \right\} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

3.3 Quasi-Pluripotence

It has previously been shown that the $\overline{D}$ and k parameters are unique for the above deflection equation (Mings, 1993). A backcalculation technique, which uses a multidirectional search algorithm to search for these parameters, has been developed. The program appears to work very well for determining $\overline{D}$ and k.

Unfortunately the above equation for $\overline{D}$ has been shown to be pluripotent (Mings, 1993). Thus there are infinitely many combinations of moduli values (E_s,E_b), which will generate the same $\bar{D}$ value. However, for the pavement system consisting of a single slab over a foundation, these equations reduce to regular plate theory equations where the plate stiffness is a function of the slab modulus only ($D = \frac{Eh^3}{12(1-\mu^2)}$). So for this case the E (modulus of the slab) is unique.

To get around the pluripotence problem when a composite plate of two materials (surface and base) is involved, empirical relationships between surface (E_s) and base (E_b) moduli have been developed. Before discussion of the development of these relationships, the concept of 'E-no base' must first be introduced. E-no base (E_{nb}) simply means I assume there is no base layer (so I am down to one layer) and calculate the E_{nb} that would generate the $\bar{D}$ value associated with a given pavement system (recall $\bar{D}$ is a unique parameter). This is similar to the method of equivalent thickness described in Chapter 2 except that instead of an equivalent thickness, an equivalent modulus is being determined for the thickness of the surface layer (h_1). E_{nb} is calculated using the following equation:

$$E_{nb} = \frac{12\bar{D}(1-\mu^2)}{h_1^3} \tag{3-3}$$

Now, to answer the question "why would anybody do that?" it will be demonstrated that the concept of E-no base provided a unique parameter which is easily calculated from $\bar{D}$. This value was then used to establish the relationships between E_s and E_b which allows for backcalculation of these parameters.

To develop these relationships, E_{nb} was calculated for known values of E_s and E_b for specified layer thicknesses. Plots of E_b vs E_{nb}/E_s were then created and the best fitting curve was generated using the EXCELL computer software. Plots were generated for different combinations of layer thicknesses. To get good correlations it was necessary to categorize the asphalt as having high (>1500000 psi), average (700000 psi < E_s < 1500000 psi), or low (<700000 psi) moduli.

There were a total of 11,000 different combinations of surface moduli, base moduli, surface thickness and base thickness cases involved in generating these equations. The ranges of values used for each of these parameters is given in Table 3-1.

Table 3-1: Ranges of Values

Parameter	minimum	maximum
surface thickness (h_1) [in]	2	10
base thickness (h_2) [in]	6	42
surface modulus (E_1) [psi]	500,000	2,000,000
base modulus (E_2) [psi]	100,000	2,000,000

The base moduli used were in the range of reasonable values for a stabilized base. To develop equations suitable for other types of bases, granular base for example, a similar procedure would have to be followed using values reasonable for a granular base. The Figure 3-1 shows the relationship generated for the case of a low moduli AC concrete for a 2-inch thick surface with a 10-inch base.

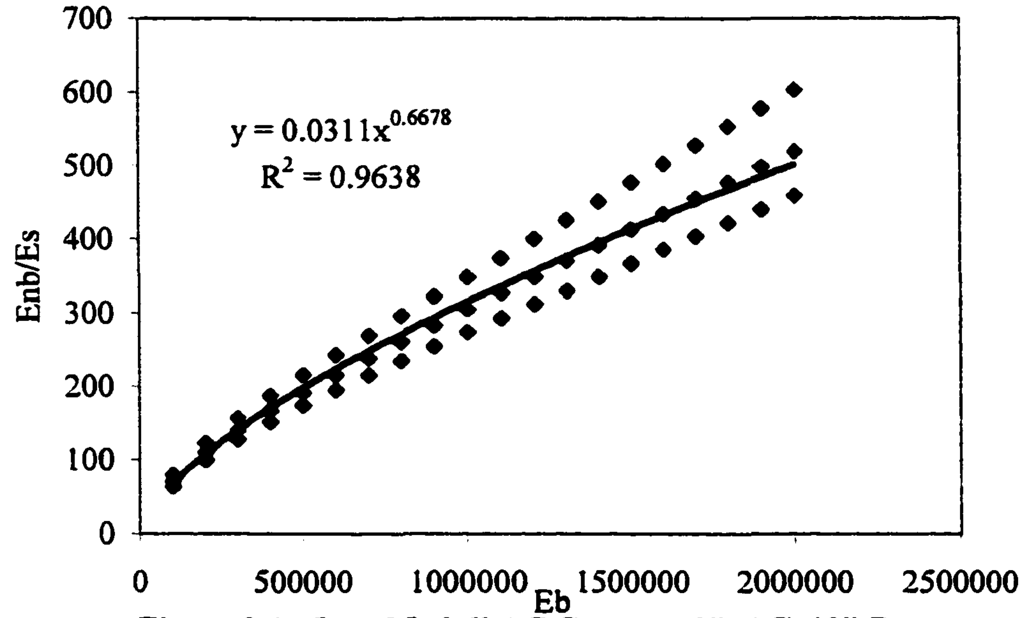


Figure 3-1: Low Moduli AC Concrete 2" AC, 10" Base

All of these relationships have the form $E_{nb}/E_s = C(E_b)^M$ where C and M are constant coefficients. Several forms of relationships were tried (linear, logarithmic, quadratic…) but the power curve was found to have the best fit. Relationships were developed for high, average and low moduli asphalts for many combinations of layer thicknesses. Appendix 3A lists the coefficients C and M and their respective R^2 values as determined for the various layer thicknesses.

These relationships were incorporated into the backcalculation program as follows. Once the program has determined the $\bar{D}$ and k values, the $\bar{D}$ value is used to calculate E_{nb}. A search routine similar to that used to determine $\bar{D}$ and k is used to search for E_b with E_s determined using the equation developed for the applicable thickness values. For example, if $h_s = 2$ and $h_b = 10$, then the E_b found from the search routine would be plugged into the equation shown if Figure 3-1 to determine E_s using the value of E_{nb} which is easily calculated from the backcalculated value of $\overline{D}$.

The search routine searches for the E_b value which will optimize the square of the difference between the backcalculated $\overline{D}$ and the $\overline{D}$ calculated using E_b. This search routine combined with the applicable equation based on thickness values enables us to determine a single (E_s,E_b) solution.

If the thickness values do not fall exactly within one of the combinations for which equations were developed, then a C and M are interpolated using linear interpolation between the two equations developed for the thickness values just above and below the actual thickness values. Recall that C and M are the constant coefficients in the equations which are all of the form $E_{nb}/E_s = C^*(E_b)^M$.

Care should be taken here to note that while I have narrowed it down to one pair, it still may not be the correct solution. This solution is only as good as the model on which it is based. If the model is sound and the equations valid, the result should be a realistic set of moduli which can be used by the Engineer.

3.4 The Program

This section presents the step-by-step process involved in the KISS program. Figure 3-2 presents a basic flow diagram for the program. The entire program, including all subroutines, is presented in Appendix 3B.

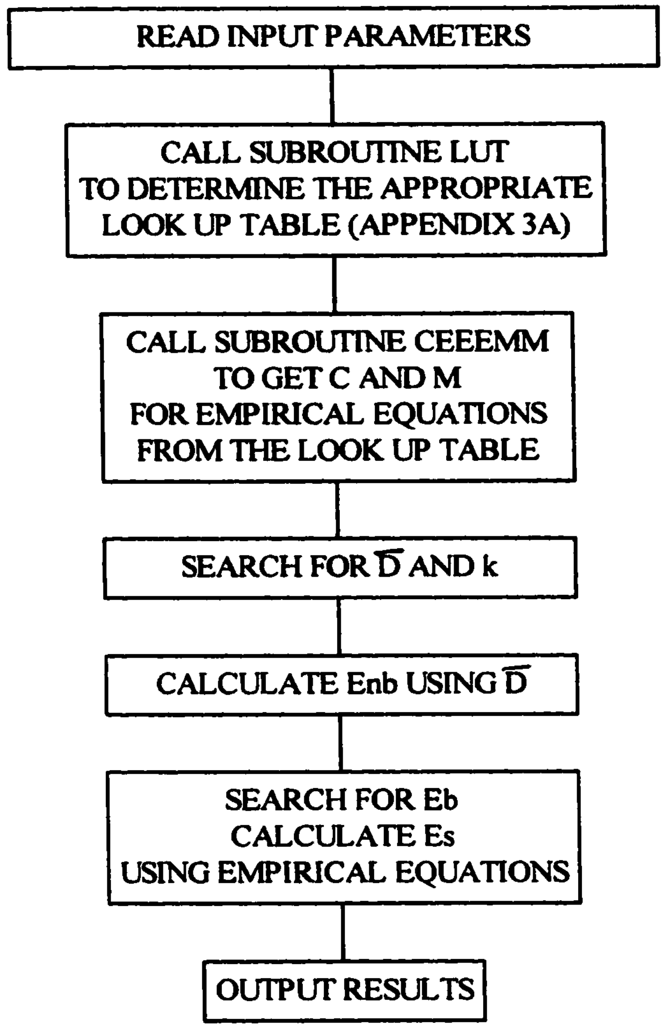


FIGURE 3-2: Flow Chart for KISS

3.4.1 Read Input Parameters

Currently the program is set up to read input values in English units. The input file is a text file which can be created with any text editor. The input file format is shown in Figure 3-3.

```
n, m
MU ,P, A, F, C, G, XO, YO, PERR
h(1)
h(2)
 .
 .
 .
h(n)
x(1), delta(1)
x(2), delta(2)
 .
 .
 .
x(m), delta(m)
```

Figure 3-3: Input File Format

Where the above parameters have the following meaning:

n = The number of layers in the system (this does not include the subgrade).

m = The number of deflections measured (typically this is 7).

MU = Poisson's ratio (Assumed equal for all layers).

P = Load intensity [psi]

A, F = plate dimensions [inches]

C, G = load dimensions [inches] (1)

XO,YO = location of center of load in global coordinates.(2)

PERR = percent error acceptable for comparing calculated and measured deflections. (This parameter may later become a default value of 0.1, so the user would not be required to input it)

h(i) = thickness of i^{th} layer.

x(j) = location of j^{th} deflection in local coordinates. (3)

delta(j) = magnitude of j^{th} deflection [inches].

NOTES:

(1) Currently the load is input as a square (rectangular) load so two parameters (length and width) are needed. Since most FWD's use a circular load, the plan is to have the user input the diameter (or radius) of the actual load and have the computer convert this to an equivalent square load by determining the size needed to give the equivalent load P.

(2) Global coordinates are considered as those having the origin in the lower left corner of the plate.

(3) Local coordinates are considered as those having the origin at the center of the applied load.

```
2 7
0.2E+00 0.11E+03 0.288E+03 0.288E+03 0.9E+01 0.9E+01 0.144E+03 144E+03 0.1E+00
0.2E+01
0.6E+01
0.00E+00  0.1071E-01
0.60E+01  0.1058E-01
0.18E+02  0.9629E-02
0.30E+02  0.7950E-02
0.42E+02  0.5919E-02
0.54E+02  0.3944E-02
0.72E+02  0.1742E-02
```

Figure 3-4: Sample Input File For A 2 inch AC Layer Over A 6 inch Base

3.4.2 Call Subroutine LUT

This subroutine initially asks the user to input the case of asphalt defined as 1 = high (E > 1,500,000 psi), 2 = average (700,000 psi < E < 1,500,000 psi), or 3 = low (E < 700,000 psi). This information may be added as part of the input file at a later time to make the program more efficient to use. The Subroutine LUT then opens the appropriate look-up table. In the event that the user does not input a 1, 2, or 3, the program gives an error message and presents the user with another opportunity to input the case.

3.4.3 Call Subroutine CEEEMM

Once the appropriate look-up table is opened, the subroutine CEEEMM determines the correct record, within the look-up table, from which to obtain the values of C and M for the empirical equation of the form $E_b = C^*(E_{nb}/E_s)^M$. The subroutine takes the values of h(1) and h(2) (thicknesses of surface and base input by the user) and determines if these values are "on the list". A value for h(1) is "on the list" if it is 2, 4, 6, 8, or 10. A value for h(2) is "on the list" if it is 6, 10, 14, 18, 22, 26, 30, 34, 38, or 42. It was determined by observing plots for C and M vs thickness that linear interpolation is appropriate for determining C and M values for intermediate thicknesses. Therefore, if h(1) or h(2) or neither are on the list, then linear interpolation is used to obtain values for C and M. The following subroutines are incorporated to accomplish this task: RECORD, LOHI, INTERP2A, INTERP2B, and INTERP4. A copy of the entire program in FORTRAN 77, including all subroutines, can be found in Appendix B.

3.4.4 Search for $\overline{D}$ and k

The main feature of the KISS program is the pattern search routine that is used first to find $\overline{D}$ and k, and then later used to find E_b. The pattern search routine used is an optimization technique which is based on a routine developed by Torczon (1989). Torczon's original routine was developed for use with parallel machines for faster results.

Here it cannot be applied for that use since most pavement design engineers do not have access to parallel machines. However, it was chosen because it has been proven to converge to a solution regardless of starting location (Torczon, 1991), and because it was easily adapted for this application. Many search routines are not guaranteed to converge to a solution and often get "stuck" and never reach a solution. The routine used is a multidirectional search algorithm that can be used to search for any number (n) of parameters. This algorithm belongs to the class of direct search methods, a class of optimization algorithms which does not compute nor approximate any derivatives of the objective function. According to Torczon, the method was inspired by the simplex methods of Spendley, Hext, and Himsworth and that of Nelder and Mead (1989). The step-by-step process is briefly outlined below. This explanation has been greatly simplified for clarity, for a more precise explanation see Torczon (1991) or Dennis and Torczon (1991).

1) State the equation to be optimized (minimized in this case). For example, in the search for $\overline{D}$ and k, the following equation is minimized.

$$Q = \sum_{j=1}^{m} (w(j) - delta(j))^2 \tag{3-4}$$

In words, this represents the sum of the squares of the differences between the measured (delta) and the calculated (w) deflections. The calculated deflections are obtained via equation 3-1. The measured deflections are those input by the user. Since the only unkowns here are $\overline{D}$ and k, Q is a function of these two parameters only.

2) Choose values with which to begin the search. The search requires simplex of n+1 sets of values. For our purposes, each set of values contains a $\bar{D}$ and k. The program contains values for starting the search so the user need not supply seed values. It has been determined, through trial and error, that refining the initial $\bar{D}$ and k values based on the results of the previous search is beneficial is obtaining a good solution. For this reason, the computer program runs through a series of several searches where the initial values are updated each time based on the results of the previous search. This will be explained in detail later.

3) Since I am searching for only 2 parameters ($\bar{D}$ and k) I only need a simplex of 3 sets of values (v_0, v_1, v_2) to begin the search. From these initial sets, the "best" set is determined by calculating Q for each set (refer to equation 3-4). The set resulting in the smallest Q is the best, This set is labeled as v_0.

4) Figures 3-5 to 3-7 (after Dennis and Torczon, 1991) demonstrate how a series of reflections, expansions, and contractions is used to arrive at a new set of values.

The first step is to reflect v_1 and v_2 through the best vertex v_0. The reflected sets of values are labeled r_1, and r_2. If either reflected set gives a smaller value of Q than v_0, then the reflection is considered successful and the algorithm tries an expansion.

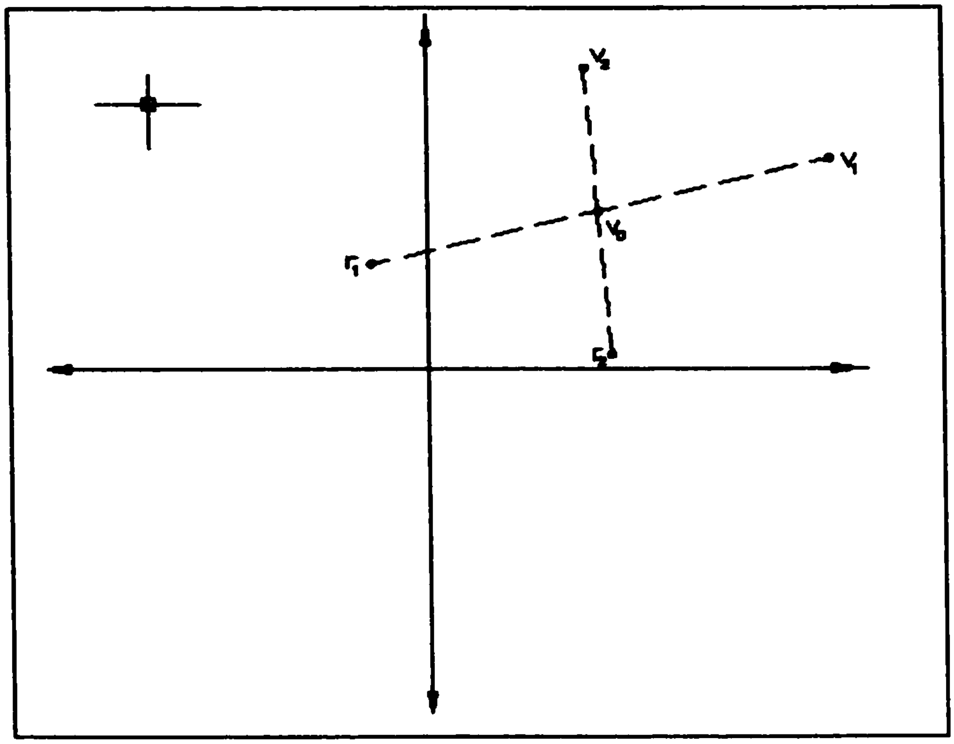


Figure 3-5: Reflection

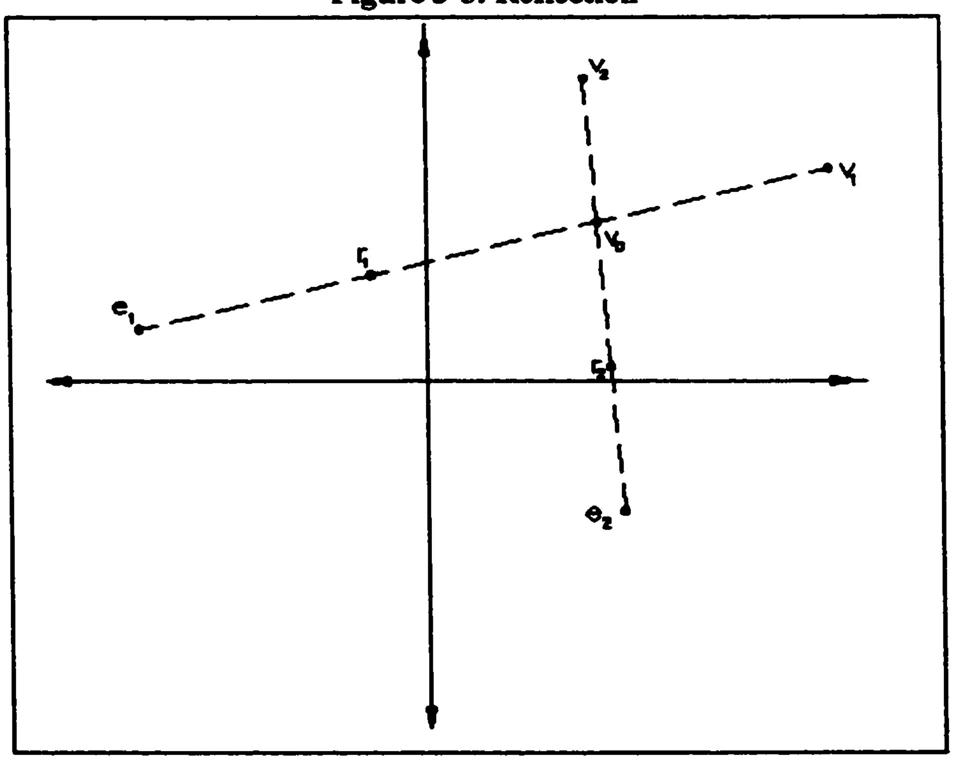


Figure 3-6: Expansion

The expansion step consists of expanding each reflected edge ($r_j - v_0$) to twice its length to give two new sets e_1 and e_2. The expansion step is tried only if the reflection step was successful. The expansion step is accepted only if one of the new sets e_1 or e_2 gives a smaller value for Q than either reflection set r_1 or r_2. So, if the expansion step is tried, the new Simplex will contain v_0 plus the values from the rotation or expansion which gave Q less than v_0. Here the method varies from Torczon's in that Torczon's method at this point either accepts or rejects both expansion or reflection values. It was found, through trial and error, to be beneficial to only accept the sets which give a smaller value of Q than v_0. So it is possible that the resulting simplex could contain a combination of reflection and expansion results such as (v_0, r_1, e_2).

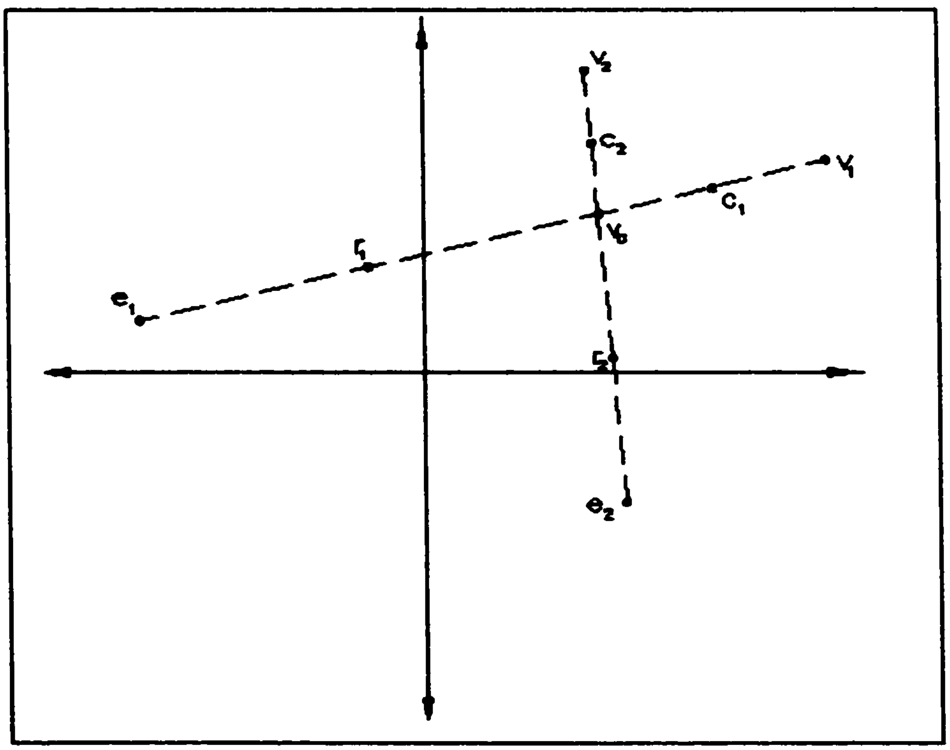


Figure 3-7: Contraction

If the reflection step was unsuccessful (neither r_1 nor r_2 gave a value of Q smaller than that from v_0) then a contraction step is performed.

The contraction sets of values c_1 and c_2 are obtained by choosing the set of values at the point half way between v_0 and the two original sets of values v_1 and v_2. If a contraction step is performed, it is automatically accepted regardless of the Q values obtained by c_1 and c_2.

Once either a rotation, expansion, or contraction is accepted, the new best set is determined to be the one which gives the smallest Q value and is relabeled as v_0 the remaining sets are relabeled as v_1 and v_2 (in no particular order).

From this point the reflection, expansion, contraction steps are continued until one of the stopping criteria is obtained. For the KISS program, the search is stopped if Q becomes smaller than 1E-10 or if the $\bar{D}$ values in the simplex get within 0.1 of each other.

5) Once convergence has been reached after the final search (recall the program runs through a series of searches). The final values of $\bar{D}$ and k are written to the output file.

The KISS program begins by running through a series of 7 searches using the following seeds for search 1:

$$\bar{D}\ (1) = 1{,}000{,}000 \qquad k(1) = 100$$

$\bar{D}$ (2) = 5,000,000   k(2) = 1000

$\bar{D}$ (3) = 2,000,000   k(3) = 50

For each of the searches 2 through 7 the $\bar{D}$ seeds are increased by a factor of 10. Through trial and error it was found unnecessary to update the k seeds so they remain the same. Once the series of 7 searches is done, the results which give the smallest Q value (refer to equation 3-4) are taken. A series of 3 more searches are performed, the first search is performed using the latest $\bar{D}$ value and plus or minus 20% of that value as seeds, the k seeds remain as the original seeds. The next search using the resulting $\bar{D}$ and plus or minus 10% of that value for seeds is performed. Again the original k seeds are used. A final search is performed using the last value of $\bar{D}$ and plus or minus 5% of that value as seeds. The results of this are taken as the final $\bar{D}$ and k values.

3.4.5 Calculate Enb using $\bar{D}$

Using the value obtained for $\bar{D}$ in the above step, Enb is calculated using equation 3-3.

3.4.6 Search for Eb

Using a search routine similar to that used to determine $\bar{D}$ and k, E_b is determined. The values of Enb, C, and M are used, in conjunction with the empirical relationships previously discussed, to determine an E_s (surface modulus) for any give E_b (base modulus) value. Through trial and error it was determined that it was not necessary to run through a series of searches to find E_b. The first search gives the same results as all subsequent searches regardless to adjustments to initial values.

The equation to be optimized for this search is the following:

$$F = \left(\bar{D} - \bar{D}_{cal}\right)^2 \tag{3-5}$$

Where $\bar{D}$ is the value obtained from the search discussed above and $\bar{D}_{cal}$ is the value obtained by plugging the current values of E_s and E_b into equation 3-2 for $\bar{D}$. Since E_s can be expressed in terms of E_b, $\bar{D}_{cal}$ (and hence F) is a function of E_b only.

The KISS program uses seed values of E_b = 200,000 and 500,000. As will be discussed in Chapter 5, changes in E_b seed value have no significant effect on the search results. The stopping criteria used for this search are if either F (calculated from equation 3-5) becomes less than 1E-10 or if the values of E_b in the search simplex get within 0.001 of each other.

3.4.7 Output results

Once all searches are complete, the program outputs the final values of $\bar{D}$, k, Q, E_s, E_b, and F.

3.5 Discussion

There are several advantages and disadvantages to using this program which will be expounded upon in this section. One of the main advantages is that I am able to take advantage of the uniqueness of $\bar{D}$ and k. I am also able to create the illusion of uniqueness for E_s and E_b through the use of the empirical equations. As stated, this uniqueness is only an illusion in that I am able to use the empirical relationships to link $\overline{D}$ to one set of E_s and E_b values. This was done by relating one set of Eb and Es values to the value of E_{nb} which is uniquely determined from a given $\overline{D}$ value.

Another advantage lies in the assumption of a rectangular load. By assuming a rectangular load the mathematics is greatly simplified because I am able to avoid the introduction of Bessel functions into the equation for deflection. Further, the nature of plate theory allows us to dispense with the assumption of pavement layers of infinite width and length.

As mentioned previously seed values used do not affect the end results of the searches for $\overline{D}$ and k or for E_b. This is a significant advantage over several of the programs discussed in Chapter 2 in which the backcalculation results are directly affected by choice of seed values.

A disadvantage is that the engineering judgment is necessary to determine if the asphalt moduli falls within the category of high, low, or average as defined previously. In Chapter 5 it will be demonstrated that an incorrect choice won't always be obvious and will not necessarily lead one to a correct choice. It is recommended to assume an average asphalt moduli for all backcalculations. This will be discussed in great detail in Chapter 5.

Chapter 4
Verification of Deflection Equations

4.1 Introduction

To verify deflections generated by Composite Plate Theory (using equation 3-1), comparisons are made for a one-layer system (slab on subgrade) and for a two-layer system (asphalt over base on subgrade).

The Composite Plate Theory deflection equation (equation 3-1) is first compared with Westergaard's equation by direct comparison and then to deflections determined by the Illi-back backcalculation program, which uses dimensional analysis and Westergaard's equation as described in Chapter 2 of this paper. These methods are only applicable to one-layer systems.

Equation 3-1 is then used to calculate deflections for two-layer systems and the results are compared to Linear Elastic Layer Theory via the computer program ELSYM5. These results are also compared with deflections determined using the finite element program SAP2000, which is used to model a composite plate over an elastic foundation.

There are several complications involved in attempting to verify that equation 3-1 gives valid deflection values. The main problem lies in the fact that the Composite Plate model makes several assumptions that differ from the more traditional models. In the paragraphs below, these differences are expounded and comparison results are given.

4.2 Comparison For One-Layer Systems

4.2.1 Introduction

Westergaard and ILLIBACK were chosen for comparison because, like the Composite Plate model, a Winkler foundation is assumed so that the parameter k (modulus of subgrade reaction) rather than E (elastic modulus) is used to characterize the subgrade. Eleven cases were tested in this comparison, and some unexpected results were found. Two different plate sizes for Composite Plate Theory were used for comparison purposes.

The moduli values used for these cases were generated by assuming moduli and k values which ranged from 500,000 to 2,000,000 psi and 85 to 150 pci respectively. Layer thicknesses from 2 to 5 inches were also assumed. These moduli, k values, and thicknesses were input into the composite plate theory program Defbowl1 which calculates deflections using composite plate theory. Since these are all one-layer systems, the composite plate theory equations simplify to the base plate theory equations. The resulting deflections, calculated at 0, 12, 24, and 36 inches from the center of the load were then input into the ILLIBACK program and E and k values were backcalculated. The resulting values were used in this analysis and are presented as cases 1 through 12 below. There were originally 12 cases input into the ILLIBACK program but the last case resulted in an error in the program and it was not possible to backcalculate E and k values for that case. This may sound convoluted but it is a case of the investigator not knowing quite where they were headed when the started the investigation. Clearly there are easier and more efficient methods of obtaining these theoretical test cases but once they had been established there was no need to redo the work.

The deflections under the center of the load (D_o) were determined via ILLIBACK, Westergaard's equation, and Equation 3-1 with the following results.

4.2.2 Results

Table 4-1: Central Deflections for Comparison With a One-Layer System.

				Do (mils)			
				Illiback	Westergaard	Composite Plate Theory	
case no.	E (psi)	k (pci)	h (in)			100'x100'	24'x24'
1	2917197	68	4	32.87	32.84	17.65	27.45
2	2409328	69	4	35.68	35.78	19.38	31.26
3	2575924	69	4	34.55	35.61	19.12	30.41
4	2738468	69	4	33.52	33.63	18.87	29.64
5	3320953	72	4	29.89	29.95	16.42	25.2
6	3719928	72	4	28.28	28.33	16.03	24.08
7	3448029	109	5	17.31	17.11	10.17	14.79
8	3821414	110	5	16.42	16.23	9.85	13.5
9	4183214	110	5	15.66	15.53	9.63	13.03
10	6696998	74	4	22.46	20.96	12.71	17.89
11	7229647	76	4	20.13	19.92	12.22	17.07

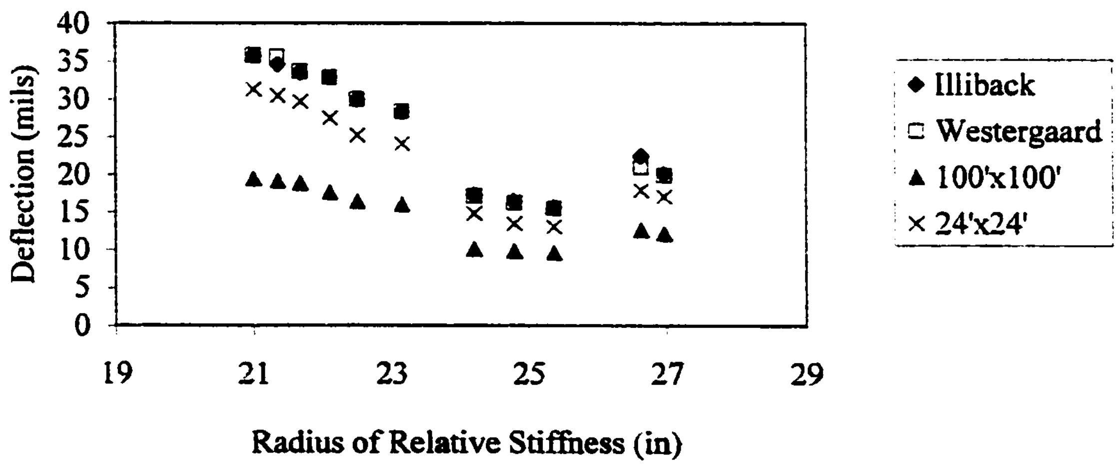


Figure 4-1: Comparison of Central Deflections

In the above Figure, the Radius of Relative Stiffness (ℓ_k) is a property relating to both the Stiffness and Modulus of subgrade reaction and is defined as follows for the dense liquid foundation model as used in the ILLIBACK program (described in Chapter 2).

$$\ell_k = \left(\frac{Eh^3}{12(1 - \mu^2)k} \right)^{\frac{1}{4}} \tag{4-1}$$

It is used here merely as a basis for comparison of the central deflection values.

In the table below % error is expressed relative to the deflections obtained by composite plate theory assuming a 24'x24' composite plate. Thus the following equation was used:

$$\%\text{Error} = \left(\frac{D_c - D_{cp}}{D_{cp}} \right) \cdot 100 \tag{4-2}$$

Where,

D_c = deflection calculated via ILLIBACK or Westergaard

D_{cp} = deflection calculated via composite plate theory for a 24'x24' composite plate.

Table 4-2: %Error Compared With 24'x24' Composite Plate

case no.	Illi-back	Westergaard
1	19.74	19.64
2	14.14	14.46
3	13.61	17.10
4	13.09	13.46
5	18.61	18.85
6	17.44	17.65
7	17.04	15.69
8	21.63	20.22
9	20.87	19.19
10	26.10	17.16
11	18.22	16.70

4.2.3 Discussion

There are several theoretical differences between Westergaard and Illiback as compared to Composite Plate Theory. Chapter 2 of this paper highlights the basic assumptions under these theories. One of the main differences is that Composite Plate Theory assumes that the edges of the plate are simply supported. Westergaard assumes that the plate is resting on the Winkler foundation, the edges are not simply supported. A further limitation to this comparison is that Westergaard only calculates the deflection directly under the load so that only central deflections could be compared, the rest of the deflection bowl is ignored.

Another difference in Westergaard's model is that the surface plate is assumed to be infinite in all lateral directions (i.e. in the X and Y directions). The Composite Plate model assumes a plate of finite dimensions in the X and Y directions. It was expected that as the composite plate increased in size, the calculated deflections would approach

Westergaard's. This is not the case. Table 4-1 and Figure 4-1 show that the deflections consistently follow the same pattern, but the smaller plate (24'x24') more closely matches Westergaard's and Illi-backs results than the larger plate (100'x100'). This result was surprising and prompted an investigation into several plate sizes with the following results.

Tabe 4-3: Central Deflections for Various Plate Sizes

	Plate size (in)							
Case no.	288	450	600	660	1200	1400	12000	288x1200
1	27.45	26.72	23.64	23.71	17.65	16.88	0.94	11.94
2	31.26	28.41	26.67	24.90	19.38	18.41	0.93	16.46
3	30.41	27.73	26.08	24.40	19.12	17.05	0.93	16.19
4	29.64	27.11	25.54	23.94	18.87	16.87	0.93	15.94
5	25.20	24.55	21.82	21.88	16.42	15.72	0.89	12.08
6	24.08	22.10	21.02	21.08	16.03	15.37	0.89	11.75
7	14.79	13.68	12.39	12.27	10.17	9.78	0.58	7.42
8	13.50	13.09	11.91	11.80	9.85	8.86	0.58	7.18
9	13.03	12.04	11.56	11.45	9.36	8.69	0.58	7.02
10	17.89	16.67	16.06	15.09	12.71	11.61	0.86	9.90
11	17.07	15.93	15.36	14.46	12.22	11.19	0.84	9.52

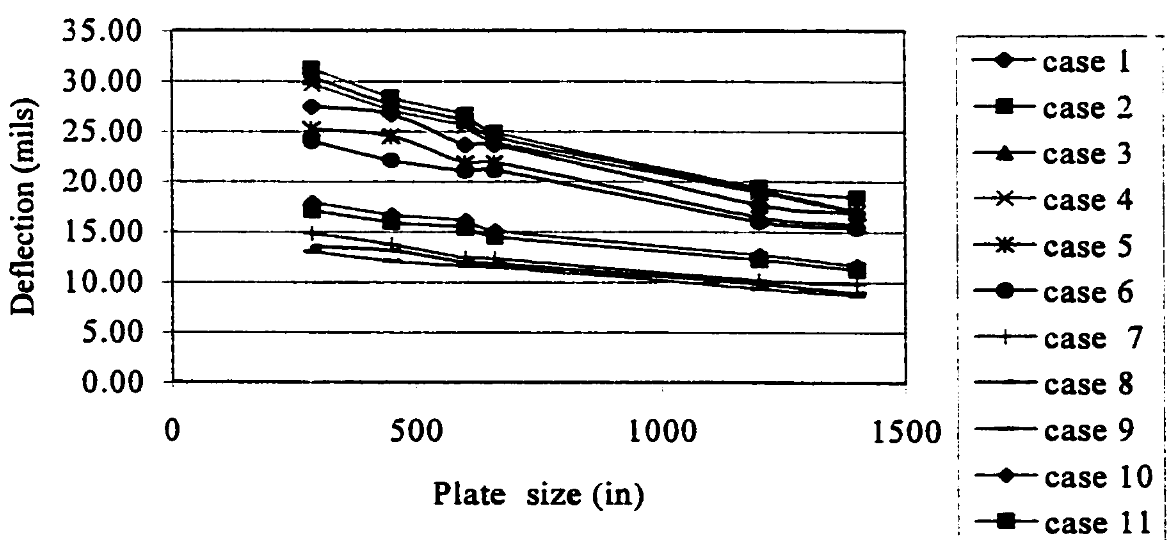


Figure 4-2: Variations in Plate Size

Note that Figure 4-3 shows only results for the square plates up to 1400 inches in length. This was done for clarity of the plot. Upon review of these results and those given previously it was determined that a 24'x24' plate best models the more realistic deflections. The effect of plate size is revisited in Chapter 5. Based on these findings, the majority of backcalculations performed using the KISS program assume a 24'x24' plate.

From Table 4-2 it can be seen that the errors were fairly large in comparing Composite Plate Theory to Westergaard's Equation and ILLIBACK. The theoretical differences explained above are the expected cause of these large discrepancies. Figure 4-1 illustrates that Westergaard and ILLIBACK deflections were in very good agreement. This is to be expected as ILLIBACK uses a finite element approximation of Westergaard's equation to determine deflections.

4.3 Comparison For Two-Layer Systems

4.3.1 Introduction

For two-layer systems, deflections calculated using Composite Plate Theory are compared to Linear Elastic Layer Theory using the ELSYM5 computer program. Like Westergaard, Elastic Layer Theory assumes a surface layer of infinite lateral dimensions. However, an advantage over Westergaard is that deflections can be calculated at many locations from the load, so I am no longer restricted to comparing deflections only under the load. Furthermore, Elastic Layer Theory (as the name implies) is capable of more than one layer over the subgrade.

Deflections for the two-layer system are also compared to deflections determined using the finite element modeling program SAP2000. SAP2000 allows modeling of a finite composite plate over a Winkler foundation. Thus minimizing the chances that theoretical assumptions could be the basis of errors between deflections. Composite Plate Theory assumes a composite plate simply supported over a Winkler foundation. Since Westergaard's equation assumes a plate resting on a Winkler foundation without the simply supported edges, SAP2000 was used to model the composite plate over the Winkler foundation both with and without the simply supported edges for the first three cases for the remaining cases, simply supported edges were assumed.

Eight cases are used for comparison, they are listed in Table 4-4.

Table 4-4: Cases for Comparison for a Two-Layer System

Case	Es psi	Eb psi	h1 in	h2 in	k pci	Esg psi	plate ftxft
1	500000	150000	3	6	100	1920	24x24
2	1000000	250000	3	6	150	2880	24x24
3	1500000	500000	3	6	100	1920	24x24
4	1500000	500000	2	6	100	1920	24x24
5	1500000	500000	4	6	100	1920	24x24
6	1500000	500000	3	10	100	1920	24x24
7	1500000	500000	3	18	100	1920	24x24
8	1500000	500000	3	6	100	1920	50x50

4.3.2 Results

The results for the first three cases are presented here in both table and figure form. Results for the remaining cases can be found in Appendix 4A.

Table 4-5: Case 1

location	deflection (in)				%Errors		
(in)	Comp. Plate	ELSYM5	SAP w/s.s.	SAP w/o s.s.	ELSYM5	SAP w/s.s.	SAP w/o s.s.
0	0.0227	0.0608	0.0228	0.0232	167.25	0.33	2.00
6	0.0222	0.0584	0.0221	0.0225	163.00	-0.61	1.09
12	0.0207	0.0543	0.0205	0.0209	161.98	-0.86	0.96
24	0.0155	0.0449	0.0171	0.0174	188.71	9.79	12.20
36	0.0093	0.0365	0.0135	0.0139	291.69	44.99	48.91
54	0.0024	0.0266	0.0088	0.0091	1002.37	263.07	276.57
72	0.0003	0.0198	0.0052	0.0054	7335.38	1838.00	1923.16

Where %Error is calculated as defined previously in equation 4-2.

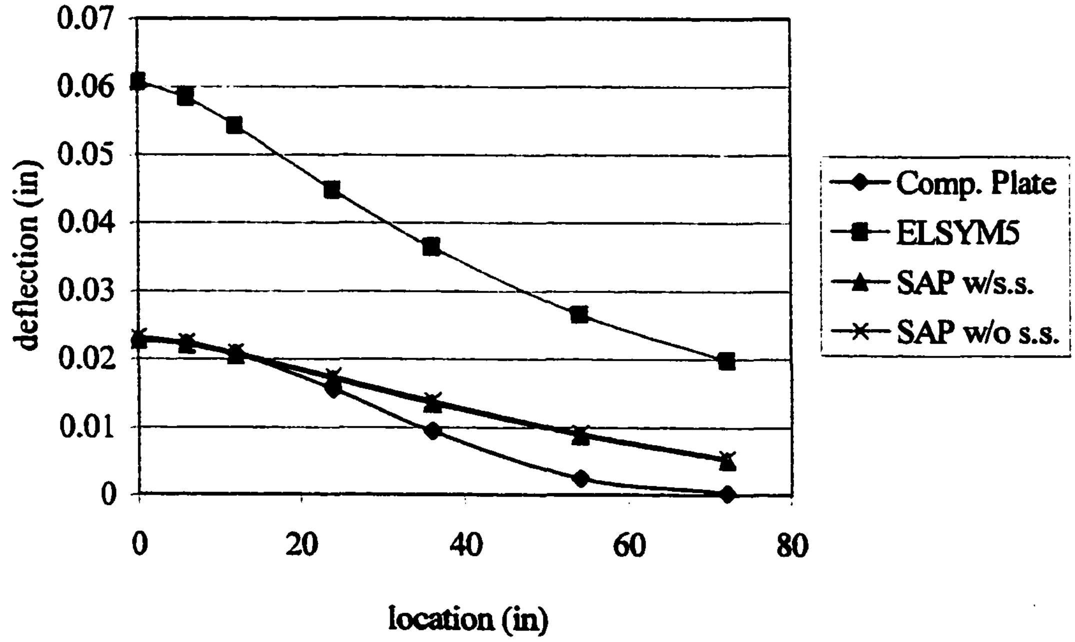


Figure 4-3: Case 1

Table 4-6: Case 2

location	deflection (in)				%Errors		
(in)	Comp. Plate	ELSYM5	SAP w/s.s.	SAP w/o s.s.	ELSYM5	SAP w/s.s.	SAP w/o s.s.
0	0.0140	0.0383	0.0140	0.0143	172.25	-0.57	1.56
6	0.0137	0.0369	0.0136	0.0139	168.51	-1.39	0.80
12	0.0129	0.0346	0.0127	0.0130	168.75	-1.50	0.83
24	0.0098	0.0289	0.0106	0.0109	195.90	8.73	11.77
36	0.0060	0.0238	0.0085	0.0088	293.96	41.01	45.84
54	0.0018	0.0177	0.0057	0.0059	874.60	213.53	228.33
72	0.0003	0.0133	0.0035	0.0037	3903.98	940.79	1002.60

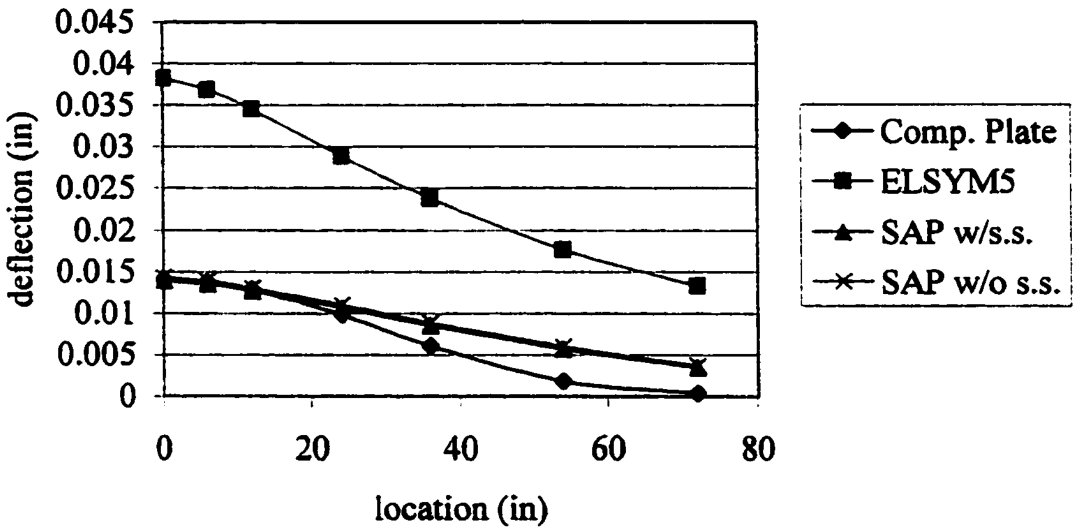


Figure 4-4: Case 2

Table 4-7: Case 3

location	deflection (in)				%Errors		
(in)	Comp. Plate	ELSYM5	SAP w/s.s.	SAP w/o s.s.	ELSYM5	SAP w/s.s.	SAP w/o s.s.
0	0.0129	0.0418	0.0123	0.0133	225.28	-4.70	3.37
6	0.0126	0.0410	0.0120	0.0130	224.59	-4.98	3.23
12	0.0122	0.0386	0.0115	0.0125	217.44	-5.62	2.92
24	0.0104	0.0349	0.0102	0.0112	236.83	-2.00	7.99
36	0.0079	0.0307	0.0087	0.0097	290.70	10.30	23.41
54	0.0040	0.0249	0.0064	0.0074	522.62	61.11	86.31
72	0.0013	0.0200	0.0044	0.0054	1396.51	231.43	302.33

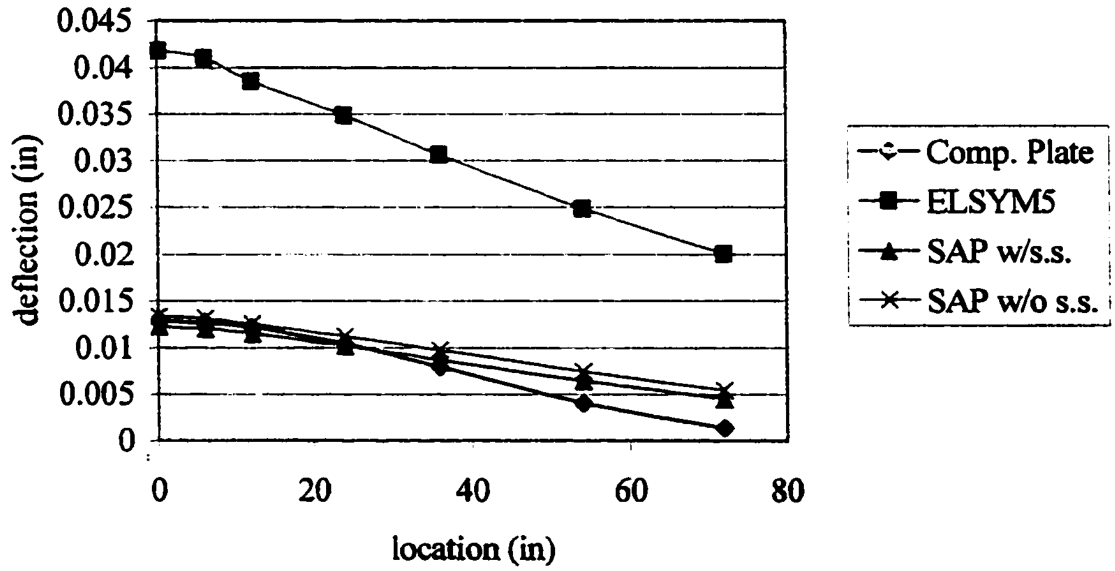


Figure 4-5: Case 3

4.3.3 Discussion

The main problem in comparing Composite Plate Theory to Elastic Layer Theory, is that the material property assumed for the subgrade is different. Elastic Layer Theory assumes an elastic modulus, E, for the subgrade. Composite Plate Theory assumes and modulus of subgrade reaction, k.

The relationship used to relate E and k for subgrade material involves the equation used in the field when deflections are measured on top of the subgrade before base and surface are placed. The equation is as follows (Ullitdz, 1987, and Darter et. al. 1992):

$$E = k[f(1 - \mu^2)a]$$

Where,

E is the elastic modulus of the subgrade (psi)

k is the modulus of subgrade reaction (pci)

a is 15 inches (standard plate size used in the field test)

f is 4/3 (this is a stress distribution factor for a parabolic distribution in a cohesive material, Ullitdz, 1987)

μ is the Poisson's ratio for the subgrade.

Deflections are calculated at x = 0, 6, 12, 24, 36, 54, and 72 inches from the center of the load. This is commonly the sensor locations used for FWD testing. Results are shown in Tables 4-4 through 4-6 and Figures 4-2 through 4-4, and in Appendix 4A.

From these results, it is evident that the Elastic Layer theory deflections do not compare very well with those from Composite Plate theory. As with the one-layer deflections, it is expected that the theoretical differences involving assumptions concerning material behavior, boundary conditions and subgrade are the underlying cause of the discrepancy between the deflections.

The results clearly demonstrate that the finite element model and Composite Plate theory agree more closely than any of the other models thus far. This is as expected since many of the theoretical differences have been eliminated. The largest errors occur in the outer deflections.

4.4 Conclusions

From the above results it can be concluded that the Composite Plate theory deflections follow similar patterns to the deflection basins of Westergaard and Illi-back. Linear Elastic theory, however, does not agree with Composite Plate theory. All three of these methods result in deflections of larger magnitude than Composite Plate theory, much larger in the case of Linear Elastic theory.

Using the SAP2000 Finite Element program to model a composite plate on a Winkler foundation it is shown that the deflections are much closer in magnitude however they do not seem to follow the same basin patterns causing larger errors in the outer deflections.

From this comparison I conclude that Composite Plate theory deflections do not agree with Westergaard's or Elastic Layer Theory deflections. I also conclude that the finite element method of approximating deflections for a composite plate on a Winkler foundation comes much closer to the deflections predicted using Composite Plate Theory equations, however they begin to differ as the distance from the load increases. Therefore these two methods of calculating deflections also do not completely agree.

Since I am comparing different methods of calculating deflections, and it is impossible to compare these results with real deflections, it is difficult to know which more closely models reality so I cannot conclude which is the better method, only that the methods do not agree. The only conclusion I can draw is that the theories and approximation techniques compared here do not completely agree. However from the SAP2000 comparisons it appears that the Composite Plate deflection calculations are realistic, however the outer deflections are in question. Yet, it is difficult to know whether to question the Composite Plate Theory deflections, or the SAP2000 deflections.

Chapter 5
Sensitivity Analysis

5.1 Introduction

The sensitivity of both the deflection equation and the backcalculation program KISS, to changes in various parameters, are investigated. Using the deflection equation from Composite Plate theory, deflections are calculated using the program MATHEMATICA. This is done for several different pavement systems, with variations of several different parameters such as layer moduli, layer thickness, modulus of subgrade reaction, and Poissons ratio.

The second sensitivity analysis performed is the sensitivity of the backcalculation program KISS to changes in various parameters such as layer thickness, seed moduli, and plate size.

5.2 Sensitivity Analysis of the Deflection Equation

5.2.1 Introduction

Composite Plate theory is discussed in Chapter 3 of this paper. The deflection equation is repeated here for reference.

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\frac{A_{mn}}{\pi^4\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2\bar{D}+k}\right\}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (5\text{-}1)$$

where

A_{mn} = the load distribution expressed as a double Fourier sine series. That is,

$$= \frac{4}{ab}\int_0^a\int_0^b P(x,y)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)dydx$$

P(x,y) is the known load distribution [lb]

m,n = the dummy variables over which summation is carried out [unitless]

a,b = the plate dimensions. [in.]

$\overline{D} = \frac{1}{3(1-\mu^2)}\sum_{i=1}^{\bar{n}} E_i\left(\zeta^3\right)_{h_i}$ represents the composite plate stiffness. [lb-in.] (5-2)

E_i = the modulus of the i^{th} layer. [lb/in^2]

$\bar{n}$ = number of materials in the composite plate [unitless]

ζ = distance to the neutral plane from the middle of the i^{th} layer [in.]

μ = Poisson's ratio [unitless]

k = modulus of subgrade reaction [lb/in^3]

Although the computer program KISS has been developed specifically for asphalt concrete pavements over a stabilized base, Composite Plate theory can be used to model many different types of pavements. Therefore, sensitivity analysis is performed on 7 different pavement scenarios, these are described in Table 5-1.

Table 5-1: Scenarios for Sensitivity Analysis of Deflection Equations

Case no.	layer 1	layer 2	layer 3	layer 4
1	AC overlay	Concrete	Subgrade	none
2	Concrete	Stabilized base	Subgrade	none
3	Concrete	Granular base	Subgrade	none
4	Asphalt	Stabilized base	Subgrade	none
5	Asphalt	Granular base	Subgrade	none
6	AC overlay	Concrete	Stabilized base	Subgrade
7	AC overlay	Concrete	Granular base	Subgrade

The effects on calculated deflections due to changes in the parameters given in Table 5-2 were analyzed.

Table 5-2: Parameters Analyzed

Symbol	Description	units
h1	thickness of surface layer	in.
h2	thickness of second layer	in.
h3	thickness of third layer (where applicable)	in.
E1	modullus of surface layer	psi
E2	modulus of second layer	psi
E3	modulus of htird layer (where applicable)	psi
k	modulus of subgrade reaction	pci
μ	Poisson's ratio	none

Table 5-3 gives the initial values and ranges used in the analysis.

Table 5-3: Initial Values and Ranges for Deflection Equation Analysis

		initial value	range
Asphalt	E (psi)	2,000,000	500,000 to 2,500,000
	h (in)	3	2 to 5
Concrete	E (psi)	4,000,000	500,000 to 5,000,000
	h (in)	6	4 to 10
Stabilized base	E (psi)	1,000,000	500,000 to 2,500,000
	h (in)	6	4 to 24
Granular base	E (psi)	35,000	25,000 to 45,000
	h (in)	6	4 to 24
Subgrade	k (pci)	100	25 to 1000
Poisson's Ratio*	m (unitless)	0.15	0.15 to 0.45

*** Recall that Poisson's ratio is assumed equal for all layers in composite plate analysis.**

There are a few exceptions to the initial values given in Table 5-3. For Case 3, an initial Poisson's ratio of 0.2 was used. For Cases 4, 5 and 7 an initial Poisson's ratio of 0.3 was used. Also for Case 5, an initial granular base thickness of 10 inches was used. All other initial values and ranges are as given in the table.

As can by seen by observation of the equation for deflection (Equation 5.1) most of these parameters only affect the parameter $\overline{D}$ (Equation 5.2). For this reason, the affect on $\overline{D}$ was analyzed as well as the overall affect on deflection. The following equations were used in the analysis.

<u>Percent Root Mean Square:</u>

$$\mathrm{RMS}(\%) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{d_i^v - d_i^o}{d_i^o}\right)^2} \times 100$$

v = varied parameter
o = original parameter
d_i = deflection at the i^{th} location
n = the number of deflection locations.

Note: For this analysis n was 49. Deflections were calculated at n = 0 to 288 inches (24 ft) every 6 inches. n = 0 being directly under the load and n = 288 being at the edge of the plate which was assumed 24'x24' for this analysis.

Percent change in parameter:

$$\frac{P_o - P_v}{P_o} \times 100$$

P_o= original parameter
P_v= varied parameter

Note: The parameters here are those given in Table 5-2.

Percent change in $\bar{D}$:

$$\frac{\left|\bar{D}_o - \bar{D}_v\right|}{\bar{D}_o} \times 100$$

$\bar{D}_o$= with original parameter
$\bar{D}_v$= with varied parameter

5.2.2 Results

Only the results for analysis on the fourth case in Table 5-1 (2 layer asphalt over a stabilized base) will be presented here since the KISS backcalculation program has only been developed for that case. The results for the remaining cases can be found in Appendix 5A. Table 5-4 presents the analysis results.

Table 5-4: Sensitivity Analysis for Asphalt Over a Stabilized Base.

Parameter varied	% change in Parameter**	% Change in $\overline{D}$	RMS%
h1	-33.33	30.65	22.30
h1	-16.67	16.08	10.58
h1	33.33	-37.39	18.14
h1	16.67	-17.77	9.56
h1	66.67	-82.91	32.48
h1	50.00	-59.05	25.77
h2	-33.33	52.87	46.30
h2	100.00	-348.65	44.14
h2	150.00	-660.23	50.75
h2	200.00	-1085.14	49.72
h2	300.00	-2340.00	48.69
E1	-75.00	47.84	39.93
E1	-50.00	27.03	19.16
E1	-25.00	11.78	7.54
E1	25.00	-9.46	5.33
E2	-50.00	35.14	26.42
E2	50.00	-25.41	13.12
E2	100.00	-45.95	21.36
E2	150.00	-63.71	27.21
k	-75.00	NA	357.72
k	-50.00	NA	146.15
k	-25.00	NA	47.08
k	200.00	NA	73.53
k	400.00	NA	100.11
k	700.00	NA	100.96
k	900.00	NA	101.96
μ	-33.33	5.21	3.20
μ	-16.67	2.93	1.78
μ	-50.00	6.91	4.29
μ	16.67	-3.70	2.16
μ	33.33	-8.33	4.73
μ	50.00	-14.11	7.75

** negative means a decrease in value by the given percentage.

NA means not applicable because k has no affect on $\overline{D}$, refer to equation 5-2.

Figures 5-1 through 5-6 shows graphical representations of the percent change in each parameter vs RMS% for all parameters in the analysis. Figure 5-7 shows all parameters in one plot to give an idea of the relative severity of the effect of changes in each parameter.

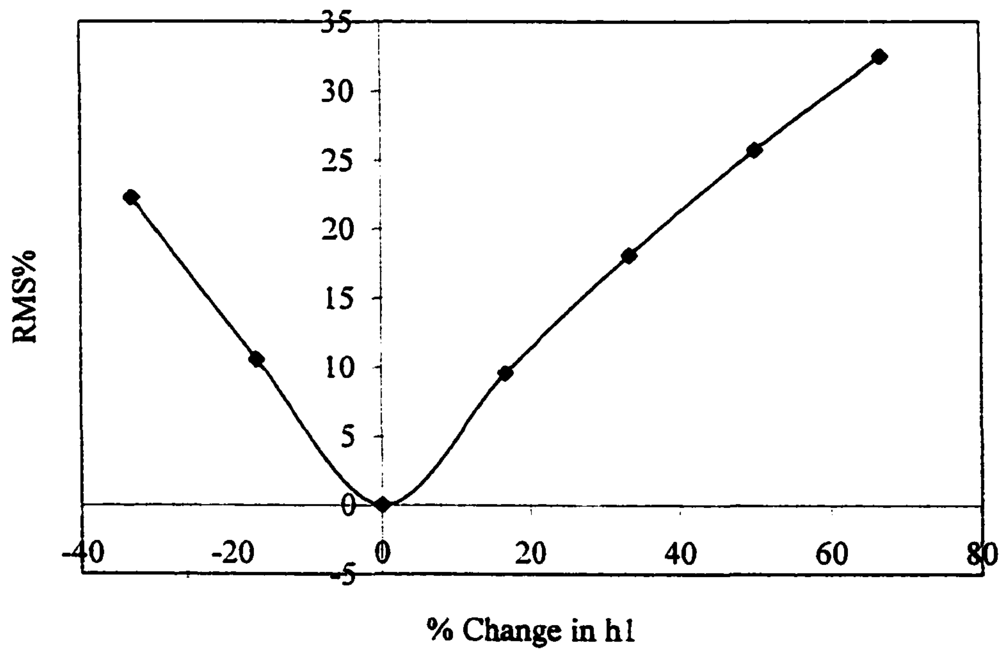


Figure 5-1

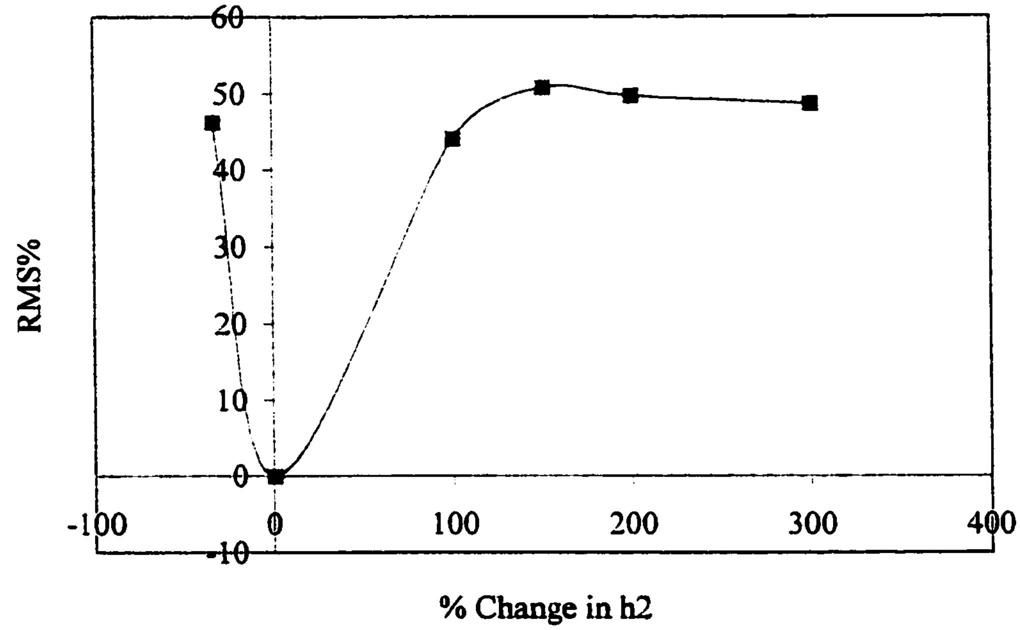


Figure 5-2

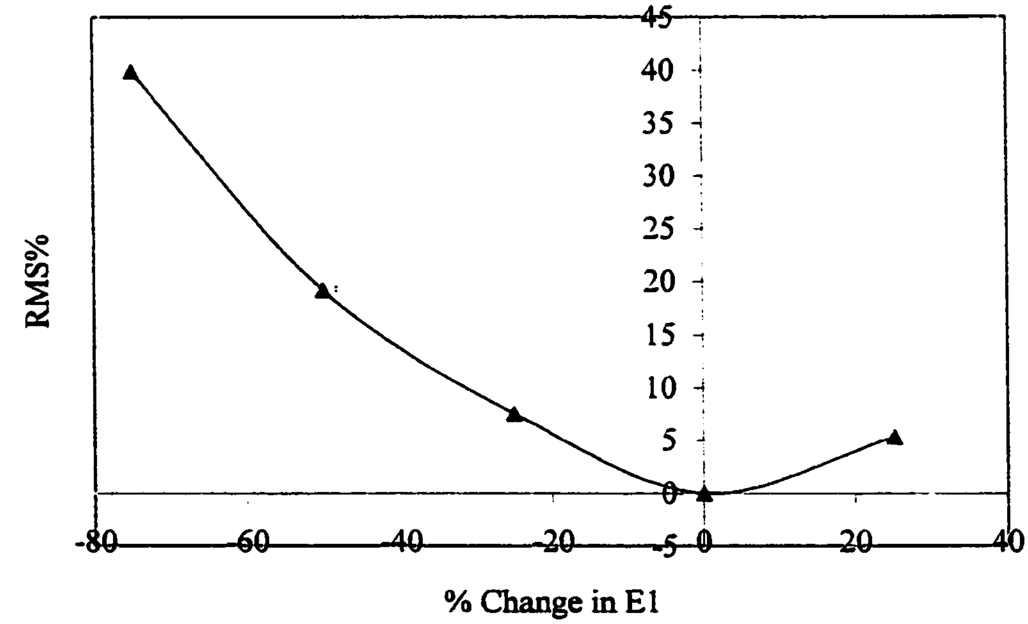


Figure 5-3

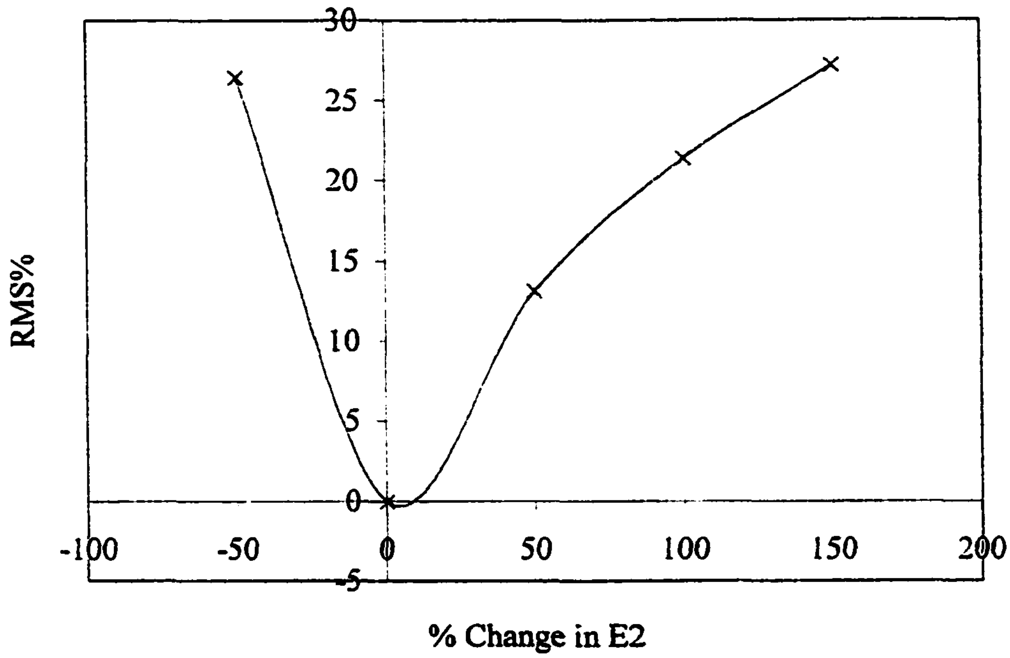


Figure 5-4

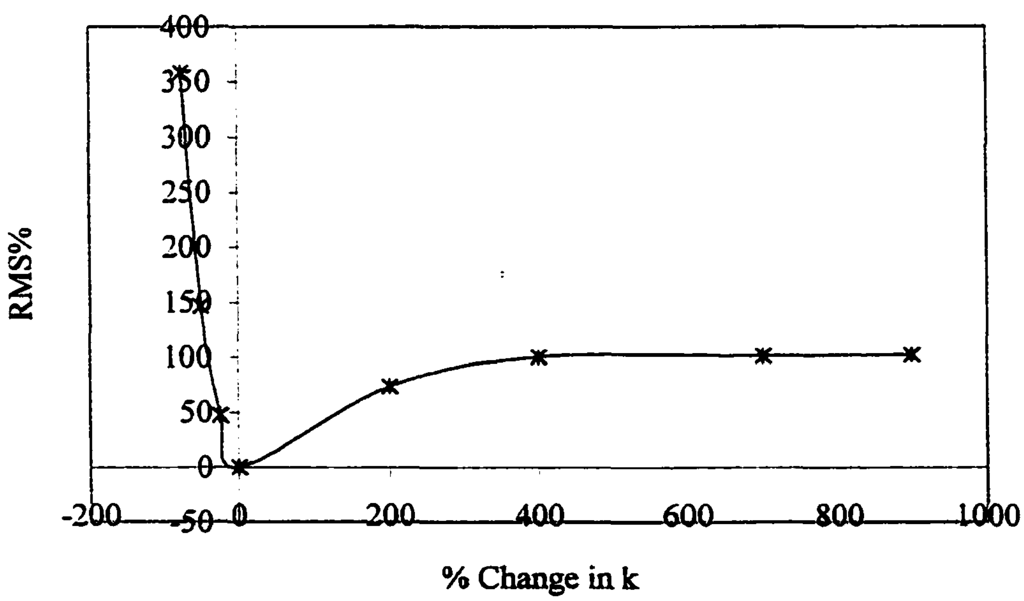


Figure 5-5

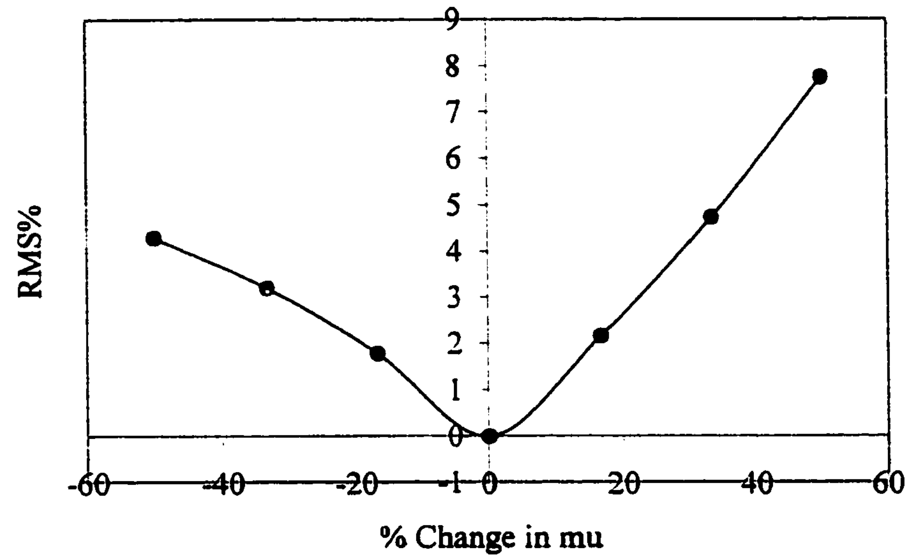


Figure 5-6

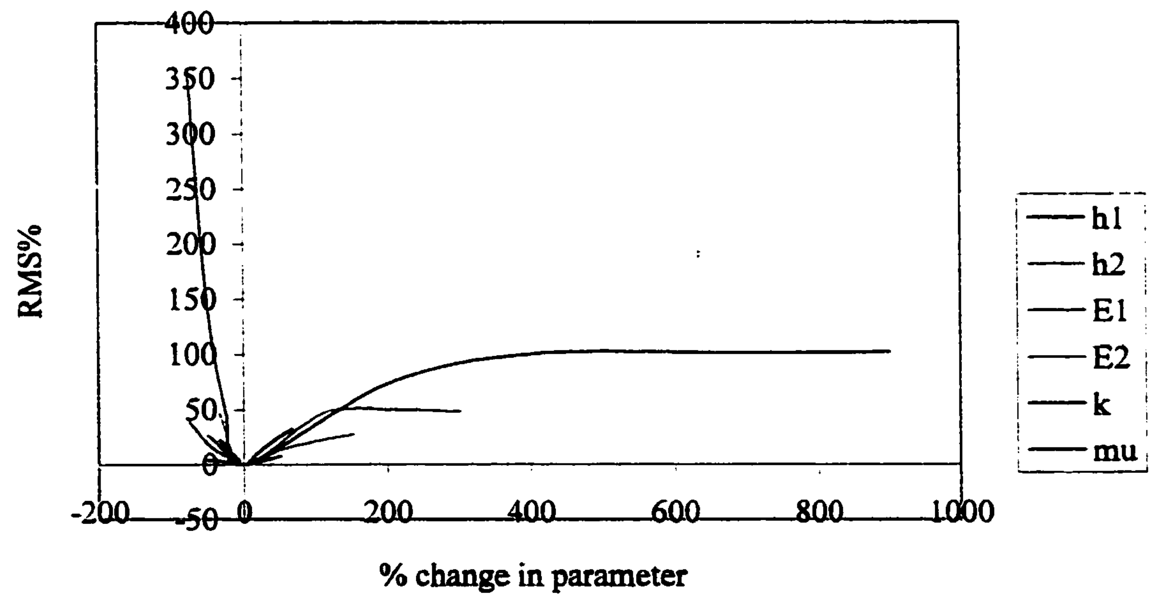


Figure 5-7

5.2.3 Discussion

The following paragraphs describe some observations made about the above results. For a decrease in parameters, k showed the most severe influence on the calculated deflections followed by h2, h1, E2, E1, and μ, this list is in order of severity of influence. The parameters k and h2 had relatively steep slopes as their values deviated in the negative direction from the original k and h2 values. The parameters h1, E2, and E1 had moderate slopes and μ had a relatively shallow slope. The steepness of the slope is an indication of the severity of the influence on the calculated deflection.

For increases in the parameters, the results were different. For up to about 120% increase in parametric values, h1 and h2 have the most severe influence (the steepest slope) with h1 being slightly worse that h2. Beyond 120%, k takes the lead. E2 has a moderate (slope) influence on the deflections and E1 and μ have minimal influence.

These results reinforce what was already suspected. It is well known that the value of μ has little influence on the calculated deflection, these results clearly demonstrate this fact. These results also demonstrate the importance of an accurate estimate of the thickness of the layers. The value of k is also shown to be important. Fortunately, as discussed in Chapter 3, the value of k is unique for these equations and the backcalculation program appears to work very well in obtaining an accurate estimate of the actual k value.

It is unknown why there is a difference in severity of influence on calculated deflections between increases and decreases of the parameters.

5.3 Sensitivity Analysis of the Backcalculation Program KISS

5.3.1 Introduction

This analysis was performed using real deflections obtained from the Federal Highway Association's (FHWA) Long Term Pavement Performance (LTPP) database. This database is described in more detail in Appendix 6A and the data for the cases used here and in Chapter 6 is also presented in that Appendix.

Using deflections from a concrete pavement over a subgrade (Case 21 described in Appendix 6A), variations in composite plate size, $\overline{D}$ and k seeds, and variations in applied stress are investigated. The affect of these variations on backcalculated moduli is discussed.

The program KISS requires the user to assume that the asphalt moduli is either high (E > 1500000 psi), average (700000 < E < 1500000 psi), or low (E < 700000 psi). Using deflections from three different asphalt pavements over stabilized bases (described as Cases 1, 4, and 6 in Appendix 6A), moduli are backcalculated for all three options.

5.3.2 Results

Table 5-5: Variations in Plate Size

plate size (ftxft)	E (psi)	k (pci)	D (lb in)
24x24	968231	452	184653042
20x20	1097191	446	209247326
500x24	2536	645	483612

Table 5-6: Variations in Applied Stress

stress (psi)	E (psi)	k (pci)	D (lb in)
88	968231	452	184653042
117	915905	469	174673883
154	888047	454	169360978

Note that the applied stress here refers to the stress applied to the surface of the pavement. The stresses given here were obtained from field tests in the LTPP database. The measured deflections from these tests were used to backcalculate the results given in the table.

Table 5-7: Variations in AC Stiffness

case 1	E1 (psi)	E2 (psi)	k (pci)	D (lb in)
high	4464025	312531	357	80581807
average	2991095	361247	357	80575369
low	1927650	455898	358	80911918
case 4				
high	3812990	133833	185	41739627
average	2617187	145540	185	41764492
low	1837419	161965	185	41749236
case 6				
high	1393026	3081823	440	193672456
average	833506	5025047	440	193672454
low	615778	6055292	440	193672455

Note that no table of results is given for variations in seed values for $\overline{D}$ and k because changes in the seed values were found to have no effect on the backcalculated values. The effect of E_b seed values was also investigated. The seed values are merely a starting place where the search routine begins, no significant effect of changing these was found.

5.3.3 Discussion

Table 5-5 demonstrates the importance of plate size assumed. The case of 500x24 foot plate was assumed because that is the size of the test sections used in the LTPP data base. This assumed plate size does not give realistic results, hence long narrow plate sizes should be avoided. Both the 20x20 and 24x24 foot plate sizes appear to give more reasonable results. For the remaining cases in this chapter and the next, a 24x24 foot plate was assumed unless otherwise stated.

The effect of variations in applied stress is less pronounced than that of plate size. This is to be expected since the changes in stress are actual and not a theoretical assumption. These different stresses were actually applied in the field and the resulting deflections were used in the backcalculation program. The differences in moduli values for different stresses demonstrate that the pavement is stress sensitive. From the results it appears that stress softening is occurring since the larger stress results in a lower modulus value. The variations in the modulus of subgrade reaction (k) appear negligible.

Table 5-7 indicates that good engineering judgment is invaluable when using the KISS backcalculation program. This is true for all backcalculation programs. As can be seen, results vary widely depending on which scenario (high, average, or low) is chosen. Further, notice that if a low asphalt moduli is assumed and the backcalculation results in a modulus which falls within the "high" category, rerunning the program assuming a high asphalt moduli is not necessarily beneficial. For example, in case 1 Table in 5-7, if low were initially assumed an asphalt modulus of 1,927,650 psi would be obtained. If the program were then rerun assuming a high asphalt moduli, the result would be a modulus value of 4,464,025 psi which is too large to be realistic. Typically an AC modulus greater than 2,000,000 psi would be unexpected.

To explain why this happens I will look at the nature of the equations used to backcalculate the layer moduli. Recall from Chapter three that the equations used to relate E_b and E_s are of the form $E_{nb}/E_s = C(E_b)^M$ where C and M are constant coefficients which have been determined empirically for various combinations of layer thicknesses (refer to Appendix 3A for a listing of C and M values), and $E_{nb} = \frac{12\bar{D}(1-\mu^2)}{h_1^3}$.

Case 1, in Table 5-7, has an asphalt thickness of 1.5 inches and a base thickness of 8.8 inches. The KISS program, in this case, would interpolate between the values of C and M for the combinations of 2 inch surface and 6 inch base layers and 2 inch surface and 10 inch base layers. Different C and M values would be obtained for each of the assumed high, average, and low asphalts as shown in the following table.

Table 5-8: C and M Values for Case 1.

asphalt	C	M
high	0.0165	0.6429
average	0.0254	0.6334
low	0.0283	0.6476

Within the KISS program, E_b is determined via an optimizing search routine, E_s is found as $E_s = E_{nb}/C(E_b)^M$. $\bar{D}$ and k are independent of the assumption of high, average, or low asphalt moduli so the backcalculated results should not vary much, regardless of what is assumed. So the differences in E_b and E_s come about due to the empirical equations used. These equations have the effect of steering the search routine in a different direction. From Figure 5-8 and Table 5-9, it is demonstrated how the assumption of low gave a high asphalt modulus and the assumption of high gave an unrealistically high result.

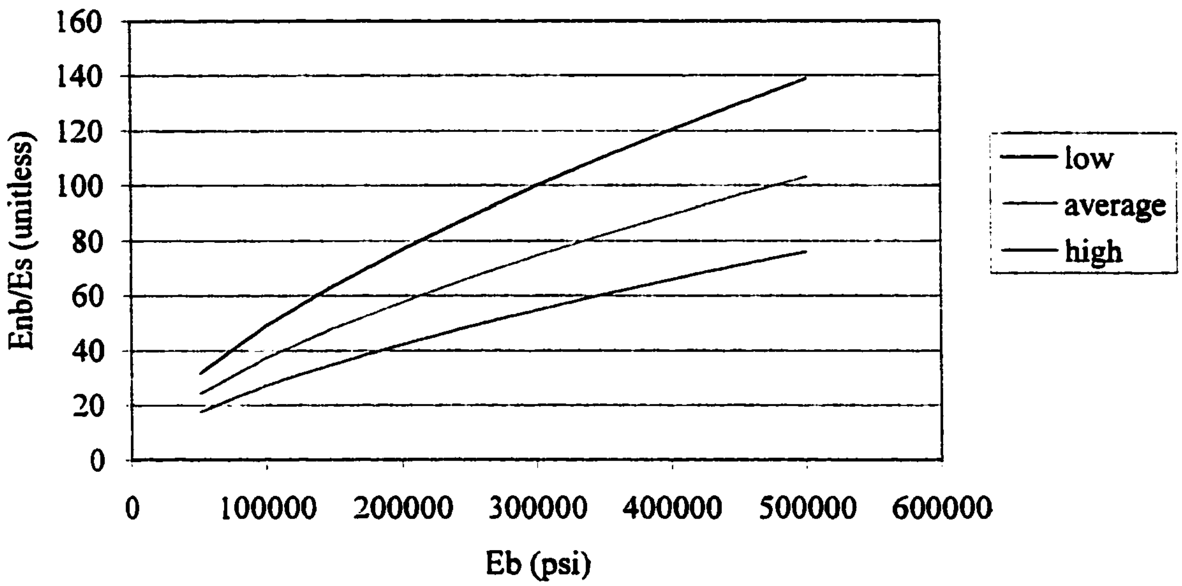


Figure 5-8: Case 1 High, Average, Low Investigation

Table 5-9 Case 1 Results

	Eb (backcalculated)	Enb/Es (C(Eb)^M)	Es Es = Enb/(Enb/Es)
high	312531	56	4469314
average	361247	84	2986826
low	455898	131	1930625

Notice that from low to high the value of E_{nb}/E_s increases, considering that E_{nb} doesn't vary much (as it is dependent on $\overline{D}$ and independent of the high, average, low assumption) the decrease in E_{nb}/E_s (for a nearly constant E_{nb}) causes the resulting E_s to increase. So in going from the assumption of low to high we only increase the E_s value from a high value to an unrealistically high value.

5.4 Conclusions

The sensitivity analysis on the deflection equation (5.1) presented in Section 5.2 of this chapter investigated not only the input parameters (layer thickness and Poisson's Ratio) but also the output parameters (elastic modulus and modulus of subgrade reaction). The results confirm the long held notion that changes in Poisson's ratio do not have a significant effect on the resulting deflections. Therefore when a Poisson's value is not known, it is ok to make an educated guess without fear of adversely affecting the results.

The results also demonstrate the importance of having good estimates of the layer thicknesses. Both h1 and h2 have a significant impact on the resulting deflections calculated.

The elastic moduli (E1 and E2) and modulus of subgrade reaction (k) are the parameters for which I am searching. Clearly these parameters have a significant effect on the calculated deflections. This can be looked at as an advantage in our search because it means that if our estimated values for k, E1, or E2 are significantly off, then our calculated deflections will also be off. Thus they will not match up with the measured deflections and our search will continue.

The sensitivity analysis on the KISS backcalculation program, presented in Section 5.3 of this paper, investigates the significance of several important factors in the backcalculation process. First and foremost is the investigation of the effect of composite plate size (meaning width and length) assumed on the backcalculated results. It is shown that the assumed plate size has a considerable effect on the resulting backcalculated moduli and modulus of subgrade reaction. Based on these results and those given in Chapter 4, it is recommended that backcalculation be performed assuming a 24x24 foot plate.

Changes in the applied stress also have an affect on the backcalculated results. This is expected to be the case when the material is stress sensitive. It is recommended that tests be performed at several load levels and that backcalculation be performed for all of the levels tested. In the future it would be prudent to introduce some of the currently known stress sensitivity equations into the backcalculation program, or possibly develop some new equations based on future research.

It is evident from the results that the choice of high, average, or low asphalt concrete scenario is significant. It appears that while breaking the asphalt up into these categories resulted in empirical equations with very good R^2 values, it gained us nothing in the resulting backcalculated values. Based on this analysis, I would recommend using only the "average" asphalt equations. Were I to do this over again, I would not break the data up into high, average and low. I would obtain the empirical equations for the entire range of AC moduli values. Although the R^2 values would be smaller, the assumption of high, average or low would not be necessary. As can be seen from these results, making an incorrect assumption will not be obvious and changing the assumption will only serve to increase or decrease the result (based on which way the assumption was changed) but will not necessarily lead to a better result. The current equations for the average modulus comes closest to what would have been obtained had the data not been broken up. Hind sight being 20/20, in the future if this program is used for design or research purposes, I would strictly assume average for all backcalculation efforts.

Chapter 6
Comparison with other programs

6.1 Introduction

This chapter presents the results of the comparison of the KISS program with other backcalculation programs using data from the SHRP LTPP (Strategic Highway Research Program – Long Term Pavement Performance) database program DataPave 2.0. LTPP is a program set up to collect long-term data on pavements all over North America. The program has more than 2400 test sections in over 900 locations. The data collection program includes inventory, material testing, pavement performance monitoring, climatic, traffic, maintenance, rehabilitation, and seasonal testing. The data are subject to an extensive series of quality control checks before being made available to the public. The DataPave software makes the LTPP database more accessible and is available free of charge from the FHWA (1999).

The backcalculation programs used in this comparison are EVERCALC, BOUSDEF, MODCOMP, and MODULUS for asphalt concrete (AC) pavements and ILLIBACK for Portland cement concrete (PCC) pavements.

Before presenting the cases for comparison and the results, the complications in making such a comparison should first be pointed out. Ullidtz and Stubstad (1985) have stated the following, ***"More important than comparing the different theories is keeping in mind that large differences exist between theoretical models and actual pavement*** *structures. Only by comparing theoretical values to measured values is it possible to determine whether a given model or approach is satisfactory".*

Unfortunately insitu moduli cannot be measured directly. Moduli can be measured in laboratory situations, however, as with backcalculated moduli, there is no way to know how close the laboratory moduli are to actual field moduli. Further, it has been demonstrated that moduli values obtained in the laboratory do not compare well with backcalculated moduli (Mahoney et.al, 1989, and Chua, 1989).

Therefore, it is important to keep in mind that the comparison is between calculated values with no "real" values as basis of comparison. Hence large differences can be expected. These differences would mainly be due to differences in theory with no basis for judging which is the most accurate. That being said, I will proceed.

Three methods of comparison were chosen for this analysis. The first is a program-to-program comparison which compares each of the backcalculation programs to the KISS program one at a time. The second method of comparison is on a case-by-case basis where all of the programs are compared at once for each case, one at a time. The third methodology for comparison was by using theoretically generated deflections. As will be shown, each of these methods of comparison gave a different perspective to the analysis.

6.2 Literature Review

As demonstrated in Appendix 2A, there have been many papers written which compare and attempt to validate the various backcalculation programs. One of the most intensive comparisons was made by the SHRP software selection and development committee (Rada et. al, 1992). In this study the programs ISSEM4, MODCOMP, MODULUS, and WESDEF were evaluated. Several different types of comparisons were done. Initially evaluators independently ran all programs using the same data sets from actual field testing. Results were judged on criteria such as reasonableness, robustness, stability, goodness of fit, suitability for SHRP's purposes. Ultimately MODULUS was chosen and modified to suit their needs.

Another such comparison was done by Mahoney et. el. (1989). In this study field test data from five sites in Washington State were used with six different techniques. Five computer programs (ELMOD, ELSDEF, EVERCALC, ISSEM4, and MODCOMP) and laboratory methods (diametral and triaxial) were used. They found more agreement than disagreement among their results.

Other in-depth studies comparing various computer programs include an intensive study by Kim and Nokes (1993) who looked at BKCHEV, BOUSDEF 2.0 EVERCALC 3.3, MODULUS 4.0, and WESDEF. This study looked at sensitivity to inputs. Maestas and Mamlouk (1992) compared ELSDEF, MODCOMP, MODULUS4, and ISSEM4 on the basis of calculated overlay thickness for 29 test sites in Arizona. The Public Works Department in Malaysia used an elimination process to compare BISDEF, DEFMET, PHONIX, Hogg's model, and PEACH. They used a Dynamic Cone Penetrometer for field measurements against which to compare their results. Their results showed that the PHONIX and PEACH programs had good potential for structural evaluation of flexible pavements (Chong, 1990). Chou and Lytton (1991) took the approach of giving deflection data to several different agencies and comparing the resulting backcalculated moduli in several ways. They compare moduli determined from deflections measured with three different NDT devices, they compared moduli determined from different load levels, and they also compare moduli backcalculated from deflections that were computer generated. Their report emphasizes the need for an expert system to aid the user.

There are also several studies in which test pavements have been instrumented so that field measurements can be used as verification for backcalculation efforts. Multi-depth deflectometers (Kim et. al. 1992, Uzan and Scullion, 1990), strain gauges (Lenngren, 1990), and geophones (Nazarian and Chai, 1992) have been used.

6.3 Cases for Comparison

Nineteen different cases were chosen from the LTPP Database. Nine of these were asphalt concrete (AC) over a stabilized base, this is the type of pavement system that the equations used in the KISS program, relating E_s to E_b, were developed for. Of the other ten cases, three were AC over a granular (untreated) base, three were for an asphalt wearing surface over an asphalt binder course, and three were two-layer cases of AC laid directly over the subgrade. The final case was for a Portland cement concrete (PCC) over an untreated base. All cases are summarized in Table 6.1 with detailed descriptions of each case are given in appendix 6A.

Table 6.1: Cases for Comparison

Case	System	Location
1	1.5" Asphalt over 8.8" asphalt stabilized base	Seminole Co., Oklahoma
2	2" Asphalt over 8" asphalt stabilized base	Muskogee Co., Oklahoma
3	2" Asphalt over 10" asphalt stabilized base	Rockingham Co., New Jersey
4	1.5" Asphalt over 9" asphalt stabilized base	Ector Co., Texas
6	4" Asphalt over 6" cement stabilized base	Laramie Co., Wyoming
7	6" Asphalt over 7.2" cement stabilized base	Alberta Canada
8	2" Asphalt over 8" asphalt stabilized base	Carter Co., Oklahoma
9	2" Asphalt over 8" asphalt stabilized base	Major Co., Oklahoma
10	3" Asphalt over 8" asphalt stabilized base	Manitoba Canada
11	7.8" Asphalt over 28" granular base	Wright Co., Minnesota
12	7.8" Asphalt over 28" granular base	Wright Co., Minnesota
14	3.5" Asphalt over 6" granular base	El Paso Co., Colorado
15	1.5" AC wearing course over 11.5" AC binder	Stephenson Co., Illinois
16	2" AC wearing course over 8" AC binder	Stafford Co., Kansas
17	2" AC wearing course over 6" AC binder	Isanti Co., Minnesota
18	8.8" Asphalt directly over subgrade	Wright Co., Minnesota
19	7" Asphalt directly over subgrade	Furnas Co., Nebraska
20	7" Asphalt directly over subgrade	Saskatchewan Canada
21	13" JPCP (jointed concrete) over subgrade	Butler Co., Arizona

Notes: 1) There are no case numbers 5 and 13, these were thrown out due to problems with the data.

2) Cases 11 and 12 are two different locations on the same road.

6.4 Comparisons

6.4.1 Program-to-Program Comparison

6.4.1.1 Methodology

This type of comparison is based on that used by Rada et. al. (1992) in SHRP's software selection procedure. The backcalculated results are compared for two programs at a time for the various layers in each case. Figures 6-1a through 6-5d show how each program compares to the KISS program. For each program there are charts showing the comparison for each of the layers in the pavement system. The numerical data is given in Tables for each layer and each program in Appendix 6B.

6.4.1.2 Results

Figures 6-1a through 6-5c are plotted using a log-log scale.

KISS vs EVERCALC

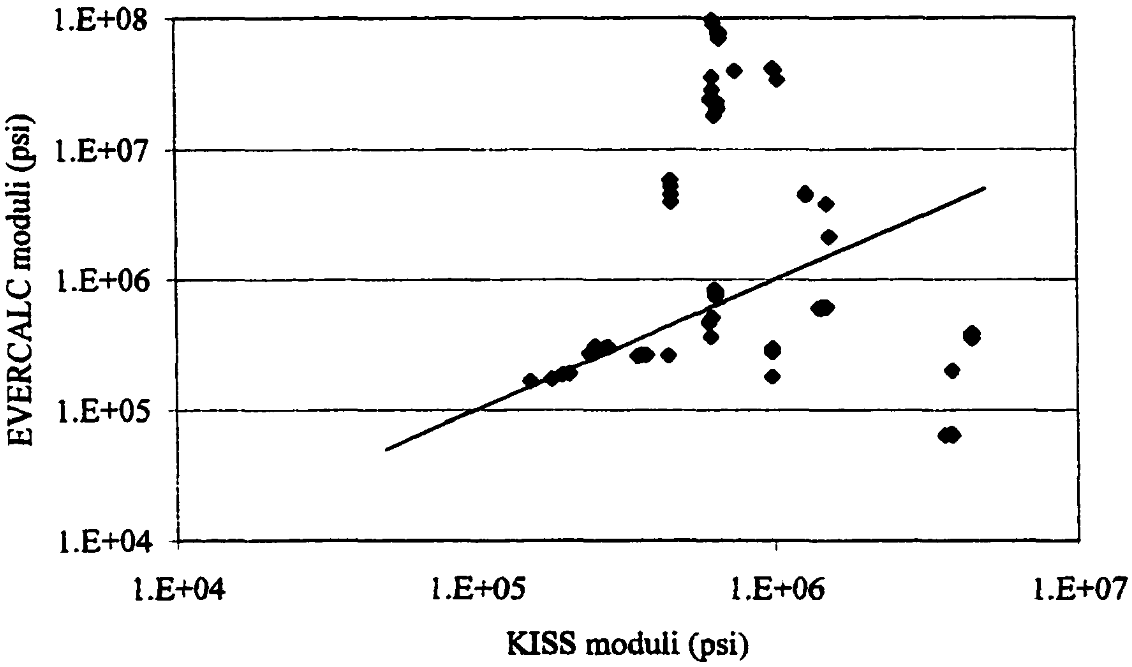


Figure 6-1a: KISS vs EVRCALC (AC Layer)

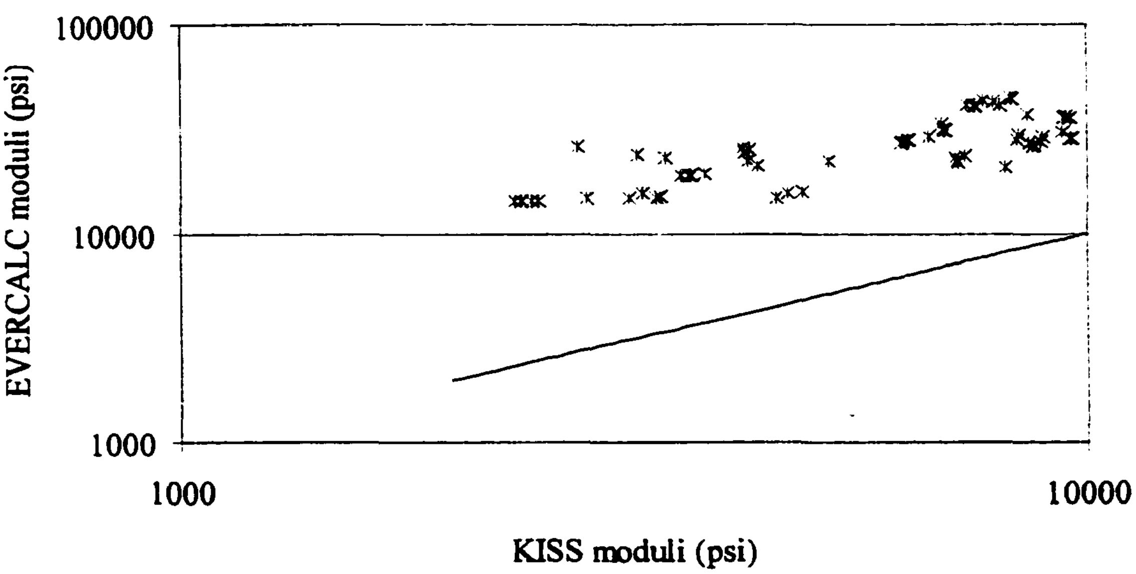


Figure 6-1b: KISS vs EVERCALC (Subgrade)

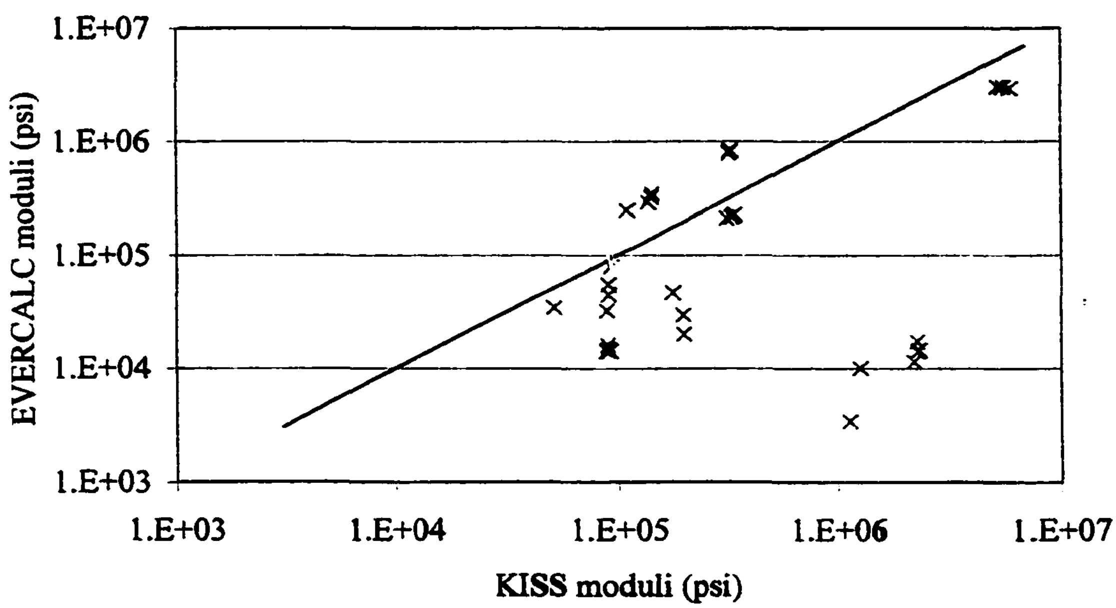


Figure 6-1c: KISS vs EVERCALC (Stabilized Base)

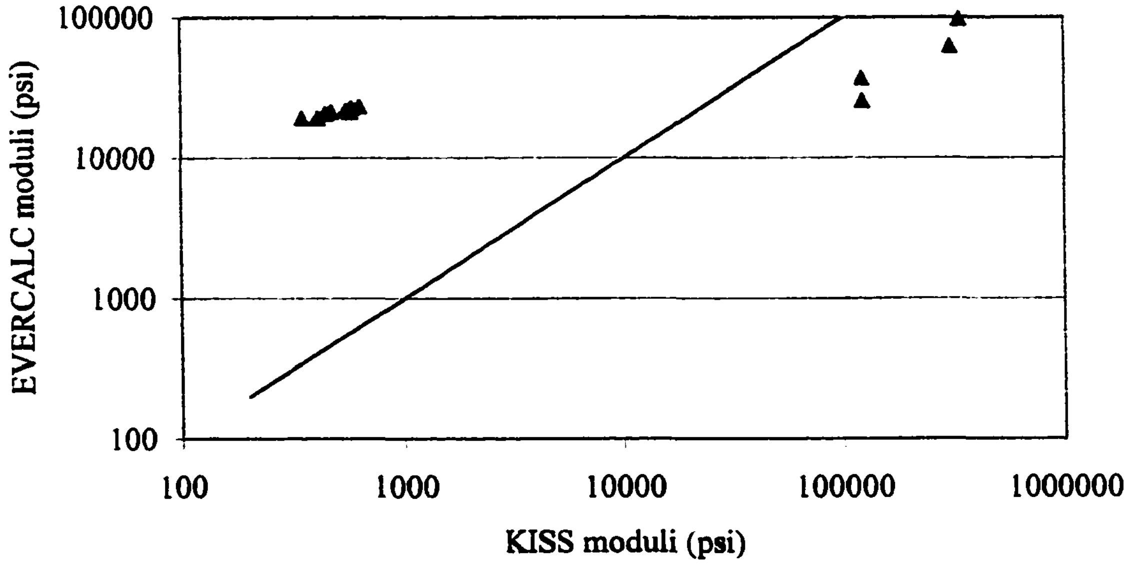


Figure 6-1d: KISS vs EVERCALC (Granular Base)

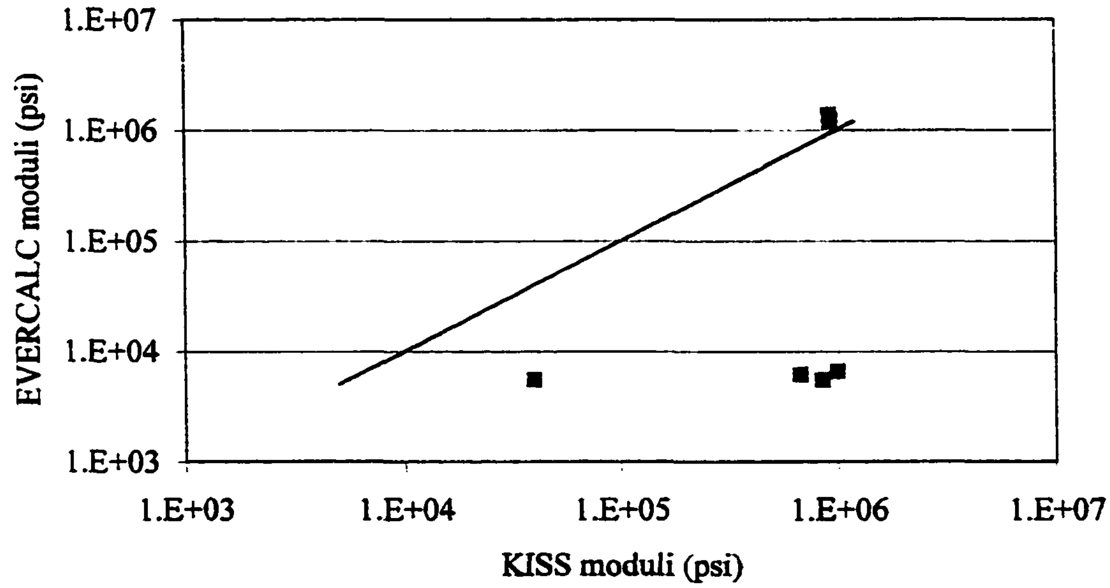


Figure 6-1e: KISS vs EVERCALC (Binder Course)

KISS vs BOUSDEF

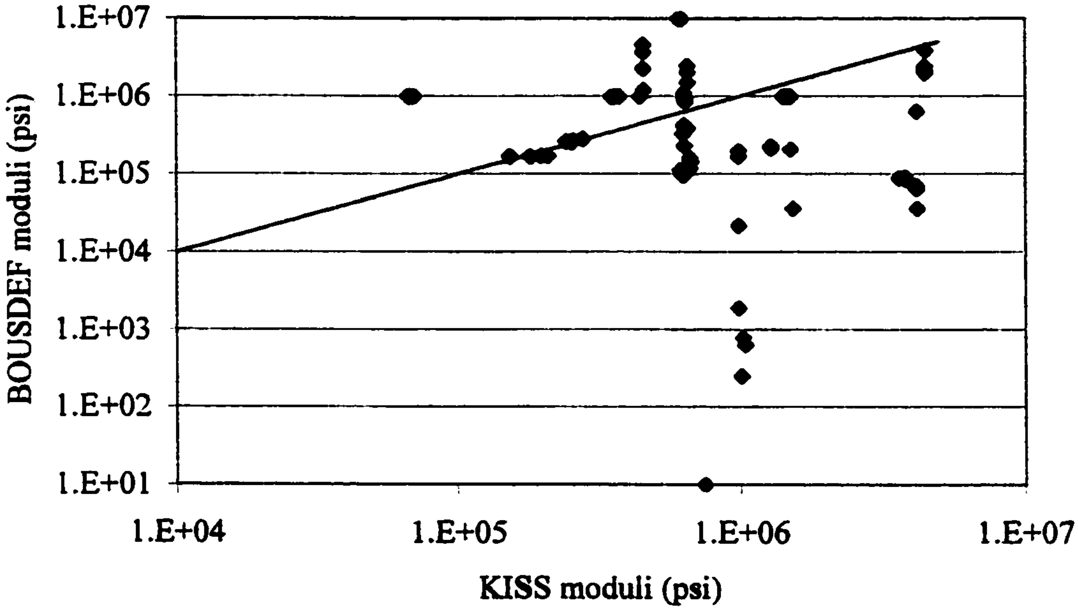


Figure 6-2a: KISS vs BOUSDEF (AC Layer)

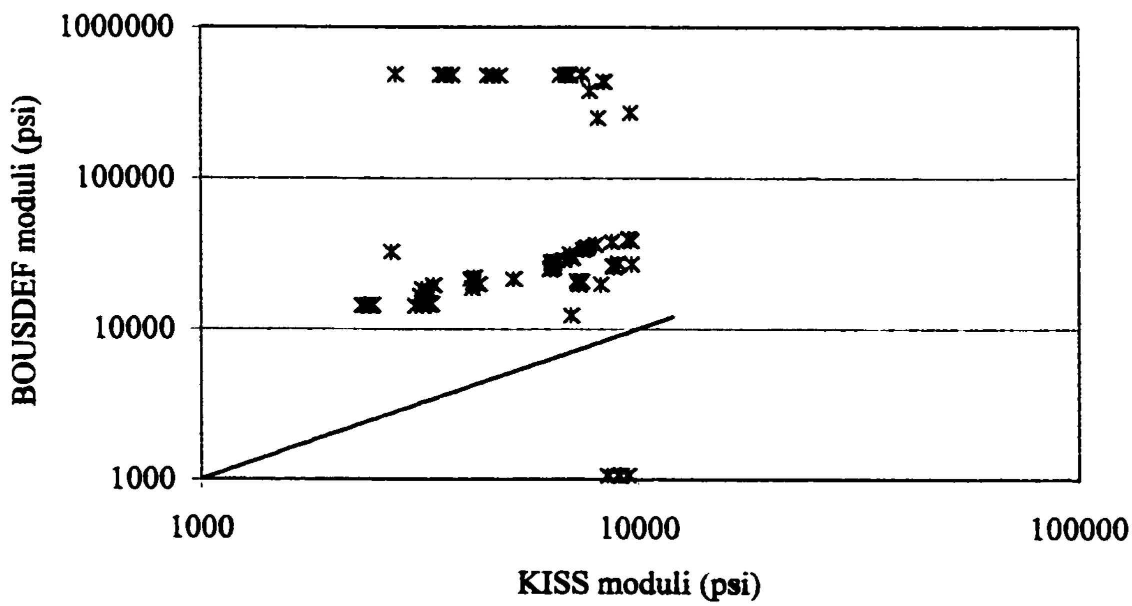


Figure 6-2b: KISS vs BOUSDEF (Subgrade)

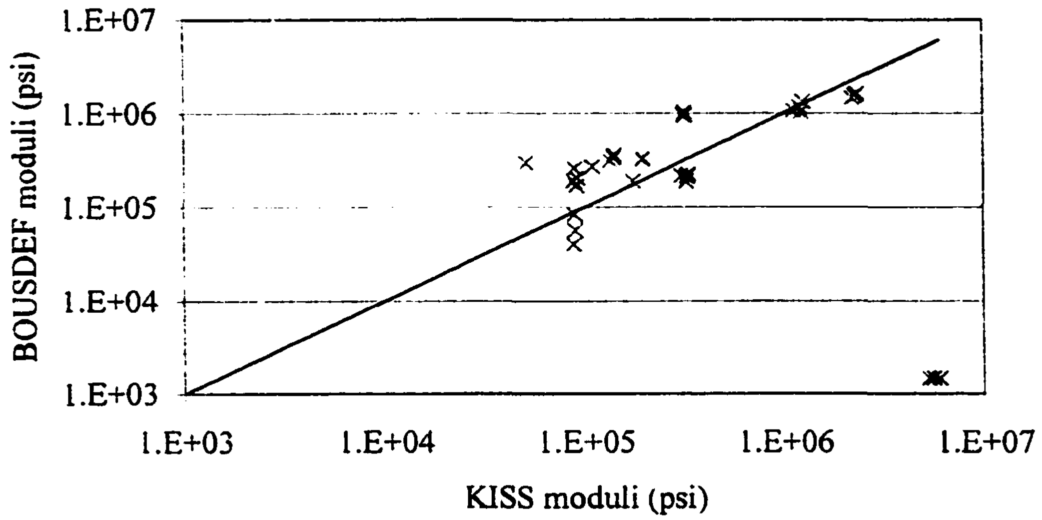


Figure 6-2c: KISS vs BOUSDEF (Stabilized Base)

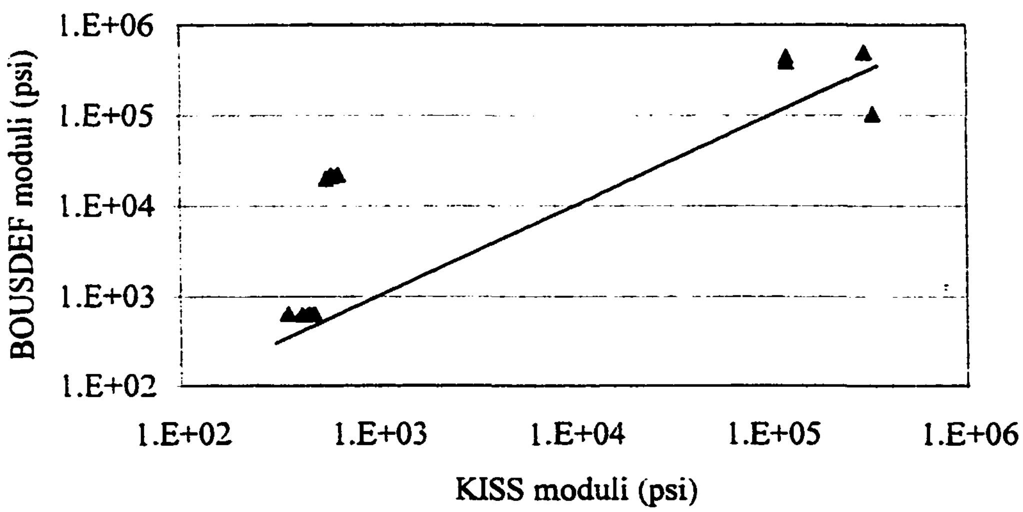


Figure 6-2d: KISS vs BOUSDEF (Granular Base)

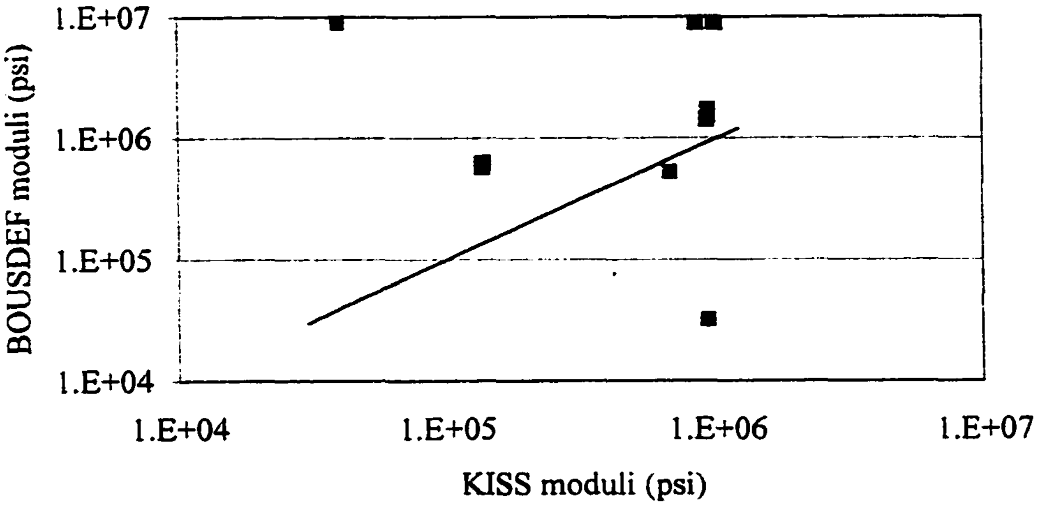


Figure 6-2e: KISS vs BOUSDEF (Binder Course)

KISS vs MODCOMP

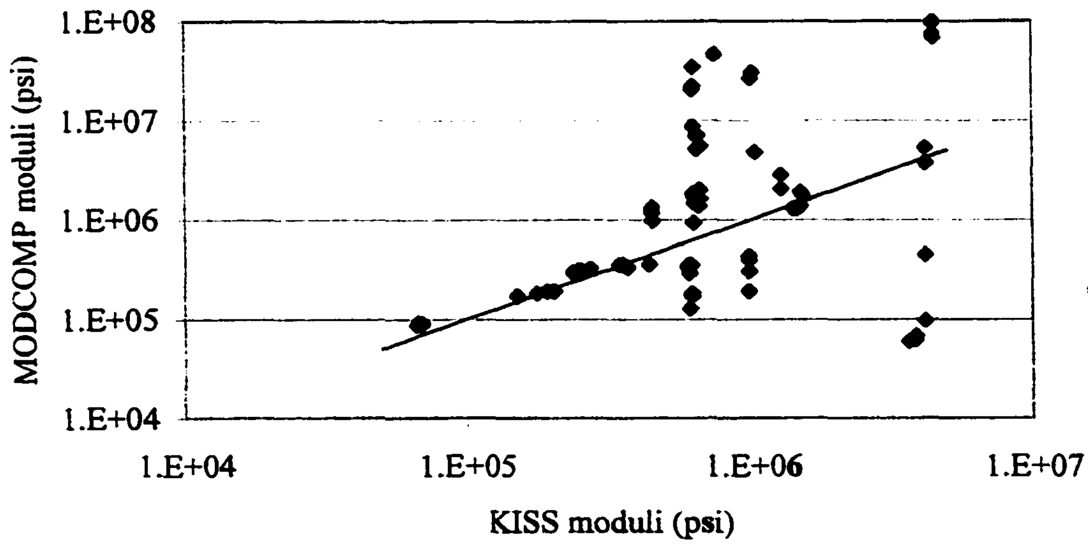


Figure 6-3a: KISS vs MODCOMP (AC Layer)

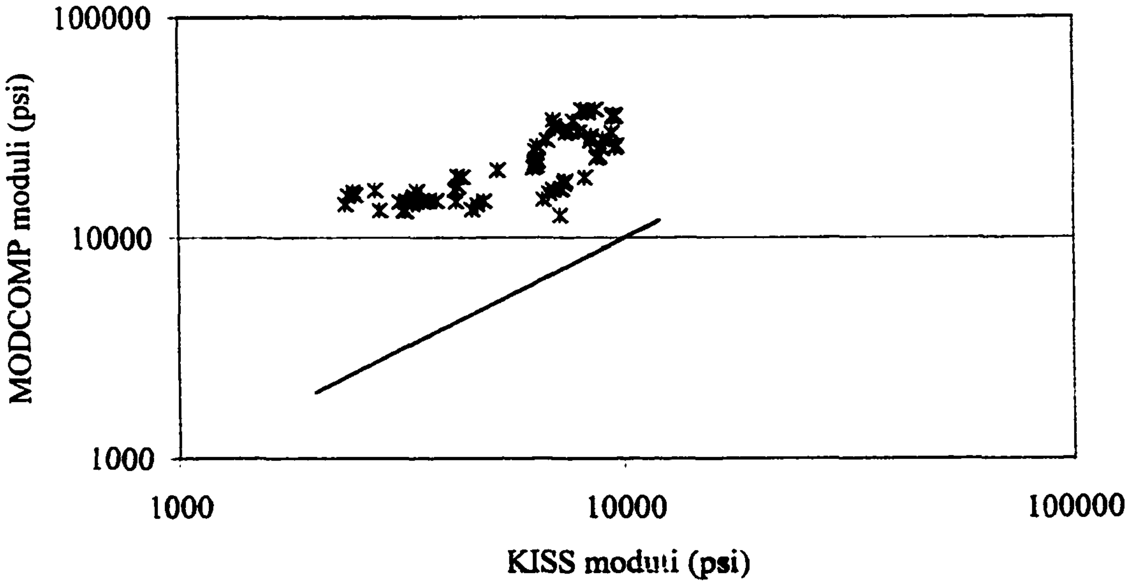


Figure 6-3b: KISS vs MODCOMP (Subgrade)

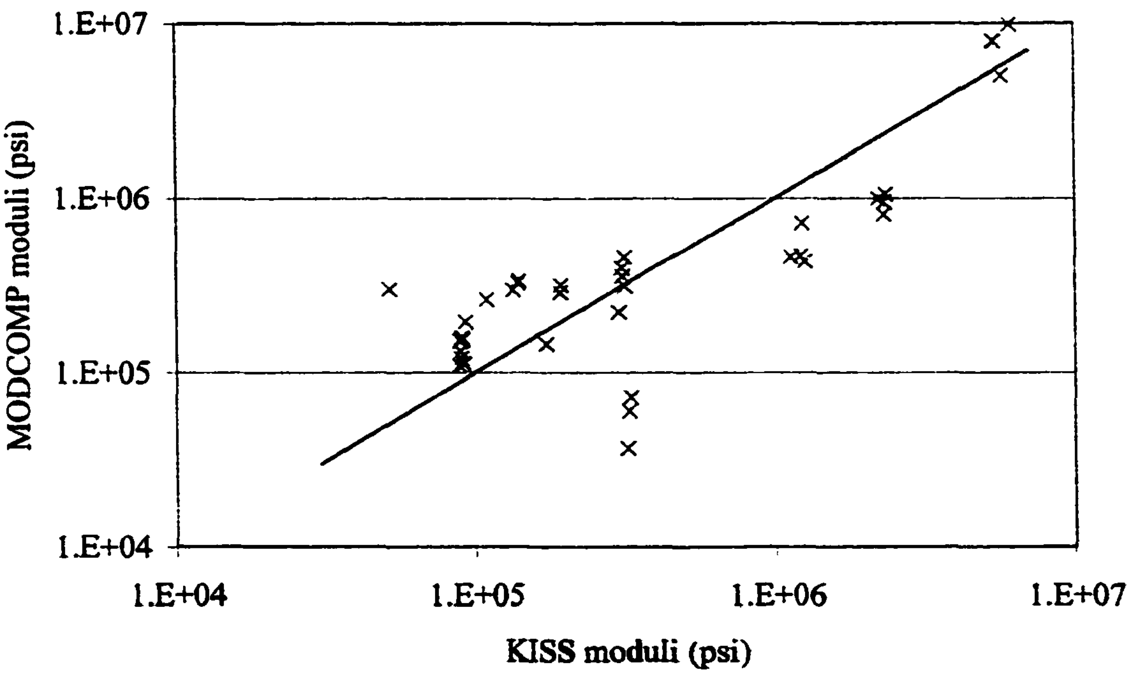


Figure 6-3c: KISS vs MODCOMP (Stabilized Base)

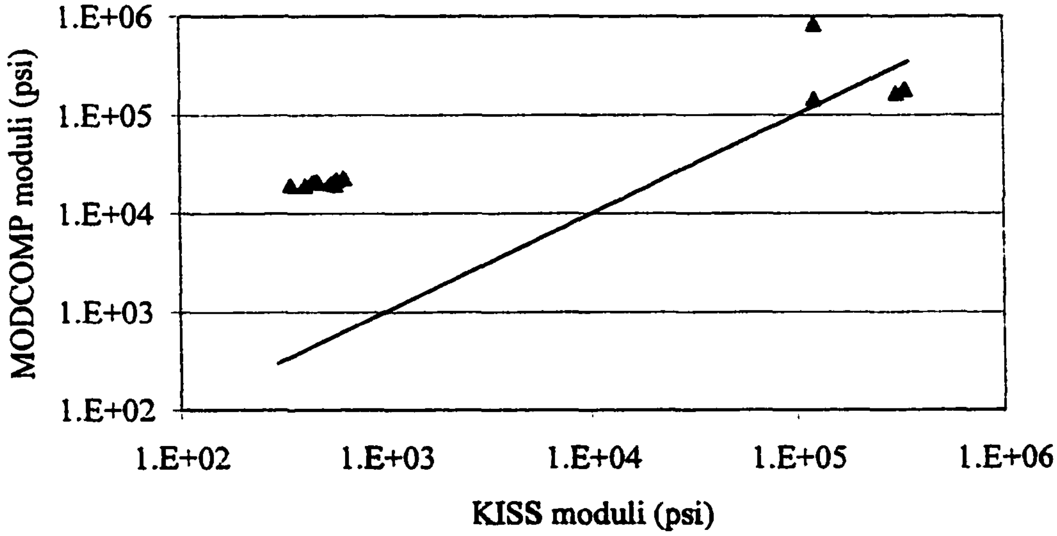


Figure 6-3d: KISS vs MODCOMP (Granular Base)

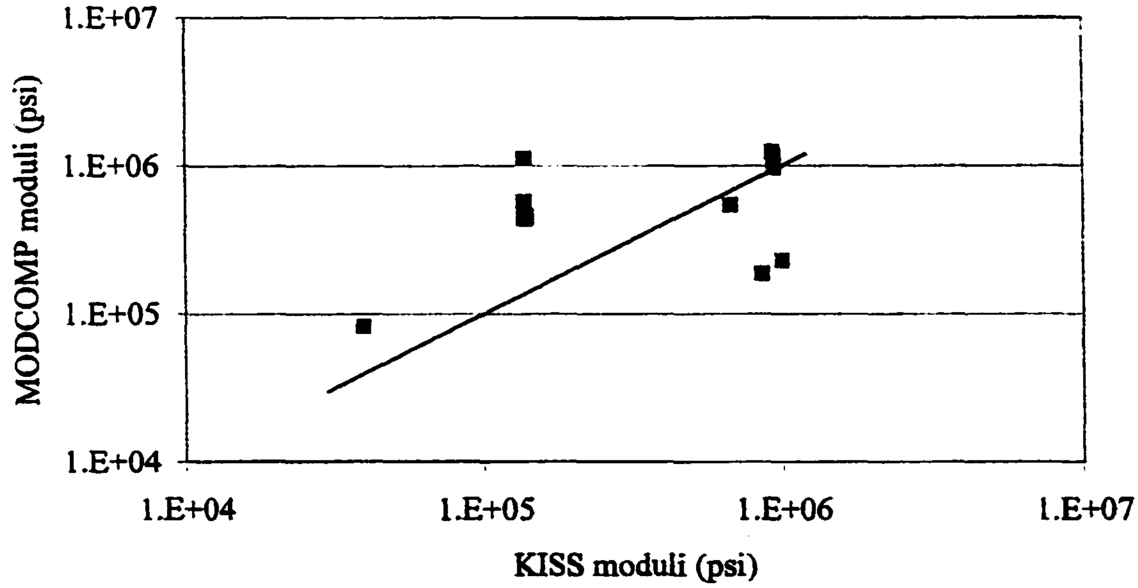


Figure 6-3e: KISS vs MODCOMP (Binder Course)

KISS vs MODULUS

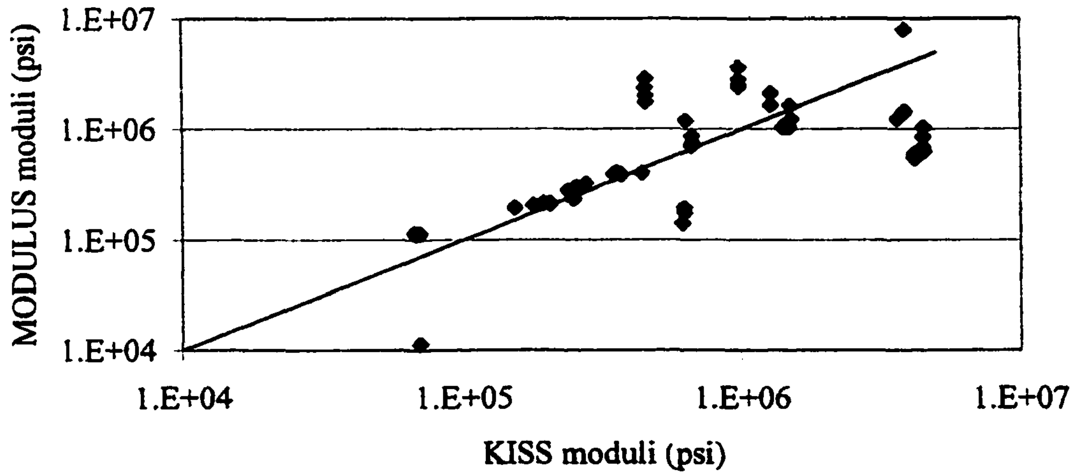


Figure 6-4a: KISS vs MODULUS (AC Layer)

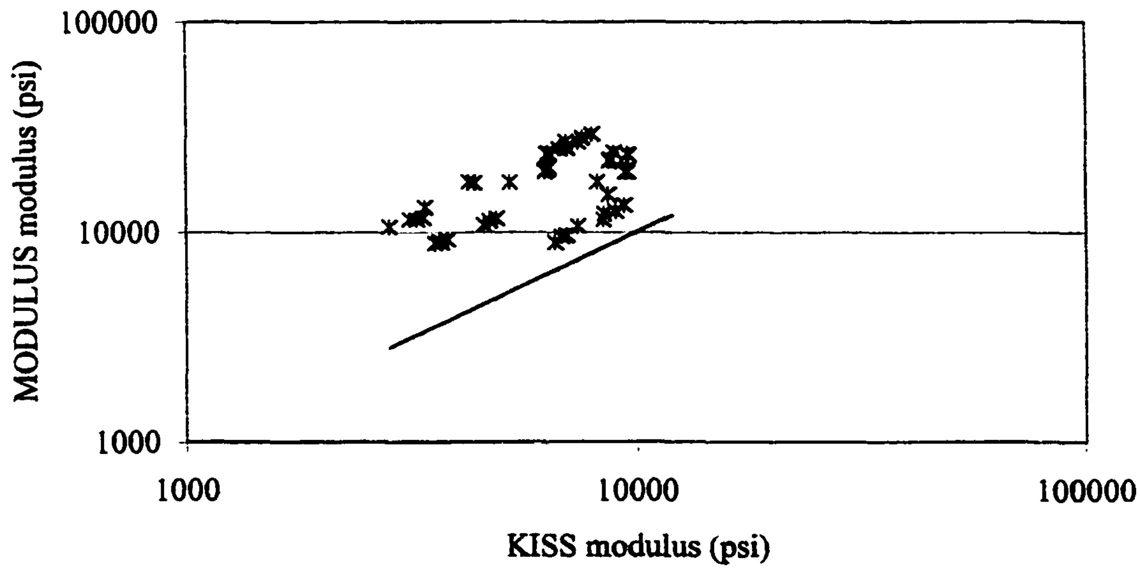


Figure 6-4b: KISS vs MODULUS (Subgrade)

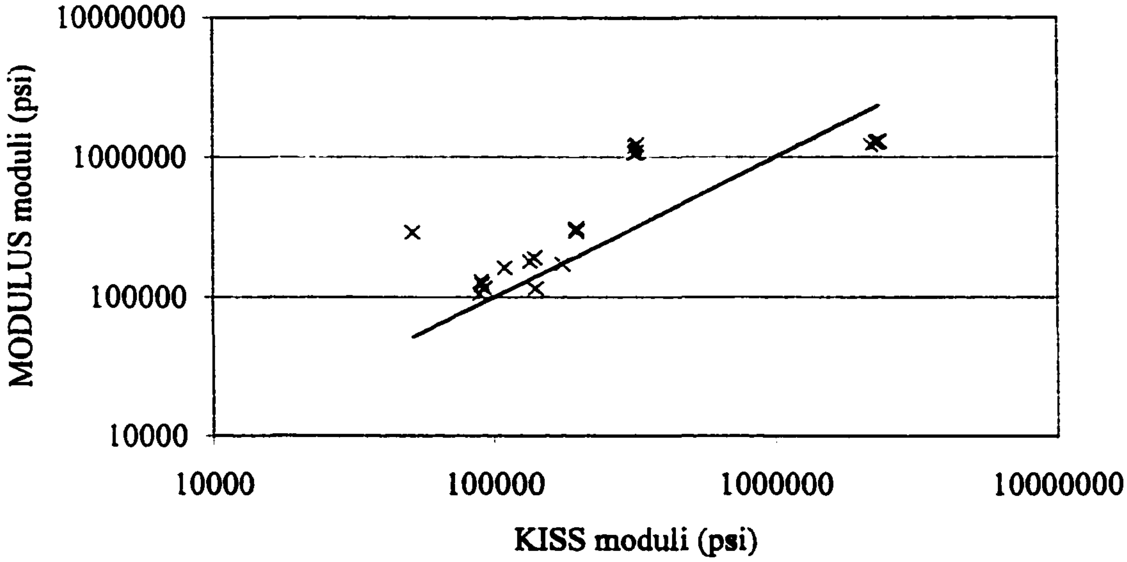


Figure 6-4c: KISS vs MODULUS (Stabilized Base)

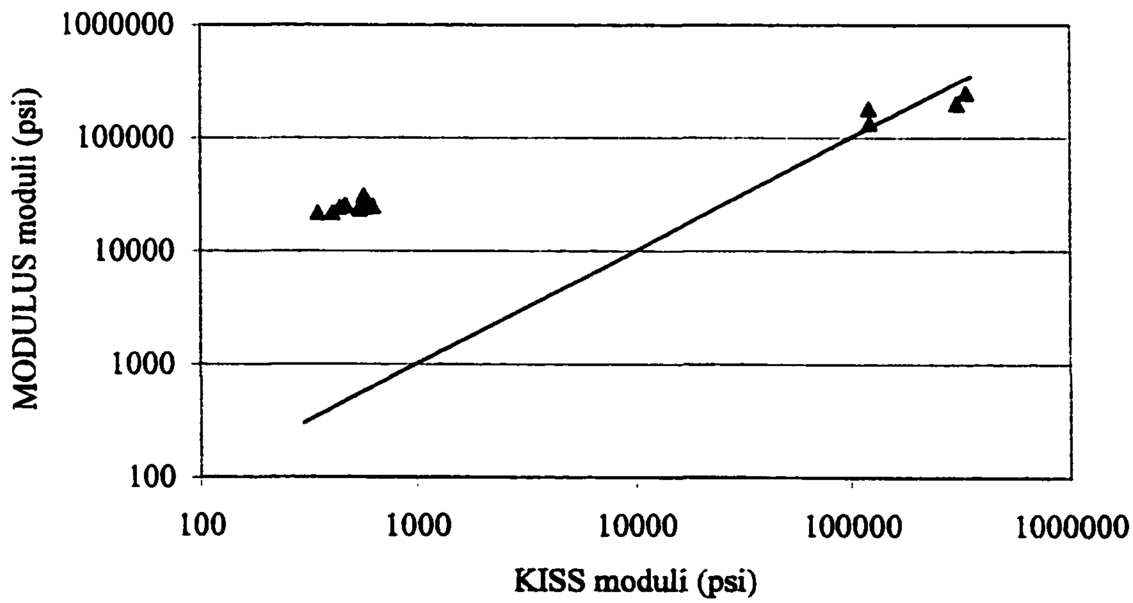


Figure 6-4d: KISS vs MODULUS (Granular Base)

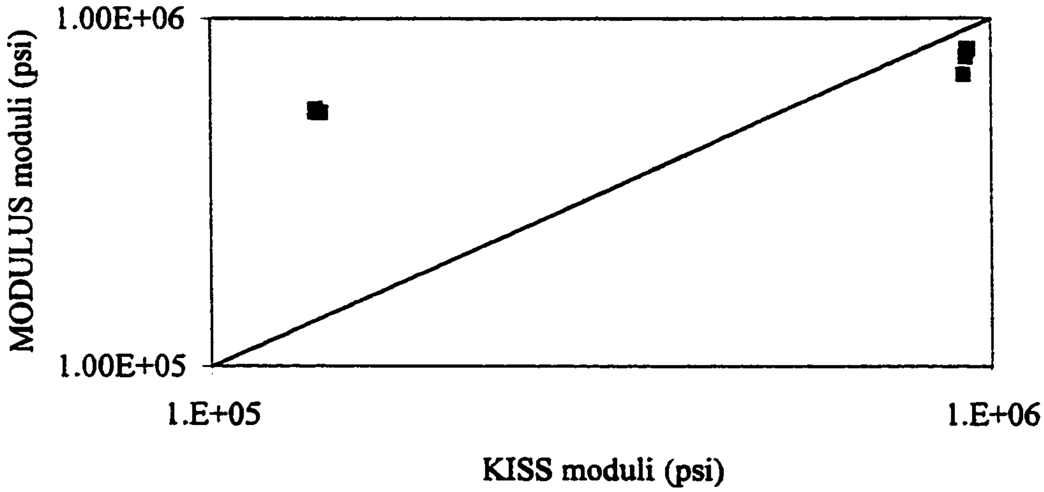


Figure 6-4e: KISS vs MODULUS (Binder Course)

KISS vs ILLIBACK

The following figures compare the KISS program to ILLIBACK modeling the subgrade as a dense liquid and elastic solid. ILLIBACK is specifically for concrete pavements so only case 21 is used in this comparison.

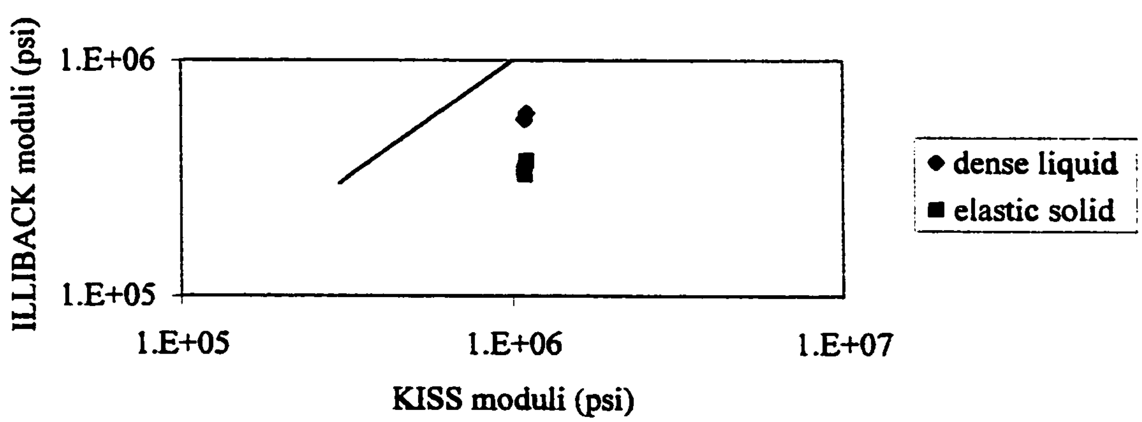


Figure 6-5a: KISS vs ILLIBACK (Concrete layer)
Both Models

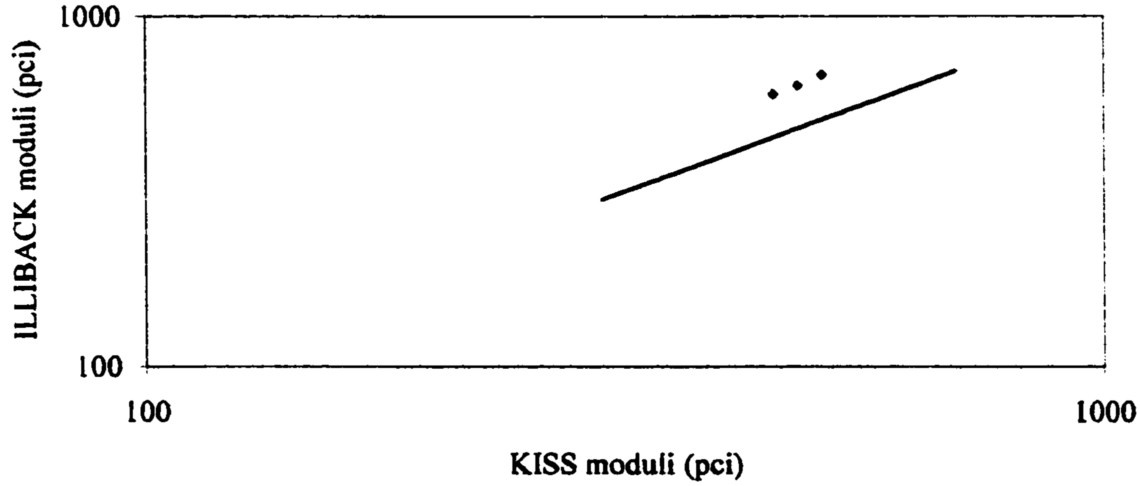


Figure 6-5b: KISS vs ILLIBACK (k Value)
Dense Liquid Model

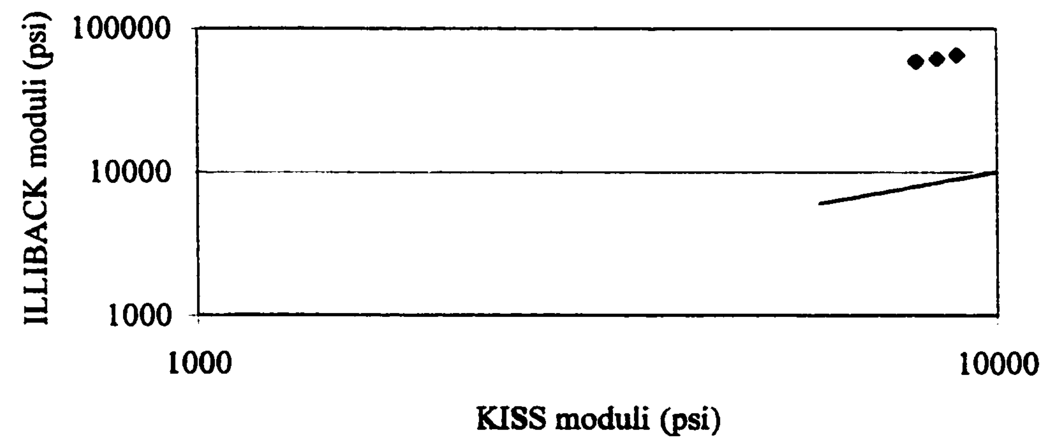


Figure 6-5c: KISS vs ILLIBACK (subgrade)
elastic solid model

6.4.1.3 Conclusions

The line drawn in each graph represents the function y = x. If the data were to line up along this line, the program results would be in perfect agreement. When the data lies below the line, the results from the KISS program are higher than those of the program being compared to. Similarly when the data lies above the line, the results from the KISS program are lower than those of the program being compared to. In the numerical results presented in Appendix 6B, a positive number in the percent difference column indicates that the results from the KISS program were lower than the comparison program. The following paragraphs discuss the conclusions drawn from observing the graphical and numerical results. The conclusions are presented for one layer at a time.

Surface layer

It is evident from Figure 6-1a that the KISS program results are typically lower than those for the EVERCALC program for the AC layer. This can also be seen in the percent difference column in the Table presented in Appendix 6B. Also in observing the Tables, it is evident that many of the AC layer moduli values backcalculated using EVERCALC were very large. In general it is expected that the actual AC layer moduli would be no larger than 2,000,000 psi. The KISS program exceeded this value in cases 1 and 4, however, the EVERCALC program exceeded this value in cases 2, 3, 8, 9, 10, 14, and 17. In several instances the EVERCALC values were larger than 10 million which is a completely unrealistic result. Although the KISS program did exceed the expected maximum in 2 of the 18 cases tested, the larger values were still within reason.

Figure 6-2a indicates that the KISS backcalculation results for the AC layer are in better agreement with the BOUSDEF program than they were for the EVERCALC program. The numbers in Appendix 6B also demonstrate this. The percent difference, on average, is much smaller between KISS and BOUSDEF than between KISS and EVERCALC. Further, there were fewer cases with unrealistically extreme backcalculated values for BOUSDEF.

The MODCOMP program had problems similar to EVERCALC in that several cases resulted in extremely large AC moduli. However there were several cases which were in very good agreement with the KISS program as can be observed in the Tables in Appendix 6B.

Of all programs in this comparison, MODULUS was in closest agreement with the KISS program for the AC layer. However, there were 5 of the 18 cases for which the MODULUS program gave no results for comparison due to program failure, these are outlined in Appendix 6B.

The ILLIBACK program is the only program for rigid pavements to be included in this comparison. The results given are for the only rigid pavement case sampled (Case 21). There were 3 load levels tested. Figures 6-5a shows the results for the Concrete layers modeled for dense liquid and elastic solid subgrades. For both models, the backcalculated modulus for the Concrete layer was consistently lower than that backcalculated by the KISS program. The ILLIBACK Concrete moduli appear to be low for a rigid pavement. The pavement in the test section was in reasonably good condition and such low moduli values were unexpected.

Subgrade layer

The comparison with subgrade layer results presented a problem. As discussed in previous chapters, the KISS program backcalculates a modulus of subgrade reaction (k) value for the subgrade while the other backcalculation programs (with the exception of ILLIBACK) backcalculate an Elastic Modulus (E) value. These are two different material properties. In Chapter 4 the methodology used for comparing the two values was discussed. Upon performing the backcalculations and making the comparisons, it appears that the methodology is not as good as I would like. The E values obtained from the backcalculated k values are consistently lower than the backcalculated E values. While this could be attributed to the theoretical differences in the backcalculation methods, it is suspected that the method of converting the k to an E is in error since the E values obtained from k's for the KISS program were consistently lower in every case reguardless of backcalculation method used. This can be observed in Figures 6-1b, -2b, -3b, -4b, and –5c (ILLIBACK with Elastic Solid subgrade model). The ILLIBACK program presented an interesting opportunity to compare the two different subgrade models. As can be seen from Figure 6-5b and the results in Appendix 6B, the k values backcalculated from ILLIBACK were higher than those obtained from the KISS program, however the differences were considerably smaller than those from the elastic solid model. Again, I believe this to be a problem with the procedure used to convert the modulus of subgrade reaction (k) to an elastic modulus (E).

Stabilized base

In the test data, cases 1 through 10 included a stabilized base. The equations used in the KISS program were developed for a pavement layer system with a stabilized base. However, there were a few cases in which the KISS program gave unexpectedly high results. These were cases 6, 8, and 9. In general relatively good agreement for the backcalculated stabilized base moduli is observed. The best agreement was found between the KISS and EVERCALC program where the average error was approximately –16%. The minus sign indicates that the EVERCALC results were numerically smaller than the KISS results. Error for these comparisons is as defined in the Tables in Appendix 6B.

Granular base

Granular base layers were included in cases 11, 12, and 14. The backcalculated granular base layer moduli from the KISS program were unexpectedly low for cases 11 and 12. From Tables 6B-4, 9, 14, and 19 in Appendix 6B the granular base layer moduli values from the other programs appear to be reasonable. The KISS results vary considerably from the other programs for granular base moduli. This could indicate that the equations used in the KISS program are not valid for granular bases as they were developed for stabilized bases. It is believed however that if a wider range of values for base moduli had been included in the data used to develop the equations, they would not have changed by a great deal. In other words these equations could possibly be used for granular bases but it has not been verified. Such verification would involve reestablishing the equations used in the KISS program with moduli values that include lower range values such as those expected from a granular base. However, the expected range of values is based on historical experience from backcalculations which, without any possibility of field verification, have become accepted range of values. So it is possible that the low values obtained by the KISS program, while outside the expected values, may be physically correct, there is unfortunately no way to verify one way or the other.

Asphalt binder course

Again, the KISS program was not written for a pavement system with an asphalt binder course. However, the backcalculated values for this layer from the KISS program were more realistic and worth comparison. Cases 15, 16, and 17 of the test data included an asphalt binder course.

The program EVERCALC was not able to produce results for case 15 due to program error, case 16 results were reasonable and case 17 results were unexpectedly low. BOUSDEF results were reasonable for cases 15 and 16 but were high for case 17 where the program got stuck on the upper limit value. MODCOMP gave reasonable results for all three cases. The results from the MODULUS program seemed low, but were within reason. For all of these programs, the percent difference from the results of the KISS program varied widely and comparison was difficult.

6.4.2. Case-by-Case Comparison

6.4.2.1 Methodology

This section compares all flexible pavement programs at once. The results are plotted for all cases of similar pavement systems. The Rigid pavement case is plotted separately because only the ILLIBACK program was used for this case.

The plots for the cases of similar pavement systems are presented below. The data for the individual cases are presented in Appendix 6C.

The cases used in this comparison are summarized in Table 6-1 with details in Appendix 6A.

6.4.2.2 Results

Cases 1 through 10 are all two-layer pavement systems with stabilized bases. The plots presented here do not include extreme values. Modulus values for AC greater than 2,000,000 psi are considered extreme and have been eliminated. Values for a stabilized base have been limited to 100,000 psi and the subgrade values were also limited to 100,000 psi. These limitations gave a reasonable range of solutions and generated plots which are easier to read. The following figures show the ranges of solutions (within the givin limits) with the average tick marked.

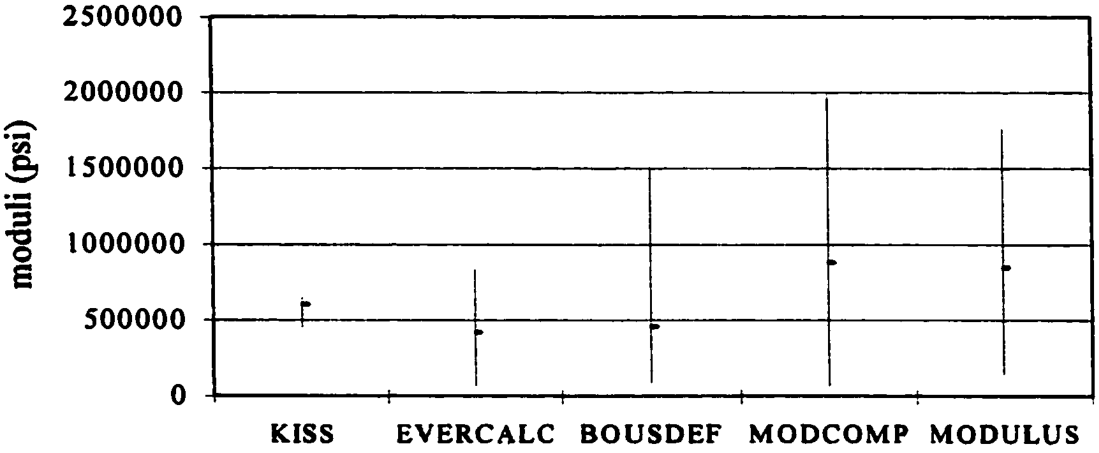


Figure 6-6a: Cases 1 to 10 (AC Layer)

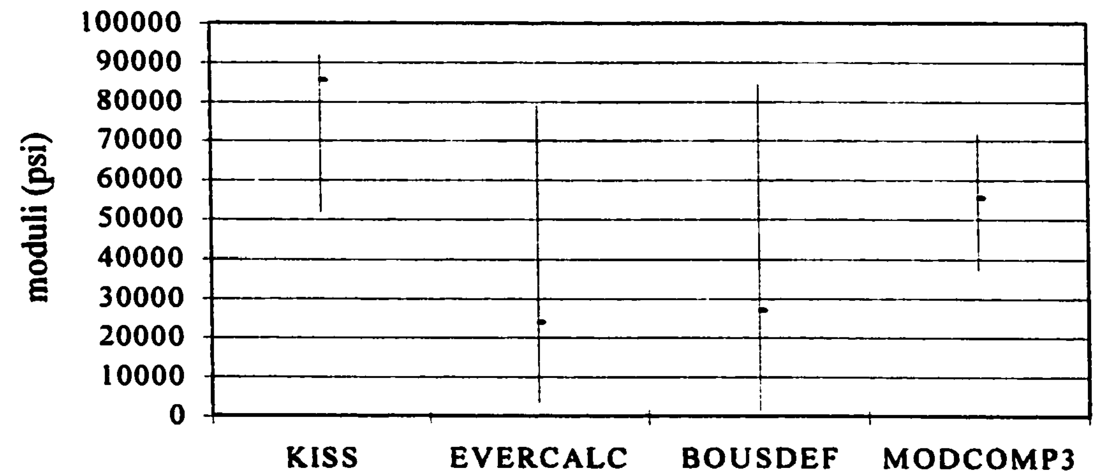


Figure 6-6b: Cases 1 to 10 (Stabilized Base)

Note: There were no results from the MODULUS program that fell within the reasonable range of solutions.

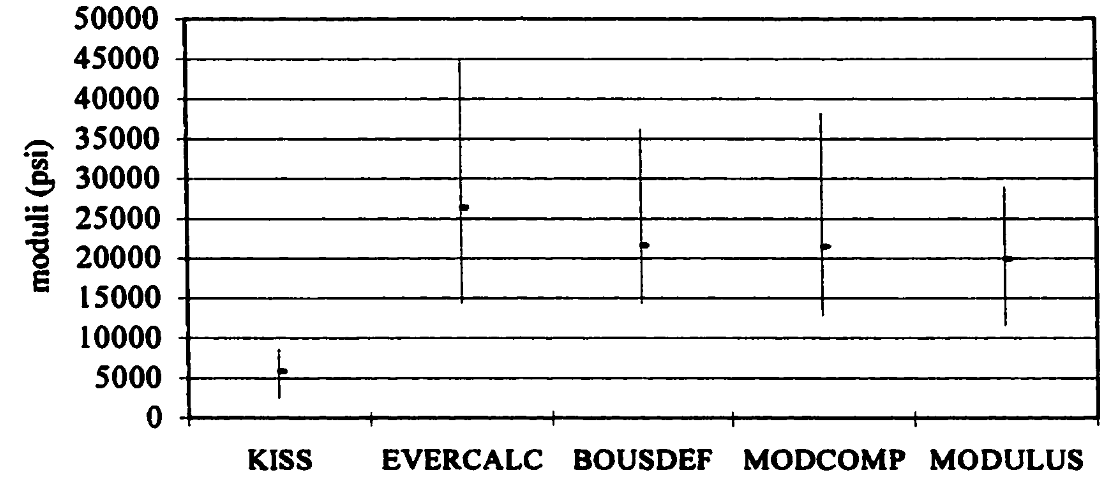


Figure 6-6c: Cases 1 to 10 (Subgrade)

Cases 11, 12, and 14 are two-layer systems with granular bases. The same limiting values were placed on moduli as described above, the granular base layer was limited to 70,000 psi.

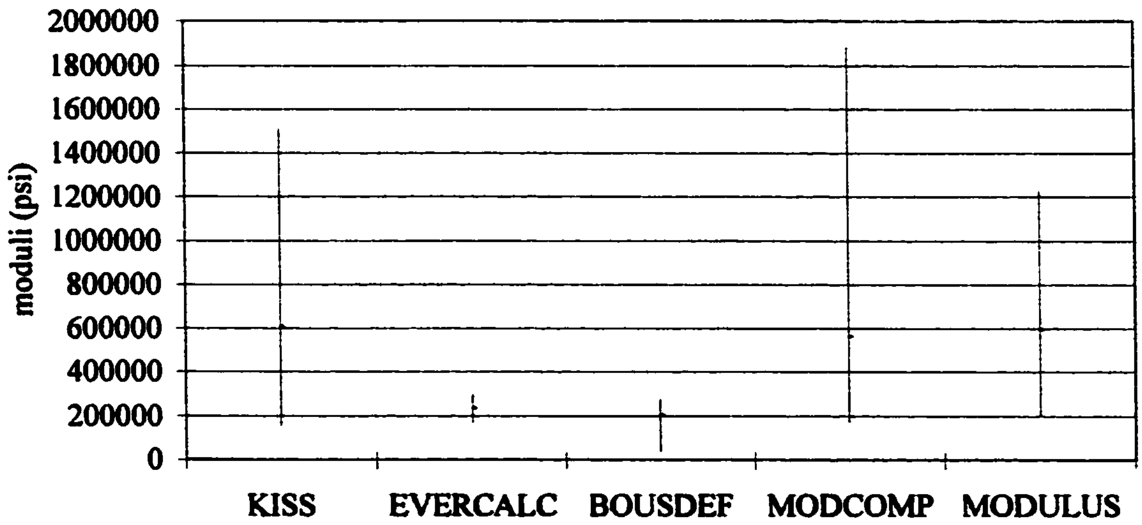


Figure 6-7a: Cases 11 to 14 (AC Layer)

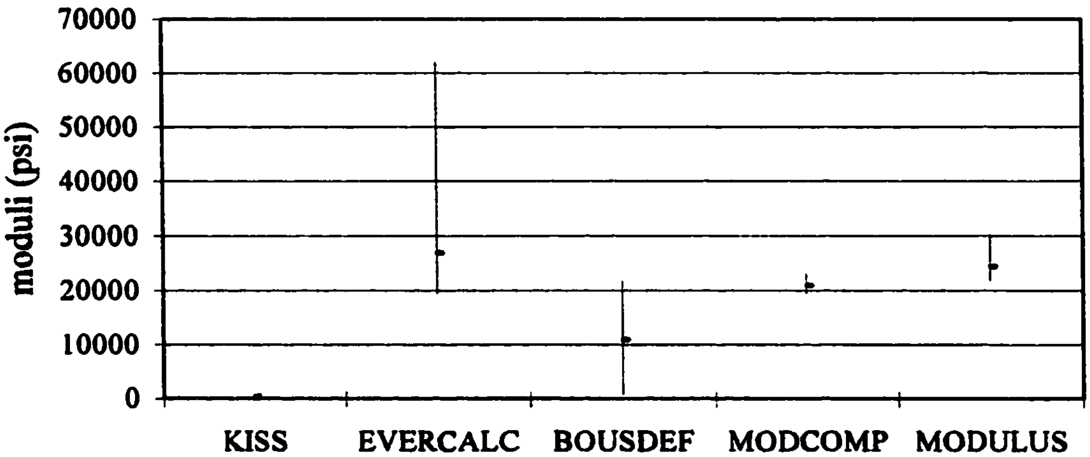


Figure 6-7b: Cases 11 to 14 (Granular Base)

Note: for this data set the range of values from the KISS program was 650 to 350 psi with an average of 503 psi. Due to the larger values from the other programs the KISS results do not show up well on the above figure.

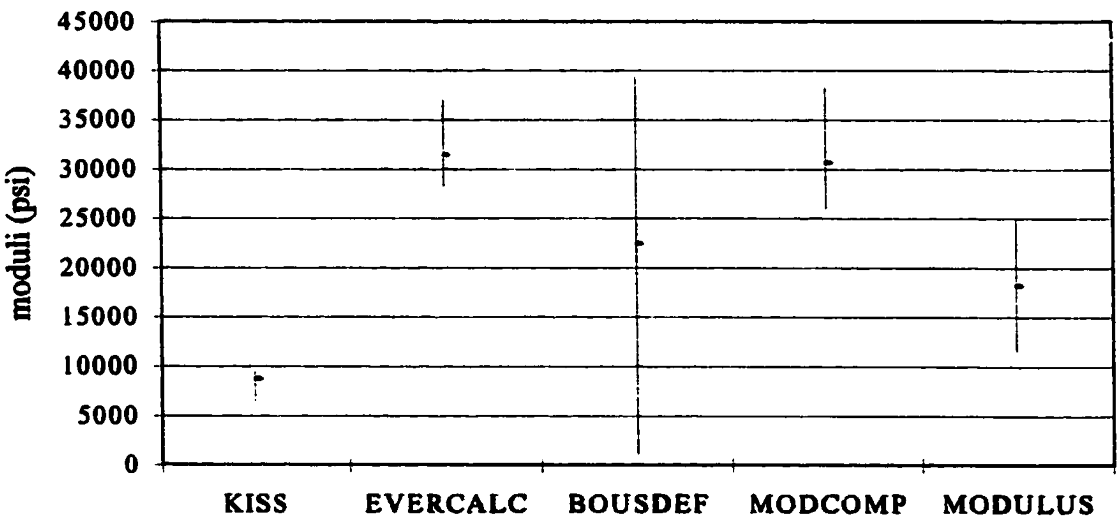


Figure 6-7c: Cases 11 to 14 (Subgrade)

<u>Cases 15, 16, and 17</u> **include an Asphalt wearing course over a thicker Asphalt binder coarse.**

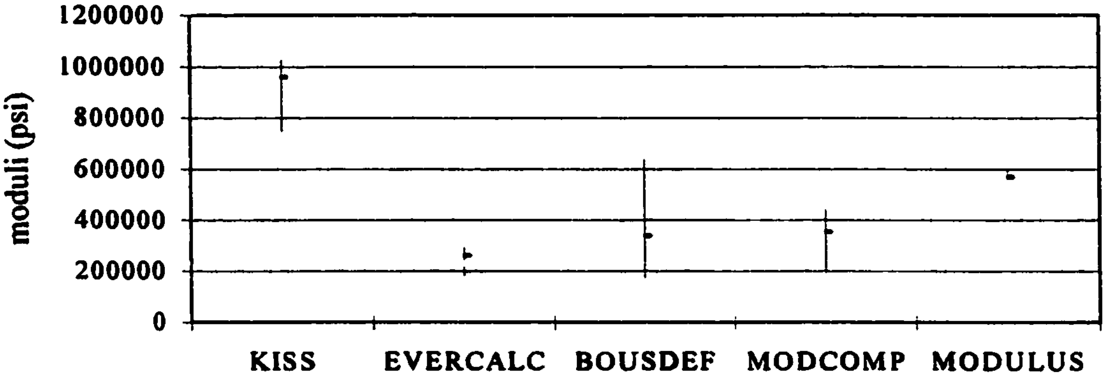


Figure 6-8a: Cases 15 to 17 (AC Wearing Course)

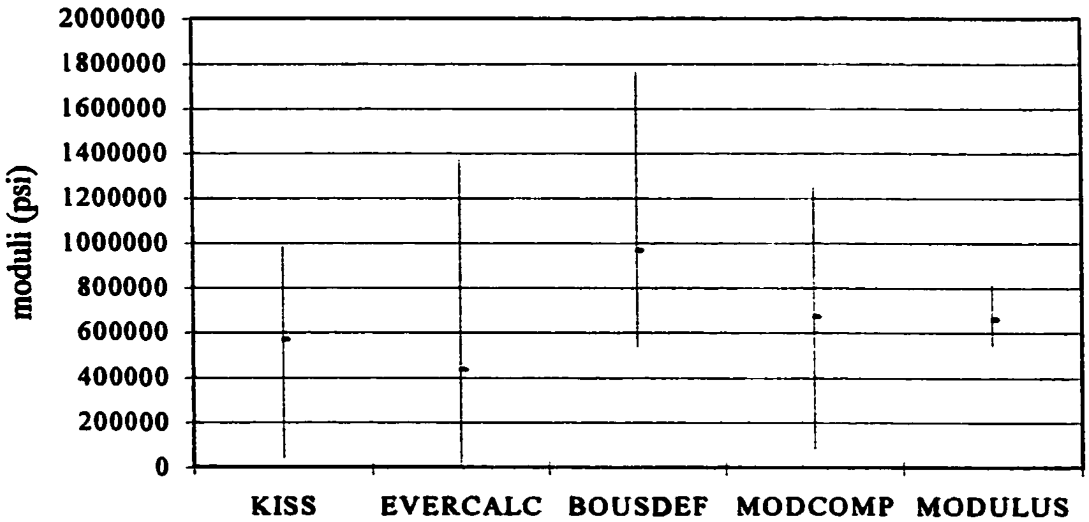


Figure 6-8b: Cases 15 to 17 (AC Binder)

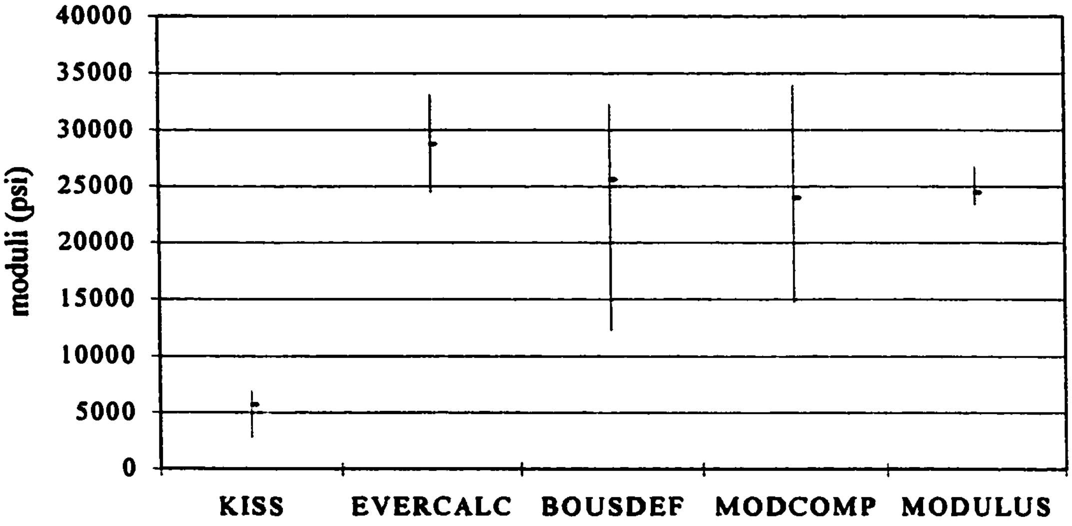


Figure 6-8c: Cases 15 to 17 (Subgrade)

Cases 18, 19, and 20 are comprised of a one-layer system of asphalt over a subgrade.

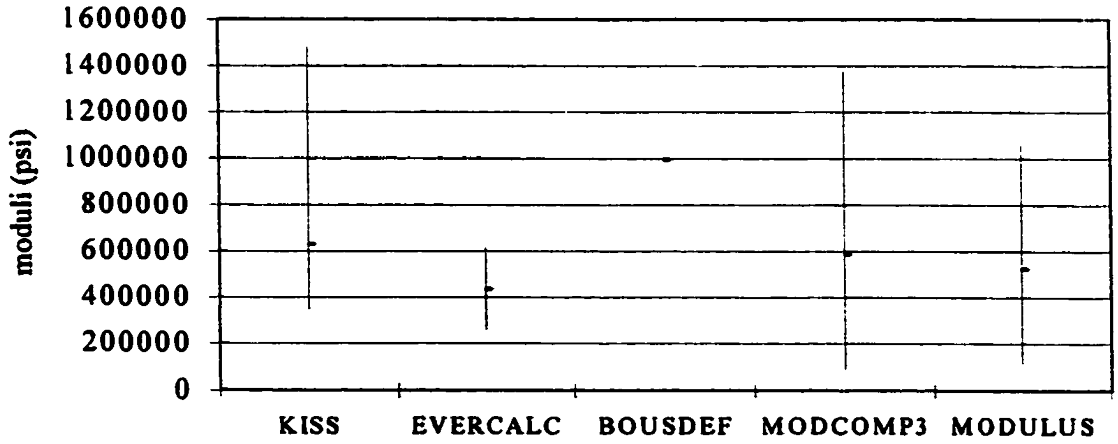


Figure 6-9a: Cases 18 to 20 (AC Layer)

Note: for this data set the range of values from the BOUSDEF program was 998779 to 995797 psi with an average of 997206 psi. Due to the small range of values these results do not show up well on the above figure.

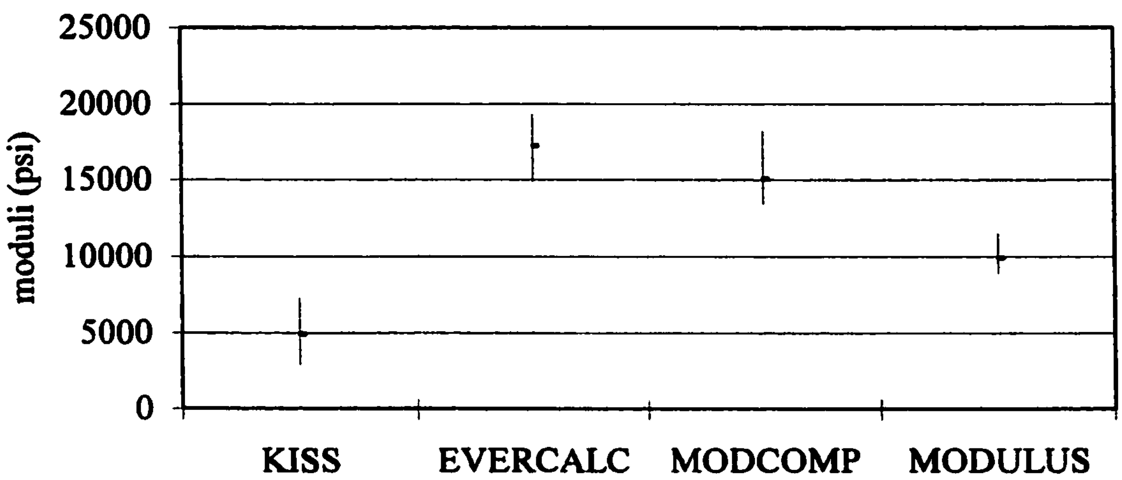


Figure 6-9b: Cases 18 to 20 (Subgrade)

Note: There were no results from the BOUSDEF program that fell within the reasonable range of solutions.

Case 21 is a concrete pavement over a subgrade. As discussed in Section 6.3.1.3 ILLIBACK was run in two different modes, one using the dense liquid foundation model (k value) and one using the elastic solid foundation model (E value). The following Figures compare AC moduli for both models with the KISS program and then compare the subgrade results by converting the results from the elastic solid model to a k value as described in chapter 4 of this paper. The comparison could have been made by converting the k values from the KISS program and dense liquid model into E values, the resulting figure would be the same, only the numbers would have differed by a fixed amount.

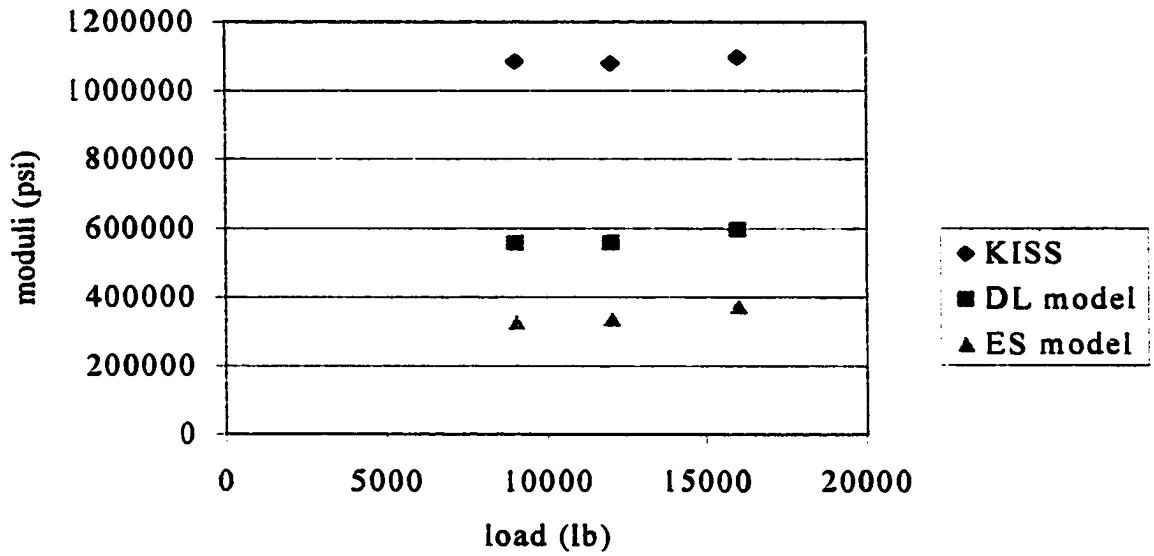


Figure 6-10a: Case 21 (Concrete Layer)

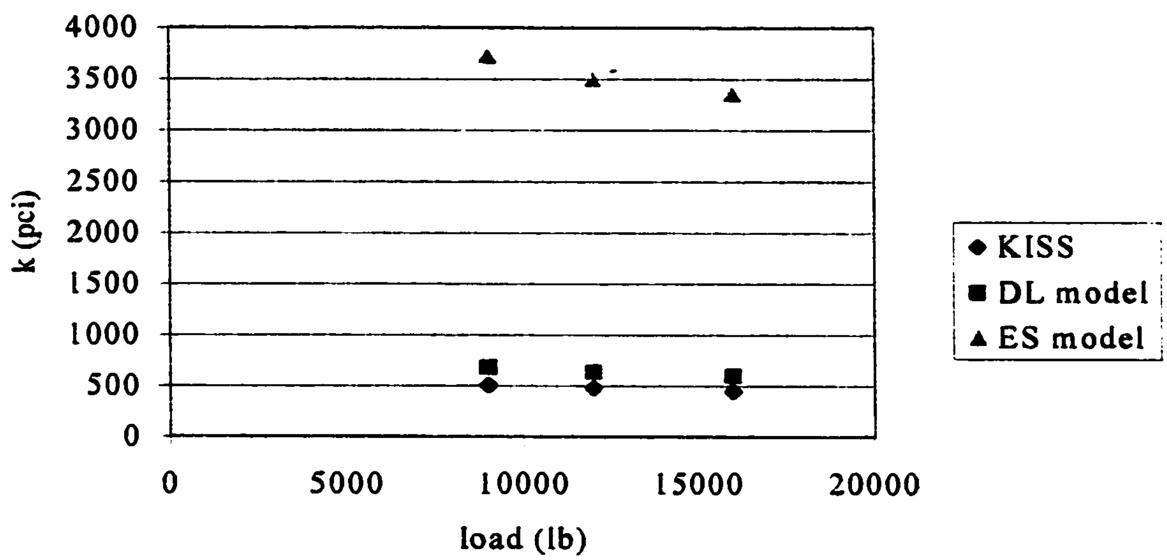


Figure 6-10b: Case 21 (k for Subgrade)

6.4.2.3 Conclusions

There were a number of difficulties encountered in working with the different programs for this comparison. Some were technical and some theoretical. The limitations on moduli values were placed because in some instances the programs gave extremely unrealistic values. As can be seen from the Figures above and the data in Appendix 6C, in some cases much or even all results from a given program were thrown out. There were instances where a program would get consistently result in the upper or lower limit regardless of how low or high the limit was made. There were instances when a program would not give any result at all. There were also complications when a program would cause the computer to lock-up. Despite these difficulties, comparisons can be made and are discussed in the following paragraphs.

Cases 1 to 10: Asphalt over a stabilized base.

In the AC layer, many of the results had to be thrown out because they were higher than the 2,000,000 psi limit. It is noted that some of the results from EVERCALC and MODCOMP were as high as 100,000,000 psi! The average modulus from these two programs was extremely high. When the values greater than 2,000,000 psi were removed, the Table in Appendix 6 C reveals that the average moduli values were lower than expected for most of the programs. In particular, the KISS program consistently obtained AC moduli values in the 650,000 psi range which seems low. The MODULUS program obtained an average closest to expected moduli values. It should be noted however that MODULUS failed to give results for many of the cases tested.

For the stabilized base layer results many of the results from all programs were above the 100,000 psi limit. All results from the MODULUS program were above this limit. This tells us that either our limit is unrealistically low, or that none of the programs work very well for a stabilized base. Unfortunately it is difficult to know which is the case.

The subgrade results from all programs were more reasonable than for the other layers. Only the BOUSDEF results from case 6 were thrown out. However, MODULUS did not give results for four of the ten cases due to program failure. In reviewing Figure 6-6c and the averages in Appendix 6 C, it is evident that the results from the KISS program were consistently lower than those of the other programs. This is suspected to be a problem in the conversion from k to E rather than a backcalculation problem. The other programs compare quite well on the average with the EVERCALC program obtaining slightly higher subgrade moduli than the others.

Cases 11 to 14: Asphalt over a granular base.

In the AC layer, again the results were lower than expected. This could be the result of faulty expectations or faulty programs, it is difficult to determine which. The average for the KISS program was similar to that for cases 1 to 10, however, consistency is not necessarily a good thing here, it is expected that the AC layer moduli for these different sections of road in different geographical locations should vary a bit more than the KISS results demonstrate.

Nearly one-third of the results from all programs for the granular base layer exceeded the 70,000 psi limit and were therefore discarded. The average for the KISS program was significantly lower than the other programs. As mentioned before, the KISS program was specifically developed for stabilized bases, the equations used to relate $\overline{D}$ to the layer moduli are invalid for these cases. BOUSDEF also obtained some surprisingly low results however all other programs performed reasonable well for the granular base layer.

All programs performed acceptably well for the subgrade layer for cases 11 to 14. Most programs resulted in a relatively wide range of subgrade moduli values. However the KISS program results fell into a narrow range of values nearly all between 8000 to 10,000 psi. Again, it is difficult to interpret this lack of variance as a good or bad sign. In general, the other programs results were higher than the KISS program for the subgrade layer, this phenomenon is discussed in the previous section.

Cases 15 to 17: AC wearing course over an AC binder.

A wide range of results were obtained for the wearing and binder courses for these cases. There were extremes of both low and high moduli values as demonstrated in Tables 6C-7 through 6C-9 in Appendix 6C. Again it should be pointed out that the equations used in the KISS program were not developed for this type of pavement system. New equations for each type of pavement system would have to be developed before reasonable results could be expected from the KISS program. In practice for a system such as this, the two AC layers would be combined and a moduli backcalculated as though they were a single layer. For comparison purposes and out of curiosity, the layers were treated separately here.

As with the previous three types of pavement systems, the programs performed reasonable well for the subgrade layers with the KISS results being significantly lower.

Cases 18 to 20: AC over a subgrade.

For these one-layer cases the KISS program is theoretically non-pluripotent meaning the solution is mathematically unique. All of the KISS program results for these three cases were within reason, but again, it is impossible to know if they match the actual field moduli values. Some of the results seem low but not out of the question. Further, the results for all four load levels are consistent. The remaining programs also performed reasonably well but the average results vary with BOUSDEF being high and EVERCALC being low.

For the subgrade layer, all BOUSDEF results were in the 480,000 psi range and were thrown out. The other programs performed acceptably with KISS being the lower result, which is consistent with previous findings.

Case 21: Portland Cement (rigid) Concrete over a subgrade.

In the concrete layer, Illiback results were significantly lower than the KISS results with the Elastic Solid model results being considerably lower than the Dense Liquid model.

In comparing the subgrade moduli, the results were the same whether k was converted to E or vice versa. For comparison purposes here, the E's for the Elastic solid model were converted to k values using the method described in Chapter 4 of this paper. From Figure 6-10b it can be seen that the KISS program and the Dense Liquid model were in good agreement with the Elastic Solid model resulting in a much higher k value. Again this could be a conversion problem rather than a backcalculation problem. On the average, the Dense Liquid model resulted in k values that were about 200 pci higher than those from the KISS program. Again it is impossible to know which is the more correct value.

6.4.3 Comparison Based on Calculated Theoretical Deflections

6.4.3.1 Methodology

The difficulty in using real test data to compare these programs should by now be obvious. Since it is impossible to know the actual layer moduli, there is no basis for comparison and I can only compare one theoretical value to another without knowing which is closer to the actual field value.

To eliminate this problem, three different methods were used to generate deflections for eight theoretical pavement systems with known moduli values. Elastic layer theory was used generate deflections via the computer program ELSYM5. Composite plate theory was used in the form of a computer program written by the author called DEFBOWL to generate deflections. Lastly the finite element program SAP 2000 was used to generate deflections. For the SAP 2000 program, the plate was modeled as a composite plate over an elastic foundation. For the first three theoretical cases, the plate was modeled both with and without simply supported edges. However it soon became obvious that it made little difference in the resulting deflections so the remaining cases were done only with simply supported edges.

The eight theoretical pavement systems and the resulting theoretical deflections used in this comparison are outlined in Appendix 6D. These are the same eight cases used in Chapter 4 for deflection comparisons and are summarized below in Table 6-2. These surface deflections were input into each of the backcalculation programs, the resulting moduli values along with some observations about the programs are included in Appendix 6E.

Table 6-2: Cases for Comparison for a Two-Layer System.

Case	Es psi	Eb psi	h1 in	h2 in	k pci	Esg psi	plate ftxft
1	500000	150000	3	6	100	1920	24x24
2	1000000	250000	3	6	150	2880	24x24
3	1500000	500000	3	6	100	1920	24x24
4	1500000	500000	2	6	100	1920	24x24
5	1500000	500000	4	6	100	1920	24x24
6	1500000	500000	3	10	100	1920	24x24
7	1500000	500000	3	18	100	1920	24x24
8	1500000	500000	3	6	100	1920	50x50

6.4.3.2 Results

Figures 6-11 a through c show the results for all programs for the surface, base, and subgrade layers. Extreme values have been removed. For the surface and base layers, all backcalculated values greater than 5,000,000 psi were removed. For the subgrade layer, all backcalculated values greater than 30,000 psi were removed.

These figures show the ranges of results for each method of calculating deflections with the actual modulus value indicated by a horizontal line.

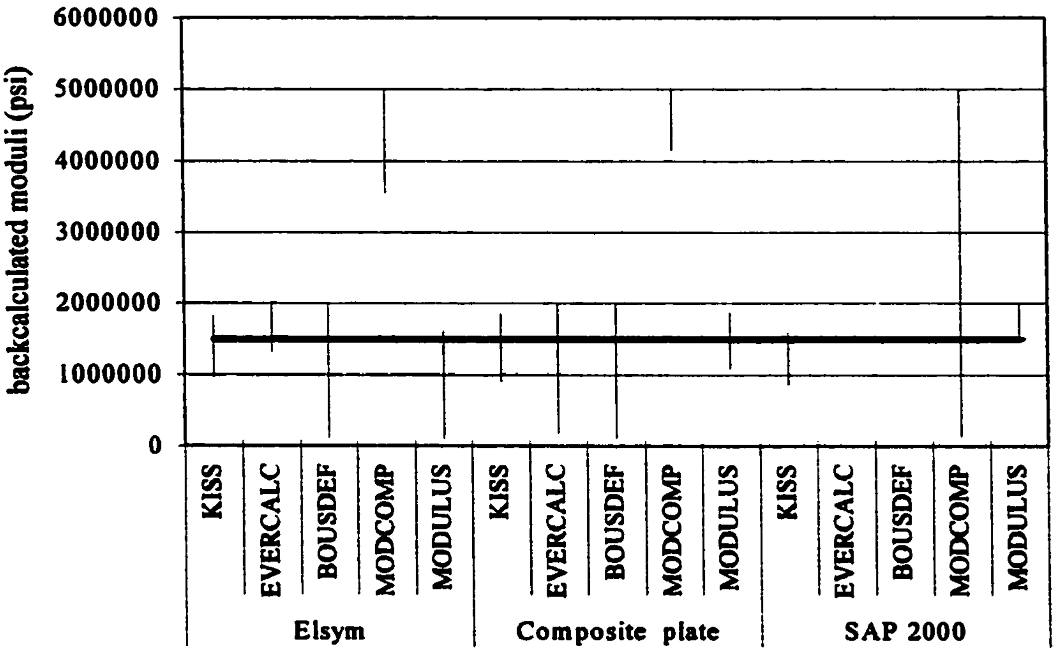


Figure 6-11a: Surface Layer Moduli

Figure 6-11a can be deceiving, for example several of the programs appear to have done well like MODULUS and BOUSDEF using SAP 2000 deflections, however, for each run each of these programs resulted in the fixed upper limit of 2,000,000 psi. Such anomalies are not evident from the Figures and can only be observed from the Tables of results in Appendix 6E. Only Cases 3 through 8 are included in Figures 6-11a through c because they all had the same initial moduli values for AC, base, and subgrade against which to compare. Cases 1 and 2 used different initial moduli.

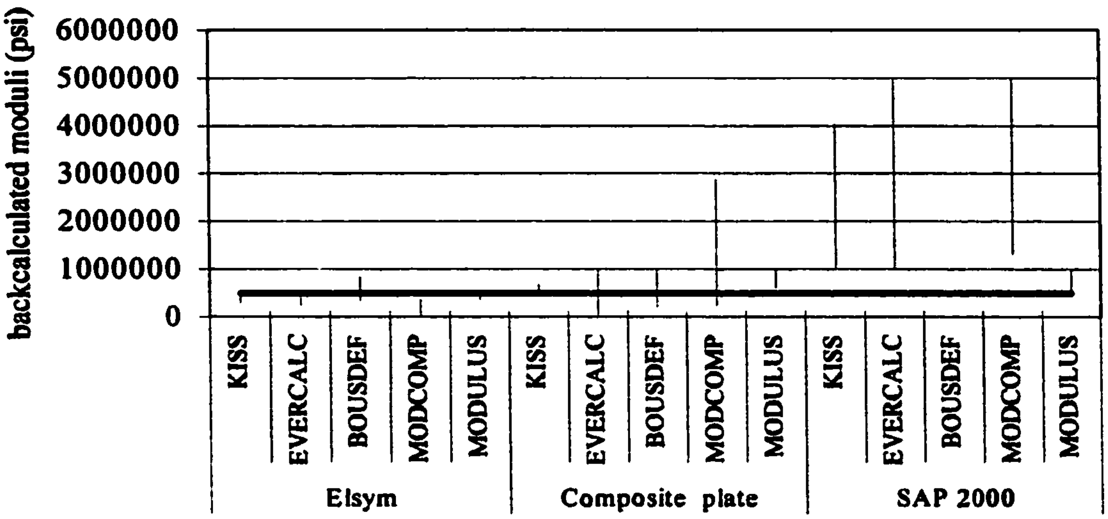


Figure 6-11b: Base Layer Moduli

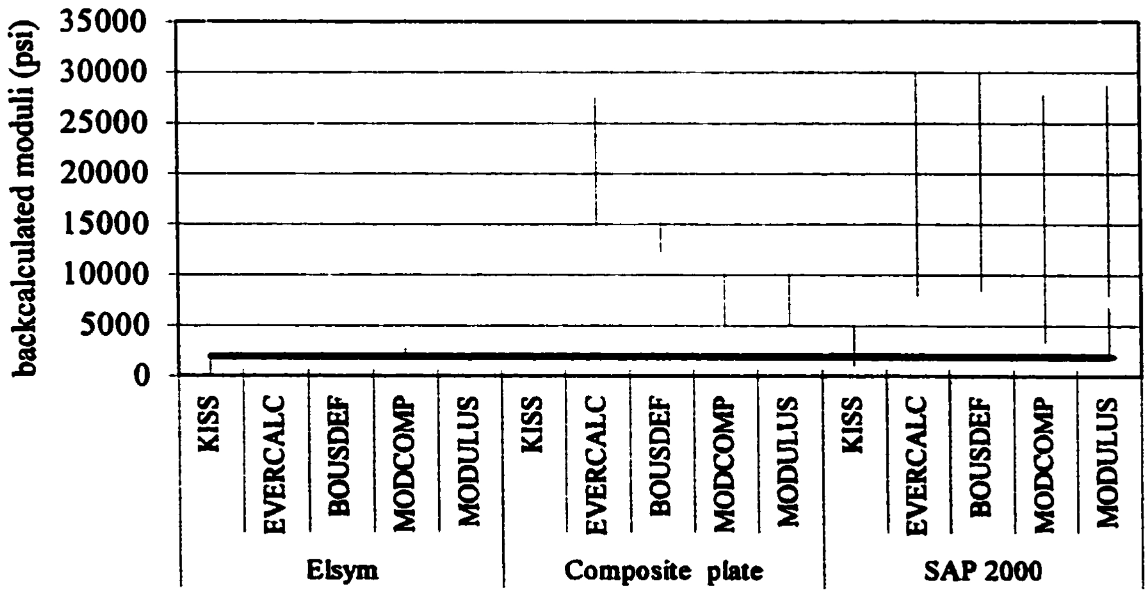


Figure 6-11c: Subgrade Moduli

The average absolute percent error was calculated as follows

$$\text{avg. abs. \% err.} = \frac{\sum_{i=1}^{n}\left|\frac{E_a - E_b}{E_a}\right|\cdot 100}{n}$$

where E_a = the actual modulus value used to calculated the deflections

E_b = the backcalculated modulus

n = the number of moduli values

This was done for each program and each method of determining deflections. For example, all results from the KISS program for AC moduli obtained using ELSYM deflections are used to determine the first entry in Table 6-3a. Tables 6-3 a through c give the average absolute percent error for each program with each type of deflection for each layer. The tables include all backcaluclated results, including the extremes that were omitted from the Figures. Appendix 6E contains all resulting errors.

Table 6-3a: Average Absolute Percent Error for Surface Layer

	Program				
deflection source	KISS	EVERCALC	BOUSDEF	MODCOMP	MODULUS
Elsym	25	65	55	370	25
Composite plate	26	49	69	2802	36
SAP w/ simple support	29	39	46	332	36
overall average	27	50	58	1072	33

Table 6-3b: Average Absolute Percent Error for Base Layer

	Program				
deflection source	KISS	EVERCALC	BOUSDEF	MODCOMP	MODULUS
Elsym	39	34	32	503	42
Composite plate	21	82	75	168	108
SAP w/ simple support	487	391	183	589	183
overall average	243	181	122	421	134

Table 6-3c: Average Absolute Percent Error for Subgrade Layer

	Program				
deflection source	KISS	EVERCALC	BOUSDEF	MODCOMP	MODULUS
Elsym	89	84	7	33	20
Composite plate	0	1507	656	220	232
SAP w/ simple support	54	554	594	342	417
overall average	48	664	403	196	217

6.4.3.3 Conclusions

It is evident from the figures and tables that there were a wide variety of results. It should be pointed out that for some of the programs, the upper limits were what kept the errors from becoming too large. For example for the surface layers, the upper limit was set at 2,000,000 psi for EVERCALC, BOUSDEF, and MODULUS, thus limiting the error. This was not possible for MODCOMP, which explains the extreme errors.

For the surface layer, the KISS and MODULUS programs had the least error. MODCOMP had extreme errors, there were a lot of problems running this program as discussed in the notes in Appendix 6E. For the base layer, again MODCOMP showed the most extreme errors. The other programs (KISS, EVERCALC, BOUSDEF, and

MODULUS) showed moderate errors with more extreme errors occurring for the deflections generated by the Sap program.

Several of the programs did very well calculating subgrade moduli using ELSYM deflections. This is to be expected as ELSYM is based on Elastic Layer theory as are most of the programs. The extreme errors obtained by MODCOMP indicate there is a problem with the program.

Also of interest is that the KISS program performed with 0% error in determining the Subgrade moduli using deflections determined via Composite Plate theory. Recall that the subgrade modulus (k value and hence E3 value) is unique for the KISS program. So it is not a surprise to get good results when the deflections are generated using the composite plate theory. Unfortunately this means little other than that the theory agrees with itself.

6.5 Overall Conclusions

As mentioned previously there are inherent difficulties in comparing different computer programs. The following is a listing of some possible reasons for the differing results from a paper by Chou and Lytton (1991):

1. The numerical routine used to calculate pavement surface deflections may be different.
2. The method of searching for new values of the layer moduli may be different.
3. Some methods try to correct for the stress dependency of the layer moduli; others do not.

4. Criteria for determining convergence may be different.
5. Moduli ranges set by individual analysts may be different.

There are many other factors that could be added to this list such as dependence on seed values, some programs determine a depth and stiffness for a rigid layer, technical problems with the programs, user error,... The list could go on and on.

Basically the only conclusions that can be made are about how one theory compares to another. Even in comparing to theoretically "known" moduli, I am comparing how the programs perform for the different theories used to calculate the deflections. Very little can be concluded about how these programs compare to reality.

Chapter 7

7.1 Summary and Conclusions

The two main objectives of this research were first to investigate existing backcalculation techniques and the theories behind them for pluripotence, second to develop a backcalculation technique that addresses the pluripotence issue and attempts to overcome it.

The first objective was accomplished in Chapter 2 of this paper where it was demonstrated that most existing backcalculation programs do have pluripotence issues of some sort. Each program was analyzed and an attempt was made to determine how pluripotence is evaded.

The second objective is the subject of Chapter 3. An in-depth discussion of Composite Plate Theory and the development of the computer program KISS is described. In this Chapter the concept of E_{nb} (no base) is described and the development of equations relating E_{nb} to E_s (surface modulus) and E_b (base modulus) is described. These equations are an integral part of the backcalculation procedure developed.

Chapters 4 through 6 are dedicated to verifying and validating the KISS program. Chapter 4 analyzes the deflection equations used for Composite Plate theory and compares them to some popular deflection equations used in backcalculation today. The deflections were found to follow similar patterns but differences in magnitude were noted. It is expected that these differences are largely due to the differences in assumptions between the various theories behind the different deflection equations. The Composite Plate theory equations are also validated using finite element analysis.

Two types of sensitivity analysis are conducted in Chapter 5. First the Composite Plate theory deflection equation is analyzed for sensitivity to various parameters. It is found that the deflections are most sensitive to layer thickness values and least sensitive to Poisson's ratio values. Thus emphasizing the importance of accurate layer thickness estimates to backcalculation results.

The KISS program was also analyzed for sensitivity. Plate size was investigated and it was determined that the plate size has a significant effect on backcalculated results. A 24x24 ft plate appears to be optimum for the KISS program. Variations in applied stress, seed values for $\overline{D}$ and k and AC assumed moduli (high, average or low) were also included in the sensitivity analysis. It was discovered that the seed values had no effect on the outcome due to the nature of the search routine used. The variations in moduli results from variations in applied stress were not as pronounced as those from variations in plate size and could be attributed to stress sensitivity of the layers. The analysis of the "moduli assumption" brought about the conclusion that perhaps breaking the data up into the high, average, and low categories was not the best approach. Although this approach allowed the development of equations with very good statistical fit, it appears to have gained nothing as far as the resulting backcalculation program is concerned. It would have been better to lump all the data together and have lower R^2 values and avoid the need to assume high, average or low. An assumption of average is recommended for all future backcalculation efforts.

Chapter 6 is devoted to the comparison of the KISS program with other popular backcalculation programs using data from the LTPP database. Three types of comparisons are made. First a Program-to-Program comparison is made in which the KISS program is compared to each of the other programs one at a time. Secondly a Case-by-Case comparison is made in which the results from all programs are compared for each pavement system from the LTPP database. Thirdly, theoretical deflections from three different sources (Composite Plate theory, Elastic Layer theory, and finite element method) are generated for eight different theoretical pavement systems. In this method, the original moduli are known and can be used as a basis for comparison. The results of these comparisons vary widely, comparison between different theoretical procedures is difficult and there were a lot of technical problems in getting the various programs to run. Often these programs disagree by quite a bit and it is difficult to tell which program is closer to the correct value.

7.2 Suggested Future Research

7.2.1 Comparison with Other Programs

There are many areas in which the KISS program could be further developed and expanded. The verification of this program in attempting to compare with other programs was disappointing due to the technical difficulties and lack of realistic results from some of the other programs. Many of the programs had difficulty because the surface layers were relatively thin (2 or 3 inches). These particular cases from the LTPP database were chosen because most pavement on Alaska roads are not thicker than this. However it might be beneficial to use samples with thicker asphalt layers just for comparison purposes, and then more programs could be included in the comparison.

There is an alternative way to compare these programs. There are several existing test pavements (Texas, Virginia, and Minnesota for example) which have devices built in to the pavement to measure stresses and strains at various locations within the system. An alternate method of comparing these programs would be to perform FWD tests and measure the deflections and stresses and strains. With this information, the moduli could be backcalculated and used to determine the stresses and strains which could then be compared. This would allow comparison with measured field value.

7.2.2 Development for Use with Other Types of Pavement Systems

The equations developed for the KISS program in relating E_{nb} to E_s and E_b only for the case of an asphalt over a stabilized base. Similar equations could be developed for Rigid (PCC) pavements and for asphalts over different kinds of bases. The development of the equations was an arduous task and would have to be done for each type of pavement system to be backcalculated for.

7.2.3 Overlay and Pavement Design Methods

The uniqueness of $\overline{D}$ and k in composite plate theory could be taken advantage of in developing pavement and overlay design methods which are based on these parameters rather than moduli values. The KISS program has been shown very reliable in backcalculated $\overline{D}$ and k for a given system and in doing so, the assumptions about the strength of the AC does not come into play and could be avoided.

7.2.4 Proposed Revisions to the KISS program.

There is much room for improvement in the KISS program as far efficiency and input is concerned. If the program is going to be used by other users for research it should be revised. The high, average, low option should be removed and only the "average" equation should be available. The program could also be upgraded to handle metric units as well as English. Further it may be possible to do away with the final search for E_s and E_b. Since, the equation for F (Equation 3-5) is a function of only one variable (when we consider E_s as a function of E_b) it may be possible to solve for E_b directly. However this needs further investigation into how complicated it would be to solve for E_b.

7.2.5 Other Issues

At the beginning of this research I tried to focus on the importance of Keeping It Simple, however in researching the various methods of backcalculation I have found that this is not easy. As a matter of fact, it's impossible. There are too many factors which affect backcalculated moduli which cannot be ignored. In the development of the KISS program many of these were ignored in an effort to focus on the pluripotence issue. Depth to a rigid (bedrock) layer, AC surface temperature affects, and nonlinearity of the base and subgrade materials are just a few of the influencing factors that were not addressed. However it is impossible to deal with all of these issues at once and they must be incorporated into the backcalculation effort one at a time. The KISS program focuses on the issue of pluripotence which is the most important factor because without a unique solution you cannot assure repeatable, consistent results. The remaining factors should be addressed only after the uniqueness issue is settled. There is widespread research into all of these factors which could be incorporated into the KISS approach to backcalculation.

REFERENCES

Alexander, D.R., Kohn, S.D., Grogan, W.P., "Nondestructive Testing Techniques and Evaluation Procedures for Airfield Pavements", *Nondestructive Testing of Pavements and Backcalculation of Moduli,* ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society of Testing and Materials, Philadelphia, 1989, pp. 502-524.

BOUSDEF User's Guide, Transportation Research Institute, Oregon State University, 100 Merryfield Hall Corvallis,OR 97331-4304, (received in 1995, no year on document).

Burmister, D.M., "The Theory of Stresses and Displacements in Layered Systems and Applications to the Design of Airport Runways", Highway Research Board, *Proceedings 23rd Annual Meeting,* 1943, Vol. 23, Eds. Roy W. Crum and Fred Burggraf, pp126-148.

Burmister , D.M., "The General Theory of Stresses and Displacements in Layered Systems I", *Journal of Applied Physics,* Vol 16, February 1945, pp.89-94.

Burmister , D.M., "The General Theory of Stresses and Displacements in Layered Systems II", *Journal of Applied Physics,* Vol 16, March 1945, pp.126-127.

Burmister , D.M., "The General Theory of Stresses and Displacements in Layered Systems III", *Journal of Applied Physics,* Vol 16, May 1945, pp.269-302.

Burmister, D.M., "Stress and Displacement Characteristics of a Two-Layer Rigid Base Soil System: Influence Diatgrams and Practical Applications", Highway Research Board, *Proceedigns 35th Annual Meeting,* 1956, Vol. 35, Eds. Fred Burggraf and Elmer M. Ward, pp773-814.

Burmister, D.M., "Evalutation of Pavement Systems of the WASHO Road Test by Layered System Methods", *Highway Research Board Bulletin,* No. 177, 1957, pp. 26-54.

Bush, A.J. III, "Nondestructive testing for Aircraft Pavements, Phase II, Development of the Nondestructive Evaluation Methodology", Final Report, FAA-RD-80-9-II, Washington, D.C., Nov 1980.

Bush III, A.J., Alexander, D.R., "Pavement Evaluation Using Deflection Basin Measurements and Layered Theory", Transportation Research Record 1022, 1985, pp. 16-29.

Chong Ket Pen, Institute of Training and Research Malaysia. Public Works Department Malaysia. "Assessment of the Available Methods of Analysis for Estimating the Elastic Moduli of Road Pavements", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Chou, Y.J., Lytton, R.L., "Accuracy and of Backcalculated Pavement Layer Moduli", Transportation Research Record 1293, p. 72-85, 1991.

Chua, K.M., "Evaluation of Moduli Backcalculation Programs for Low-Volume Roads", *Nondestructive Testing of Pavements and Backcalculation of Moduli,* ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society of Testing and Materials, Philadelphia, 1989, pp. 398-414.

Dynatest Consulting, Inc., and Soils and Materials, Inc., Pavement Deflection Analysis: Participant Workbook, prepared for the FHWA National Highway Institute, 1993.

ELMOD4 Training Manual published by Dynatest International, California. 1-805-648-2230, usa@dynatest.com.

EVERCALC User's Guide, Volume 3 – Pavement Analysis computer Software and Case Studies, Section 2.0, EVERCALC –Pavement Backcalculation, February 1995.

FHWA Publication no. FHWA-RD-99-1125, "The Key to a world of Pavement Information, DataPave 2.0", U.S. Department of Transportation Federal Highway Administration, 1999.

Hall, K.T., Mohseni, A., "Backcalculation of Concrete-Overlaid Portland Cement Concrete Pavement Layer Moduli", Transportation Research Record 1293, pp. 112-123, 1991.

Hoffman, M.D., "Mechanistic Interpretation of Nondestructive Pavement Testing Deflections", Ph.D. dissertation, University of Illinois at Urbana-Champaign, 1980.

Hogg, A.H.A., "Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastion Foundation of Infinite Depth", *Philosophical Magazine*, Series 7, March (25), 1938, pp 576-582.

Irwin, L.H., "Instructional Guide for Back-Calculation and the Use of MODCOMP3, version 3.6" Cornell University, Local Roads Program, March 1994, CLRP Publication No. 94-10.

Ioannides, A.M., "ILLI-BACK" user's manual, copyright 1988.

Ioannides, A.M., "Dimensional Analysis in NDT Rigid Pavement Evaluation", *Journal of Transportation Engineering*, Vol. 116, No.1, January, 1990, pp. 23-35.

Ioannides, A.M., "Concrete Pavement Backcalculation Using ILLI-BACK 3.0", Nondestructive Testing Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 103-124.

Kim, Ok-Kee and Nokes, William A., "Evaluation of Backcalculation Methods to Predict Pavement Layer Moduli", Report No. FHWA/CA/TL-94/11, California Dept. of Transportation, Sacramento, CA 95814, August 1993.

Kim, Y.R., Khosla, N.P., Satish, S., Scullion, T., "Validation of Moduli Backcalculation Procedure Using Multidepth Deflectometers Installed in Various Flexible Pavement Structures", Transportation Research Record 1377, pp. 128-142, 1992.

Lee, Sang-Won, "Backcalculation of pavement moduli by use of pavement surface deflections", Ph.D. Thesis, University of Washington, 1988.

Lee, S.W., Mahoney, J.P., Jackson, N.C., "Verification of Backcalculation of Pavement Moduli", Transportation Research Record 1196, 1988, pp. 85-95.

Lenngren, C.A., "Pavement Strain Versus Backcalculated Data", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Losberg, A., "Structurally Reinforced Concrete Pavements", Doktorsvhandlingar Vid Chalmers Tekniska Hogskolo, Goteborg, Sweden, 1960.

Love, A.E.H., The Mathematical Theory of Elasticity, 3rd ed., Cambridge University Press, 1923.

Lytton, R. L., "Backcalculation of Pavement Layer (same as ref79)Properties," Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G. Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 7-38.

Maestas, J.M., Mamlouk, M.S., "Comparison of Pavement Deflection Analysis Methods Using Overlay Design", Transportation Research Record 1377, 1992.

Mahoney, J.P., Coetzee, N.F., Stubstad, R.N., Lee, S.W., "A Performance Comparison of Selected Backcalculation Computer Programs", Nondestrucive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 452-467.

May, R.W., and Von Quintus, H.L., "The Quest for a Standard Guide to NDT Backcalculation", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, 505-520.

Mings (Atkinson), Judith A., "Uniqueness of Material Properties as Derived From Backcalculations", A Masters Thesis, University of Alaska Fairbanks, 1993.

Nazarian, S., Chai, Y.E., "Comparison of Theoretical Case study and In Situ Behaviors of a Flexible Pavement Section",Transportation Research Record 1377, pp.172-182, 1992.

Rada, G.R., Richter, C.A., and Stephanos, P.J., "Layer Moduli from Deflection Measurements: Software Selection and Development of Strategic Highway Research Program's Procedure for Flexible Pavements", Transportation Research Record 1377, 1992.

Rada, G.R., Richter, C.A., and Jordahl, P., "SHRP's Layer Moduli Backcalculation Procedure", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 38-52.

Rhode, G.T., Scullion, T., "MODULUS 4.0: Expansion and Validation of the Modulus Backcalculation system", Research Report 1123-3, Texas Transportation Institute, Texas A&M University System, College Station, 1990.

Scullion, T., Uzan, J., Paredes, M., "MODULUS: A Microcomputer-Based Backcalculation System", TRR 1260, pp. 180-191, 1990.

Sivaneswaran, N., Kramer, S.L., Mahoney, J.P., "Advanced Backcalculation Using A Nonlinear Least Squares Optimization Technique", Transportation Research Record 1293, pp. 93-102, 1991.

Stolle, D., and Hein, D., "Parameter Estimates of Pavement Structure Layers and Uniqueness of the Solution," Nondestrucive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 313-322.

Torczon, V., "Mulit-Directional Search: A Direct Search Algorithm for Parallel Machines" PhD Thesis for Rice University, May 1989.

Torczon, Virginia, "On the Convergence of the Multidirectional Search Algorithm", SIAM Journal Optimization, Vol.2, No.1, pp.123,145, February 1991.

Dennis, J.E., Jr., Torczon, V., "Direct Search Methods on Parallel Machines", Siam Journal Optimization, Vol.1, No. 4, pp. 448-474, November, 1991.

Ullidtz, P., Pavement Analysis, Developments in Civil Engineering, 19, Technical University of Denmark, Elseveir, New York, 1987.

Ullidtz, P., Modelling Flexible Pavement Response and Performance, Technical university of Denmark, Narayana Press, Gylling, 1998.

Ullidtz, P., Battiato, G., Larsen, B.K., and Stubstad, R.N., "Verification of the Analytical-Empirical Method of Pavement Evaluation Based on FWD Testing", Paper accepted for the 6th International Conference on Structural Design of Asphalt Pavements, Ann Arbor, 1987.

Ullidtz, P., Stubstad, R.N., "Analytical-Empirical Pavement Evaluation Using the Falling Weight Deflectometer", Transportation Research Record 1022, 1985.

Uzan, J., Scullion, T., Michalek, C.H., Paredes, M., Lytton, R.L., "A Microcomputer Based Procedure for Backcalculating Layer Moduli from FWD Data", Research Report 1123, Project 2-18-87-1123, Texas Transportation Institute, July 1988.

Uzan, J., Lytton, R.L., Germann, F.P., "General Procedure for Backcalculating Layer Moduli, Nondestrucive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 217-228.

Uzan, J., Scullion, T., "Verification of Backcalculation procedures", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Van Cauwelaert, F.J., Alexander, D.R., White, T.D., Barker, W.R., "Multilalyer Elastic Program Backcalculating Layer Moduli in Pavement Evaluation", Nondestructive Testing of Pavments and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp.171-188.

Westergaard, H.M., "Stresses in Concrete Pavements Computed by Theoretical Analysis", *Public Roads*, 7(2), 1926, pp 25-35.

Westergaard, H.M., "Stresses in Concrete Runways of Airports", Proceedings 19th Annual Meeting, Highway Research Board, Washington, D.C., 1939, pp 197-295.

Zhou, H., Hicks, R.G., Connor, B., "Improved Overlay Design Procedure For the State of Alaska, Volume IV, Computer Programs", June 1989.

Zhou, H., Hicks, R.G., Bell, C. A., "BOUSDEF: A Backcalculation Program for Determining Moduli of a Pavement Structure", *TRR 1260*, pp. 166-179, 1990.

Appendix 2A
Literature Review

This appendix gives an extensive (although not exhaustive) listing of references both included and not included in this paper. The references are separated by topic. Of course there is some overlap in topics but I tried to put them under the main subject of the paper. The papers are listed in chronological order starting with the oldest and ending with the most recent under each category.

New and Improved Backcalculation Methods

Michelow, J., Analysis of Stresses and in an n-layered Elastic System Under a Load Uniformly Distributed on a Circular Area, California Research corporation, Richmond, California, Sept. 24, 1963. (CHEVRON, deflection calculation for linear elastic layer theory)

Stubstad, R.N., Sharma, J., "Deriving Mechanistic Properties of Pavements from Surface Deflections", Proceedigns, The International Conference of Computer Applications in Civil Engineering, Roorkee, India, 1979. (ISSEM4)

Ullidtz, P., Peattie, K.R.,"Pavement Analysis by Programmable Calculators", Journal of Transportation Engineering, ASCE, vol 106, Sept. 1980, pp. 581-597. (Method of equivalent Thickness)

Bush, A.J. III, "Nondestructive testing for Aircraft Pavements, Phase II, Developing Nondestructive Evaluation Methodology", Final Report, FAA-RD-80-9-II, Washington, D.C., Nov 1980. (CHEVDEF)

Uzan, J., Witczak, M.W., "Composite Subgrade Modulus for Rigid Airfield Pavement Design Based Upon Multilayer Theory", Third International Conference on Concrete Pavement Design and Rehabilitation, Purdue University, West Lafayette, Indiana, April 23-25, 1985, pp. 157-166.

Bush III, A.J., Alexander, D.R., "Pavement Evaluation Using Deflection Basin Measurements and Layered Theory", Transportation Research Record 1022, 1985, pp. 16-29. (BISDEF)

Uddin, W., Meyer, A.H., Hudson, W.R., "Aflexible Pavement Structural Evaluation System Based on Dynamic Deflections" Proceedings, Association of Asphalt Paving Technologists, Vol. 54, 1985, pp.432-453. (FPEDD-1)

Ullidtz, P., Stubstad, R.N., "Analytical-Empirical Pavement Evaluation Using the Falling Weight Deflectometer", Transportation Research Record 1022, 1985, pp. 36-44. (ELMOD)

Husain, S., George, K.P., "In Situ Pavement Moduli from Dynaflect Deflection", Transportation Research Record 1043, 1985, pp. 102-112.

Ullidtz, P., Pavement Analysis, Developments in Civil Engineering, 19, Technical University of Denmark, Elseveir, New York, 1987. (ELMOD)

Ullidtz, P., Battiato, G., Larsen, B.K., Stubstad, R.N., "Verification of the Analytical-Empirical Method of Pavement Evaluation Based on FWD Testing", Paper accepted for 6^{th} International Conference of Structural Design of Asphalt Pavements, Ann Arbor, 1987, pp. 521-532. (ELMOD)

Lee, Sang-Won, Ph.D. Thesis, "Backcalculation of pavement moduli by use of pavement surface deflections", University of Washington, 1988. (EVERCALC)

Stubstad, R.N., "Description of and Users Guide for The Dynatest ISSEM4 Computer Program", unpublished, written for Dynatest Consulting, inc., April, 1988. (Quasi-finite element approach, incorporates non-linear analysis)

Lee, S.W., Mahoney, J.P., Jackson, N.C., "Verification of Backcalculation of Pavement Moduli", Transportation Research Record 1196, 1988, pp. 85-95. (EVERCALC)

Uzan, J., Scullion, T., Michalek, C.H., Paredes, M., Lytton, R.L., "A Microcomputer Based Procedure for Backcalculating Layer Moduli from FWD Data", Research Report 1123, Project 2-18-87-1123, Texas Transportation Institute, July 1988. (MODULUS)

Ioannides, A.M., "Finite Difference Solution Plate on Elastic Solid", Journal of Transportation Engineering, Vol. 114, No.1, January, 1988, pp. 57-75. (ILLI-BACK)

Ioannides, A.M., "ILLI-BACK" user's manual, copyright 1988.

Bush III, A.J., Brown, R.W., Bailey, C.E., "An Evaluation Procedure For Rigid Airfield Pavements", Proceedings 4th International Conference on Concrete Pavement Design and Rehabilitation, April 18-20 1989, Purdue University, West Lafayette, Indiana 47907, USA, pp 419-429.

Uzan, J., Lytton, R.L., Germann, F.P., "General Procedure for Backcalculating Layer Moduli" Nondestrucive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 217-228. (MODULUS)

Van Cauwelaert, F.J., Alexander, D.R., White, T.D., Barker, W.R., "Multilalyer Elastic Program Backcalculating Layer Moduli in Pavement Evaluation", Nondestructive Testing of Pavments and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp.171-188. (WESDEF)

Ioannides, A.M., "Dimensional Analysis in NDT Rigid Pavement Evaluation", Journal of Transportation Engineering, Vol. 116, No.1, January, 1990, pp. 23-35. (ILLI-BACK)

Kobisch, R., Triki, R., "Determination of Three-Layer Pavements Moduli From the Analysis of Deflections Basins Measured with the 03/04 Lacroix Deflectograph", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway. (SYPHAX, French)

Lytton, R.L., Germann, F.P., Chou, Y.J., Stoffels S.M., "Determining Asphaltic Concrete Pavement Structural Properties by Nondestructive testing", NCHRP Report 327, June 1990. (Use of expert systems in conjunction with MODULUS)

Zhou, H., Hicks, R.G., Bell, C. A., "BOUSDEF: A Backcalculation Program for Determining Moduli of a Pavement Structure", TRR 1260, pp. 166-179, 1990.

Scullion, T., Uzan, J., Paredes, M., "MODULUS: A Microcomputer-Based Backcalculation System", TRR 1260, pp. 180-191, 1990.

Rhode, G.T., Scullion, T., "MODULUS 4.0: Expansion and Validation of the Modulus Backcalculation system", Research Report 1123-3, Texas Transportation Institute, Texas A&M University System, College Station, 1990.

Royal Institute of Technology, Stockholm, Sweden, "A Simple Correlation Approach to Interpretation of Deflection Bazin Data", International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Inoue, T., Matsui, K, "Structural Analysis of Asphalt Pavement by FWD and Backcalculation of Elastic Layered Model", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Himeno, K, Maruyama, T., Abe, N, Hayashi, M., "The Use of FWD Deflection Data in Mechanistic Analysis of Flexible Pavements", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway, pp. 401-410. (LMBS from Japan)

Zhou, H, Hicks, R.G., Bell, C.A., "Development of Backcalculation Program and its Verification", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway, pp. 391-400. (BOUSDEF)

Sivaneswaran, N., Kramer, S.L., Mahoney, J.P., "Advanced Backcalculation Using A Nonlinear Least Squares Optimization Technique", Transportation Research Record 1293, pp. 93-102, 1991. (EVERCALC4.0)

Sargand, S.M., Hazen, G.A., Wilson, B.E., Russ, A.C.,"Evaluation of Resilient Modulus by Back-Calculation Technique", Final report for Ohio Dept of Transportation, and Federal Highway Administration, 1991. (FEM)

Uzan, J., Briggs, R., and Scullion, T., "Backcalculation of Design Parameters for Rigid Pavements", Transportation Research Record 1377, pp 107-114, 1992.

Yuan, D., Nazarian, S., "Rapid Determination of Layer Properties of Pavements from Surface Wave Method", TRR 1377, pp 159-166, 1992

Zaghloul S., and White, T., "Use of a Three-Dimensional, Dynamic Finite Element Program for Analysis of Flexible Pavement", Transportation Record 1388, 1993, pp. 60-69.

Aouad, M.F., Stokoe II, K.H., Briggs, R.C., "Stiffness of Asphalt Concrete SASW Surface Layer from Stress Wave measurements", TRR 1384, 1993, pp. 19-35.

Wang, F., Lytton, R.L., "System Identification Method for Backcalculating Pavement Layer Properties", Transportation Research Record 1384, pp.1-7, 1993.

Harichandran, R.S., Mahmood, T., Raab, A.R., and Baladi, G.Y., "Modified Newton Algorithm for Backcalculation of Pavement Layer Properties", Transportation Research Record 1384, pp.15-22, 1993. (MICHBACK)

Noureldin, A. S., "New Scenario for Backcalculation layer thickness of Layer Moduli of Flexible Pavements", Transportation Research Record 1384, pp.23-28, 1993.

Chou, Y.J., "Knowledge-Based System for Flexible Pavement Structural Evaluation", Journal of Transportation Engineering, Vol 119, pp 450-466, 1993.

Khazanovich, L., "Structural Analysis of Multi-Layered Concrete Pavement Systems", PhD Thesis, University of Illinois at Urbana-Champaign, 1994.

Rada, G.R., Richter, C.A., and Jordahl, P., "SHRP's Layer Moduli Backcalculation Procedure", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 38-52. (MODULUS 4.2)

Stubbs, N., Torpunuri, V.S., Lytton, R.L., and Magnuson, A.H., "A Methodology to Identify Material Properties in Pavements Modeled as Layered Viscoelastic Halfspaces (Theory)," Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198(F 1), H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 53-67. (Uses Dynamic Analysis)

Harichandran, R.S., Mahmood, T., Raab, A.R., Baladi, G.Y., "Backcalculation of Pavement Layer Moduli, Thicknesses and Stiff Layer Depth Using a Modified Newton Method," Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 68-82. (MICHBACK)

Ioannides, A.M., "Concrete Pavement Backcalculation Using ILLI-BACK 3.0", Nondestructive Testing Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 103-124. (ILLI-BACK)

de Almeida, J.R., Brown, S.F., and Thom, N.H., "A Pavement Evaluation Procedure Incorporating Material Non-Linearity", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 218-232. (LEAD/FEAD based on PADAL developed in the UK)

Irwin, L.H., "Instructional Guide for Back-Calculation and the Use of MODCOMP3, version 3.6" Cornell University, Local Roads Program, March 1994, CLRP Publication No. 94-10.

Ioannides, A.M., Khazanovich, L., "Backcalculation Procedures for Three-Layered Concrete Pavements", Proceedings, Fourth International Conference on the Bearing Capacity of Roads and Airfields, Vol 1, pp77-91, University of Minnesota, July 17-21, 1994.

Meier, R.W., Rix, G.J., "Backcalculation of Flexible Pavement Moduli Using Artificial Neural Networks", Transportation Research Record 1448, 1994, pp.75-82.

Khazanovich, L., Ioannides, A.M., "DIPLOMAT: Analysis Program for Both Bituminous and Concrete Pavements", Transportation Research Record 1482, 1995, pp. 52-60.

EVERCALC User's Guide, Volume 3 – Pavement Analysis computer Software and Case Studies, Section 2.0, EVERCALC –Pavement Backcalculation, February 1995.

BOUSDEF User's Guide, Transportation Research Institute, Oregon State University, 100 Merryfield Hall Corvallis,OR 97331-4304, (received in 1995, no year on document).

Shuo, Li, Fwa, T.F., Tan, K.H., "Closed-Form Back-Calculation of Rigid-Pavement Parameters", Journal of Transportation Engineering, Vol. 122 No. 1, Jan/Feb 1996. (NUS-BACK, Singapore)

Fwa, T.F., Shi, X.P., Tan, S.A., "Use of Pasternak Foundation Model in Concrete Pavement Analysis", Journal of Transportation Engineering, Vol. 122, no. 4, July/August, 1996, pp 323 –328, paper no. 10082. (New Foundation Model)

Shuo, L., Fwa, T.F., Tan, K.H., "Back-Calculation of Parameters for Slab on Two-Layer Foundation System", *Journal of Transportation Engineering,* Vol. 123, No. 6, November, December, 1997, pp. 484-488. (NUS-DEF3, Singapore)

Khazanovich, L., Roesler, J., "DIPLOBACK: Neural-Network-Based Backcalculation Program for Composite Pavements", Transportation Research Record 1570, 1997, pp. 143-150.

Fwa, T.F., Tan, C.Y., Chan, W.T., "Backcalculation Analysis of Pavements-Layer Moduli Using Genetic Algorithms", Transportation Research 1570, 1997, pp.134-142. (NUS-GABACK, Singapore)

Meier, R.W., Alexander, D.R., Freeman, R.B., "Using Artificial Neural Networks as a Forward Approach to Backcalculation", Transportation Research Record 1570, 1997, pp. 126-133. (A "smart" version of WESDEF)

Livneh, M., "Single-Measurement Estimation of IN Situ Asphalt-Layer Moduli with Protable Falling Weight Deflectometer", Transportation Research Record 1570,pp. 118-125. (Isreal, for asphalt layer only)

Lee, Y., Kim, Y.R., Ranjithan, S.R., "A Dynamic Analysis-Based Approach to Determine Flexible Pavement Layer Moduli Using Deflection Basin Parameters", Paper submitted for TRB 77th Annual Meeting Jan 11-15, 1998. (Uses Artificial Neural Networks)

Davids, W.G., Turkiyyah, G.M., Mahoney, J.P., "EverFE- a Rigid Pavement 3d Finite Element Analysis Tool",(abstract only), University of Washington. (YEAR???)

Kim, Y., Kim, Y.R., "Prediction of Layer Moduli from FWD and Surface Wave Measurements Using Artificial Neural Network", Paper submitted for TRB 77th Annual Meeting, January 11-15, 1998, Washington, D.C. (Uses SASW)

Kang, Y.V., "Multifrequency Back-Calculation of Pavement-Layer Moduli", *Journal of Transportation Engineering,* Vol. 124, No. 1, January/February, 1998, pp. 73-81. (BKGREEN, Tiawan, uses Green's functions)

Ioannides, A.M., and Khazanovich, L., "General Formulation for Multilayered Pavement Systems", Journal of Transportation Engineering, Vol. 124, No1, January/February, 1998. Paper no. 14070. pp. 82-90. (DIPLOMAT)

Fwa, T. F., Tan, K. H., and Li, S., " Graphical Solutions for Back-Calculation of Rigid Pavement Parameters", Journal of Transportation Engineering, Vol. 124, No. 1, January/February 1998. (NUS-BACK)

Lin, P., Wu, Y., Juang, C.H., Huang, T., "Back-Calculation of Concrete Pavement Modulus Using Road-Rater Data", *Journal of Transportation Engineering,* Vol. 124, No. 2, March/April 1998, pp. 123-127. (BACK for Jointed pavements, Taiwan)

Tutumluer, E., Thompson, M.R., "Anisotropic Modeling of Granular Bases in Flexible Pavements", Transportation Research Record 1577, 1997, pp. 18-26. (GT-PAVE, Finite Element)

Li, S., Fwa, T. F., Tan, K. H., "Parameters Back-Calculation for Concrete Pavement with Two Slab Layers", Journal of Transportation Engineering, Vol. 124, No. X6, November/December, 1998. (NUS-BACK2)

Factors effecting Backcalculation (including Temperature/Seasonal effects, Non-linearity, Depth to Rigid Layer, Thickness, Static vs Dynamic analysis, etc...)

Hooke, R., Jeeves, T.A., " 'Direct Search' Solution of Numerical and Statistical Problems, Journal of the Association for Computing Machinery, Vol. 8, pp. 212-229, 1961. (Pattern search routine used in MODULUS program)

Schuefel, Hans-Paul, Numerical Optimization of Computer Models, Pub. John Wiley & Sons (YEAR???), pp. 42 to 47. (Describes Hooke and Jeeves Pattern search used in the MODULUS program)

Sharpe, G.W., Southgate, H.F., Deen R.C., "Pavement Evaluation by Using Dynamic Deflections", Bureau of Highways, Kentucky Department of Transportation, Lexington, Transportation Research Record N700, 1979, pp. 34-46.

Ioannides, A.M., Thompson, M.R., and Barenberg, E.J., "Westergaard Equations Reconsidered", Transportation Research Record 1043, 1985, pp. 13-23.

Mamlouk, M.S., "Use of Dynamic Analysis in Predicting Field Multilayer Pavement Moduli", Transportation Research Record 1043, 1985, pp. 113-121.

Khedr, S., "Deformation Characteristics of Granular Base Course in Flexible Pavements", Transportation Research Record 1043, 1985, pp. 131-148.

Carmichael III, R.F., Stuart, E., "Predicting Resilient Modulus: A Study to Determine the Mechanical Properties of Subgrade Soils", Transportation Research Record 1043, 1985, pp. 145-148.

Carmichael, III, R.F., Stuart, E., "Predicting Resilient Modulus: A Study to Determine the Mechanical Properties of Subgrade Soils", Transportation Research Record 1043, 1985, pp. 145-148.

Uddin, W., Meyer, A.H., Hudson, W.R., "Rigid Bottom Considerations for Nondestructive Evaluation of Pavements", Transportation Research Reccord 1070, 1986, pp. 21-29.

Uzen, J., Sides, A., "The Effect of Contact Area Shape and Pressure Distribution on Multilayer Systems Response", Transportation Research Record 1117, pp. 21-24, 1987.

Rwebangira, T., Hicks, R.G., Truebe, M., "Sensitivity Analysis of Backcalculation Procedures Transportation Research Record 1117, pp. 25-37, 1987.

Chandra, D., Lytton, R.L., Yang, W., "Effects of Temperature and Moisture on Low-Volume roads", NTIS order number PB90-147463/AS, study 2-18-87-473, December, 1988.

Ioannidese, A.M., and Donnelly, J.P., "Three Dimeensional Analysis of Slab on Stress-Dependent Foundation", Transportaiton Research Record 1196, 1988, pp.72 to 84. (3-D Finite Element Analysis)

Irwin, L.H., Yang, W.S., Stubstad, R.N., "Deflection Reading Accuracy and Layer Thickness Accuracy in Backcalculation of Pavement Layer Moduli", Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM STP1026, A.J. Bush III and G.Y. Baladi, Eds., American Society of Testing and Materials, Philadelphia, 1989, pp. 229-244.

Stolle, D., and Hein, D., "Parameter Estimates of Pavement Structure Layers and Uniqueness of the Solution," Nondestrucive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 313-322.

Briggs, R.C., Nazarian, S., "Effects of Unknown Rigid Subgrade Layers on Back-Calculation of Pavement Moduli and Projections of Pavement Performance,' Transportation Research Record 1227, 1989, pp. 183-193.

Bush III, A., Bentsen, R., "Nondestructive Evaluation Equipment for Airfield Pavements", Transportation Research Record, 1260, pp. 192 – 215, 1990.

Mork, H., "a Norwegian/Swedish in-depth Deflection Study - Backcalculation of Moduli", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway. (Stress Dependence)

Bergstedt, B., "Temperature Correction of Falling Weight Deflectometer data", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Deng Xue-Jun, Huang Xiao-Ming, "Dynamic Analysis Evaluation of Concrete Pavement - Theory, Method", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway, pp 541-547.

Rohde, G.T., Yang, W., Smith, R.E., "Inclusion of Depth to a Rigid Layer in Determining Pavement Layer Properties", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Magnuson, A.H., Lytton R.L., and Briggs, R.C., "Comparison of Computer Predictions and Field Data for Dynamic Analysis of Falling Weight Deflectometer Data", Transportation Research Record 1293, 1991, pp. 61-71.

Touma B. C., Crovetti, J.A., Shahin, M.Y., "Effect of Various Load Distributions on Backcalculated Moduli Values in Flexible Pavements", Transportation Research Record 1293, 1991, pp. 31-41.

Rohde, G.T., and Smith, R.E., "Determining Depth Apparent Stiff Layer From FWD data", Texas Transportation Institute, Research Report 1159-1, October, 1991

Parker Jr., F., "Estimation of Paving Materials Design Moduli from Falling Weight Deflectometer Measurements", TRR 1293, pp. 42-51, 1991.

Ong, C.L., Newcomb, D.E., Siddharthan, R., "Comparison of Dynamic and Static Backcalculation Moduli for Three-Layer Pavements",Transportation Research Record 1293, pp. 86-92, 1991.

Hossain, A.S.M.M., Zaniewski, J.P., "Detection and Determination of Depth of Rigid Bottom in Backcalculation of Layer Moduli from Falling Weight Deflectiometer Data", TRR 1293, pp. 124-135, 1991.

Hossain, M., and Zaniewski, J.P., Variability in Estimation of Structural Capacity of Existing Pavements from Falling Weight Deflectometer Data", Transportation Research Record 1355.

Ioannides, A.M., Khazanovich, L, Becque, J.L., "Structural Evaluation of Base Layers in Concrete Pavement Systems", Transportation Research Record 1370, 1992, pp. 20-28.

Thompson, M.R., "Report of the Discussion Group on Backcalculation Limitations and Future Improvements", Transportation Research Record 1377, 1992.

Siddharthan, R., Sebaaly, P.E., Javaregowda, M.,"Influence of Statistical Variation in FallingWeight Deflectometers on Pavement Analysis",Transportation Research Record 1377, 1992.

Zaniewski, J.P., Hossain, M.,"Effect of Thickness and Temperature corrections on prediction of pavement structural capacity using falling weight deflectometer data", Transportation Research Record 1377,1992.

Briggs, R.C., Scullion, T., Maser, K.R., "Asphalt Thickness Variation on Texas Strategic Highway Research Program Sections and Effect on Backcalculated Moduli", Transportation Research Record 1377,1992.

Mahoney, J.P., Winters, B.C., Jackson, N.C., Peirce "Some Observations about Backcalculation and Use of a Stiff Layer Condition", Transportation Research Record 1384, 1993, pp. 8-14.

Seng, C, Stokoe II, K. R., Roesset, J.M., "Effect depth to bedrock on the accuracy of backcalculated moduli obtained with dynaflect and fwd tests", Texas DOT Report No. FHWA/TX-93+1175-5, January 1993.

Ketcham, S.A., "Dynamic Response Measurements and Identification Analysis of a Pavement During Falling-Weight Deflectometer Experiments", Transportation Research Record 1415, 1993, pp.78-87. (Wave Propagation Model, using FWD data)

Khazanovich, L., Ioannides, A.M., "Structural Analysis of Unbonded Concrete Overlays Under Wheel and Environmental Loads", Transportation Research Record 1449, 1994, pp. 174-181.

Noureldin, A.S., "Influence of Stress Levels and Seasonal Variations on In Situ Pavement Layer Properties", Transportation Research Record 1448, 1994, pp. 16-24.

Uzan, J., "Advanced Backcalculation Techniques", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 3-37. (Static vs Dynamic analysis)

Hall, K.T., and Darter, M.I., "Improved Methods for Asphalt-Overlaid Concrete Pavement Backcalculation and Evaluation" Nondestructive Testing of Pavements Backcalculation of Modelui (Second Volume) stp1198, Harold L. Quintus, Albert J. Bush, III, and Gilvert Y. Baladi, Eds., American Society for Teesting and Materials, Philadelphin, 1994, pp. 83-102. (Guidelines for interpretation of Backcalculation results)

Zaghloul, S.M., White, T.D., Drnevich, V.P., and Coree, B., "Dynamic Analysis of FWD Loading and Pavement Response Using a Three-Dimensional Dynamic Finite Element Program", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, pp. 125-138. (ABAQUS)

May, R.W., and Von Quintus, H.L., "The Quest for a Standard Guide to NDT Backcalculation", Nondestructive Testing of Pavements and Backcalculation of Moduli (Second Volume), ASTM STP 1198, H.L. Von Quintas, A.J. Bush, III, and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1994, 505-520.

Ullidtz, P., The Technical University of Denmark, Coetzee, N.F., Dynatest Consulting, Inc., "Analytical Procedures in NDT Pavement Evaluation", Offered for presentation at the 1995 TRB session on Structural Modelling Applications in Pavement Analysis and Design.

Nazarian, S., Boddapati, K.M., "Pavement-Falling Weight Deflectometer Interaction Using Dynamic Finite-Element Analysis", Transportation Research Record 1482, 1995, pp. 33-43.

Rowe, G.M., Brown, S.F., Sharrock, M.J., Bouldin, M.G., "Viscoelastic Analysis of Hot Mix Asphalt Pavement Structures", Transportation Research Record 1482, 1995, pp. 44-51.

Inge, Jr., E.H., Kim, Y.R., "Prediction of Effective Asphalt Layer Temperature", Transportation Research Record 1473, 1995, pp. 93-100.

Foinquinos, R., Roesset, J.M., Stokoe II, K.H., "Response of Pavement Systems to Dynamic Loads Imposed by Nondestructive Tests", Transportation Research Record 1504, 1995, pp. 57-67.

Roesset, J.M., Stokoe II, K.H., Seng, C., "Determination of Depth to Bedrock from Falling Weight Deflectometer Test Data", Transportation Research Record 1504, 1995, pp. 68-78.

Baig, S., Nazarian, S., "Determination of Resilient Modulus of Subgrades Using Bender Elements", Transportation Research Record 1504, 1995, pp. 79-86. (A new laboratory test)

Nazarian, S., Boddapati, K.M., "Pavement-Falling Weight Deflectometer Interaction Using Dynamic Finite-Element Analysis", Transportation Research Record 1482, 1995, pp. 33-43.

Ullidtz, P., and Coetzee, N.F., "Analytical Procedures in Nondestructive Testing Pavement Evaluation", Transportation Research Record 1482, 1995, pp. 61-66.

Kim, Y.R., Hibbs, B.O., Lee, Y., "Temperature correction of Deflecitons and Backcalculated Asphalt Concrete Moduli", Transportation Research Record 1473, 1995, pp. 55-62.

Newcomb, D.E., Van Deusen, D.A., Jiang, Y., Mahoney, J.P., "Considerations of Saturated Soil Conditions in Backcalculation of Pavment Layer Moduli", Transportation Research Record 1473, 1995, pp. 63-71.

Yuan, D., Nazarian, S.,Chen, D., Hugo, F., "Use of Seismic Pavement Analyzer in Monitoring Degradation of Flexible Pavements Under Texas Mobile Load Simulator", A paper for possible inclusion in TRB special session on "Emerging Accelerated and Nondestructive Paving Testing Technologies", YEAR???

Fwa, T.F., Shi, X.P., and Tan, S.A., "Analysis of Concrete Pavements by Rectangular Thick-Plate Model", *Journal of Transportation Engineering*, March/April, 1996. pp. 146 – 154.

Papagiannakis, A.T., Amoah, N., Taha, R., "Formulation for Viscoelastic Response of Pavements under Moving Dynamic Loads", Journal of Transportation Engineering, Vol. 122, No. 2, March/April, 1996, pp. 240-245.

Fwa, T. F., Shi, X. P., Tan, S. A., "Analysis of Concrete Pavements by Rectangular Thick-Plate Model", Journal of Transportation Engineering, March/April 1996, pp 146-154. (Analyses Westergaard's solutions)

Wu, C., Shen, P., "Dynamic Analysis of Concrete Pavements Subjected to Moving Loads", *Journal of Transportation Engineering*, Vol. 122, No. 5, September/October, 1996, pp 367-373.

Zhou, H., Rada, G.R., Elkins, G.E., "Investigation of Backcalculated Moduli Using Deflections Obtained at Various Locations in a Pavement Structure", Transportation Research Record 1570, Auguast 1997, pp. 96-107.

Vepa, T.S., George, K.P., "Deflection Response Models For Cracked Rigid Pavements", *Journal of Transportation Engineering,* Vol. 123, No. 5, September/October, 1997, pp. 377-384. (Dynamic analysis and cracks and joints)

Shoukry, S.N., Martinelli, D.R., Selenzneve, O.I., "Dynamic Performance of Composite Pavements Under Impact", Transportation Research Record 1570, 1997, pp. 163-171.

Park, S.W., Kim, Y.R., "Temperature correction of Backcalculated Moduli and Deflections Using Linear Viscoelasticity and Time-Temperature Superposition", Transportation Research Record 1570, 1997, pp. 108-117.

FHWA/LTPP, "Analyses Relating to Pavement Material Characterizations and Their Effects On Pavement Performance", Publication no. FHWA-RD-97-085, January 1998.

Helwany, S., Dyer, J., Leidy, J., "Finite-Element Analyses of Flexible Pavements", *Journal of Transportation Engineering,* Vol. 124, No. 5, September/October, 1998, pp. 491-499. (Analysis of different loading configurations)

Grogan, W.P., Freeman, R.B., Alexander, D.R., "Impact of FWD Testing Variability of Pavement Evaluations", *Journal of Transportation Engineering,* Vol. 124, No. 5, September/October, 1998, pp.437-442.

Long, B., Hossain, M., Gisi, A.J., "Seasonal Variation of Backcalculated Subgrade Moduli", Transportation Research Record 1577, 1997, pp. 70-80.

Non-destructive testing (Impact loads, SASW, etc...)

Wiseman,, G., Greenstein, J., Uzan, J., "Application of Simplified Layered Systems to NDT Pavement Evaluation", Transportation Research Record 1022, 1985, pp. 29-36. (Compares vibratory and impact loading devices)

Wang, M.C., "Pavement Response to Road Rater and Axle Loadings", Transportation Research Record 1043, 1985, pp. 149-157.

Hossain, M., Zaniewski, J., "Characterization of Falling Weight Deflectometer Deflection Basin", Transportation Research Record, 1293, pp. 1-11, 1991.

Uzan, J., Lytton, R, "Experiment Design Approach to Nondestructive Testing of Pavements", Journal of Transportation Engineering, Vol. 115, No. 5, September, 1989. pp. 505-520. (Analysis of measurement error)

George, K.P., "Resilient Modulus of Subgrade Soils Using Gyratory Testing Machine", FHWA Report no. MSHD-RD-92-096, April 1992.

Bay, J.A., Stokoe II, K.H., Jackson, J.D., "Development and Preliminary Investigation of Rolling Dynamic Deflectometer", Transportation Research Record 1473, 1995, pp. 43-54.

Mohsen, M.S., Saleh, B., "Evaluation of In-Situ Pavement Modulu Using High Resolution Tiltmeter", *Journal of Transportation Engineering,* Vol. 124, No. 5, September/October, 1998, pp. 443-447. (Amman, Jordan)

Comparison of Backcalculation programs with each other and with lab and field results

Lytton, R. L., "Backcalculation of Pavement Layer Properties," Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G. Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 7-38.

Tam, W.S., and Brown, S.F., "Back-Analyzed Elastic Stiffnesses: comparison Between Different Evaluation Procedures", *Nondestructive Testing of Pavements and Backcalculation of Moduli,* ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society ofr Testing and Materials, Philadelphia, 1989, pp. 189-200.

Mahoney, J.P., Coetzee, N.F., Stubstad, R.N., Lee, S.W., "A Performance Comparison of Selected Backcalculation Computer Programs", Nondestrucive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 452-467.

Chua, K.M., "Evaluation of Moduli Backcalculation Programs for Low-Volume Roads", *Nondestructive Testing of Pavements and Backcalculation of Moduli,* ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society ofr Testing and Materials, Philadelphia, 1989, pp. 398-414.

Mouratidis, A., Iliou, N., "Structural Evaluation Asphalt Pavements from Laboratory Material Testing", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway, pp 531-540.

Lenngren, C.A., “Pavement Strain Versus Backcalculated Data”, Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Chong Ket Pen, Institute of Training and Research Malaysia. Public Works Department Malaysia. "Assessment of the Available Methods of Analysis for Estimating the Elastic Moduli of Road Pavements", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Uzan, J., Scullion, T., "Verification of Backcalculation procedures", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway.

Chou, Y.J., Lytton, R.L., "Accuracy and of Backcalculated Pavement Layer Moduli", Transportation Research Record 1293, p. 72-85, 1991.

Hall, K.T., Mohseni, A., "Backcalculation of Concrete-Overlaid Portland Cement Concrete Pavement Layer Moduli", Transportation Research Record 1293, pp. 112-123, 1991.

Maestas, J.M., Mamlouk, M.S., "Comparison of Pavement Deflection Analysis Methods Using Overlay Design", Transportation Research Record 1377, 1992.

Rada, G.R., Richter, C.A., and Stephanos, P.J., "Layer Moduli from Deflection Measurements: Software Selection and Development of Strategic Highway Research Program's Procedure for Flexible Pavements", Transportation Research Record 1377, 1992.

Nazarian, S., Chai, Y.E., "Comparison of Theoretical Case study and In Situ Behaviors of a Flexible Pavement Section",Transportation Research Record 1377, pp.172-182, 1992. (geophones)

Kim, Y.R., Khosla, N.P., Satish, S., Scullion, T., “Validation of Moduli Backcalculation Procedure Using Multidepth Deflectometers Installed in Various Flexible Pavement Structures”, Transportation Research Record 1377, pp. 128-142, 1992.

Kim, Ok-Kee and Nokes, William A., “Evaluation of Backcalculation Methods to Predict Pavement Layer Moduli”, Report No. FHWA/CA/TL-94/11, California Dept. of Transportation, Sacramento, CA 95814, August 1993. (also analyses many factors effecting backcalculation)

Applications of Backcalculation Results

Elliott, R.P., Thompson, M.R., "ILLI-PAVE Mechanistic Analysis of ASHO Road Test Flexible P avements", Transportation Research Record 1043, 1985, pp. 39-49.

Thomson,M.R., Elliott, R.P., "ILLI-PAVE-Based Response Algorithms for Design of Conventional Flexible Pavements", Transportation Research Record 1043, 1985, pp. 50-56.

Brown, S.F., Tam, W.S., Brunton, J.M., "Structural Evaluation and Overlay Design: Analysis and Implementation, "Proceedings, Sixth International Conference on Structural Design of Asphalt Pavements, The University of Michigan, Ann Arbor, 1987, Vol I, pp. 1013-1028.

Hall, K., Darter, M., Carpenter, S., Connor, J., "Development of a Demonstration Prototype Expert System for Concrete Pavement Evaluation", Transportation Research Record 1117, pp. 58-65, 1987.

Zhou, H., Hicks, R.G., Huddleston, I.J., "Evaluation of the 1986 AASHTO Overlay Design Method", Transportation Research Record 1215,pp 299-316, 1989.

Zhou, H, Hicks, R.G., Connor, B., "Improved Overlay Design Procedure For the State of Alaska, Volume IV, Computer Programs", June 1989.

Alexander, D.R., Kohn, S.D., Grogan, W.P., "Nondestructive Testing Techniques and Evaluaiton Procedures for Airfield Pavements", *Nondestructive Testing of Pavements and Backcalculation of Moduli,* ASTM STP 1026, A.J. Bush III and G.Y. Baladi, Eds., American Society ofr Testing and Materials, Philadelphia, 1989, pp. 502-524. (Application of BISDEF)

Ullidtz, P., Stubstad, R. N., "Why Bother About Moduli?", Third International Conference on Bearing Capacity of Roads and Airfields, 3-5 July 1990, Trondheim, Norway, pp 659-667.

Darter, M.I., Smith, K.D., and Hall, K.T., "Concrete Pavement Backcalculation Results from Field Studies", Transportation Research Record 1377, 1992, pp. 7-16. (rigid pavements)

Zhou, H., Huddleston, J., Lundy, J., "Implementation of Backcalculation in Pavement Evaluation and Overlay Design in Oregon", Transportation Research Record 1377, pp. 150-158, 1992. (BOUSDEF)

Dynatest Consulting, Inc., and Soils and Materials, Inc., "Pavement Deflection Analysis: Participant Workbook", prepared for the FHWA National Highway Institute, 1993.

Wu, C., Todres, H.A., "Evaluation of Roller-Compacted Concrete Pavements Using Nondestructive Load Testing", Transportation Research Record 1473, 1995, pp. 82-92. (Application of ECOPP program)

Hall, K.D., Watkins, Q.B., "Comparison of AASHTO and ROADHOG Flexible Pavement Overlay Design Procedures", Transportation Research Record 1482, 1995, pp.94-101.

Killingsworth, B.M., Zollinger, D.G., "Sensitivity Analysis of Input Parameters for Pavement Design and Reliability", Transportation Research Record 1482, 1995, pp. 111-122.

Chen, D., Zaman, M., Laguros, J., Soltani, A., "Assessment of Computer Programs for Analysis of Flexible Pavement Structure", Transportation Research Record 1482, 1995, pp. 123-133.

Rada, G.R., Shepherd, K.L., Zeigler, T.D., Smith T.E., "Montana's Automated Deflection Analysis Procedure", Transportation Research Record 1570, 1997, pp. 151-162. (An application of MODULUS)

Khanal, P.P., Mamlouk, M.S., "Program BIMODPAV for Analysis of Flexible Pavements", Journal of Transportation Engineering, Vol. 123, No. 1, January/February, 1997, pp. 43-50. (Deals with non-isotropic properties of asphalt)

Appendix 3A

Table 3A-1: Coefficients for Empirical Relationships

		For High AC Moduli			For Average AC Moduli			For Low AC Modduli		
h1	h2	C	M	R^2	C	M	R^2	C	M	R^2
2	6	0.0068	0.6383	0.976	0.0146	0.6071	0.9031	0.0219	0.6005	0.9674
2	10	0.0207	0.6448	0.9768	0.03	0.6446	0.8963	0.0311	0.6678	0.9638
2	14	0.0407	0.6569	0.9767	0.0456	0.6773	0.8921	0.0398	0.7153	0.9625
2	18	0.064	0.6714	0.9758	0.0611	0.7049	0.89	0.0488	0.07503	0.9623
2	22	0.0887	0.6863	0.9748	0.0767	0.728	0.8891	0.059	0.7772	0.9627
2	26	0.1143	0.7005	0.974	0.0929	0.7477	0.8889	0.0704	0.7967	0.9632
2	30	0.1403	0.7139	0.9733	0.1097	0.7646	0.8891	0.0832	0.8162	0.9638
2	34	0.1668	0.7263	0.9728	0.1274	0.7793	0.8894	0.0974	0.8308	0.9643
2	38	0.1938	0.7378	0.9724	0.1462	0.7922	0.8899	0.1132	0.8431	0.9648
2	42	0.2213	0.7483	0.9722	0.1661	0.8036	0.8904	0.1311	0.8535	0.9606
4	6	0.0025	0.6105	0.979	0.0058	0.5699	0.9098	0.0126	0.5328	0.9684
4	10	0.0047	0.636	0.9762	0.0112	0.597	0.9051	0.0191	0.5796	0.9683
4	14	0.0095	0.6396	0.9762	0.0174	0.6175	0.9226	0.0244	0.6197	0.9663
4	18	0.0165	0.6427	0.9767	0.0261	0.6356	0.8978	0.0289	0.6532	0.9645
4	22	0.0253	0.6474	0.9769	0.0339	0.6533	0.895	0.0332	0.6812	0.9633
4	26	0.0353	0.6535	0.9768	0.0417	0.6696	0.893	0.0375	0.7048	0.9627
4	30	0.0463	0.6604	0.9765	0.0495	0.6847	0.8915	0.0419	0.725	0.9624
4	34	0.058	0.6677	0.9761	0.0572	0.6984	0.8904	0.0464	0.7424	0.9623
4	38	0.0701	0.6752	0.9756	0.065	0.711	0.8897	0.0513	0.7577	0.9624
4	42	0.0824	0.6826	0.9751	0.0728	0.7226	0.8893	0.0564	0.7711	0.9626
6	6	0.0028	0.5545	0.9786	0.0047	0.5367	0.9118	0.0094	0.5041	0.9692
6	10	0.0027	0.6193	0.9783	0.0065	0.5761	0.9069	0.0138	0.5411	0.9685
6	14	0.0042	0.6346	0.9765	0.0101	0.5934	0.9058	0.0181	0.5722	0.9685
6	18	0.0068	0.6383	0.976	0.0146	0.6071	0.9031	0.0219	0.6005	0.9674
6	22	0.0105	0.64	0.9762	0.0196	0.62	0.9006	0.0252	0.6257	0.966
6	26	0.0152	0.642	0.9766	0.0248	0.6326	0.8983	0.0282	0.648	0.9648
6	30	0.0207	0.6448	0.9768	0.03	0.6446	0.8963	0.0311	0.6678	0.9638
6	34	0.0269	0.6483	0.9769	0.0352	0.6561	0.8946	0.0339	0.6854	0.9632
6	38	0.0336	0.6524	0.9769	0.0405	0.667	0.8933	0.0368	0.7011	0.9627
6	42	0.0407	0.6569	0.9767	0.0456	0.6773	0.8921	0.0398	0.7153	0.9625
8	6	0.0043	0.4954	0.9729	0.0052	0.5013	0.9093	0.0085	0.4828	0.9714
8	10	0.0024	0.5897	0.9796	0.0051	0.5571	0.9111	0.011	0.5195	0.9684
8	14	0.0028	0.6226	0.978	0.0069	0.5788	0.9084	0.0144	0.5451	0.9685
8	18	0.0039	0.6337	0.9766	0.0096	0.5916	0.9061	0.0176	0.5685	0.9685
8	22	0.0057	0.6374	0.9761	0.0129	0.6021	0.9041	0.0206	0.5902	0.9679
8	26	0.0081	0.639	0.9761	0.0165	0.612	0.9021	0.0232	0.6103	0.9669
8	30	0.0111	0.6402	0.9763	0.0203	0.6216	0.9003	0.0256	0.6286	0.9658
8	34	0.0146	0.6417	0.9765	0.0241	0.631	0.8986	0.0278	0.6454	0.9649
8	38	0.0185	0.6437	0.9768	0.028	0.6401	0.897	0.03	0.6607	0.9642
8	42	0.0229	0.6461	0.9769	0.032	0.649	0.8957	0.032	0.649	0.8957
10	6	0.0069	0.4427	0.9647	0.0064	0.4672	0.9036	0.0085	0.4633	0.9726
10	10	0.0028	0.5545	0.9786	0.0047	0.5367	0.9118	0.0094	0.5041	0.9692
10	14	0.0024	0.6034	0.9793	0.0055	0.5654	0.9103	0.012	0.5276	0.9683
10	18	0.0029	0.6243	0.9778	0.0071	0.5803	0.9082	0.0147	0.5475	0.9686
10	22	0.0038	0.6331	0.9767	0.0093	0.5905	0.9063	0.0173	0.5662	0.9686
10	26	0.0051	0.6367	0.9761	0.0119	0.5991	0.9047	0.0197	0.5839	0.9681
10	30	0.0068	0.6383	0.976	0.0121	0.6206	0.9091	0.0219	0.6005	0.9674
10	34	0.0089	0.6394	0.9761	0.0176	0.6149	0.9016	0.0239	0.616	0.9666
10	38	0.0114	0.6404	0.9763	0.0206	0.6226	0.9001	0.0258	0.6304	0.9657
10	42	0.0142	0.6416	0.9765	0.0237	0.6301	0.8987	0.0276	0.6438	0.965

Appendix 3B
The Program

This appendix presents the computer program KISS and the required subroutines written in Fortran77.

KISS.FOR

```
* This program was developed to backcalculate moduli values for the case of asphalt
* concrete over a stabilized base.
*
      implicit real*8 (a-h,o-z)
      double precision MU,kay
      double precision h(100),Ebseed(100),D(100),k(100),
     c          x(100),delta(100),Dseed(100),kseed(100)
      character*25 name0,name1
*
* opening batch file
*
      PRINT*, 'Enter name of batch file '
      READ*, name0
      OPEN (UNIT = 9, FILE = name0, STATUS = 'OLD')
      READ (9,*) l
*
*  opening input and output files
*
      OPEN (UNIT=11, FILE='out/4_10.out', STATUS='NEW')
*
      DO 30 iz = 1, l
        READ (9,*) name1
        OPEN (UNIT=10, FILE=name1, STATUS='OLD')
*
        READ (10,*) n,m
        READ (10,*) MU,P,A,F,C,G,XO,YO,PERR
        DO 20 j = 1,n
          read (10,*) h(j)
   20   CONTINUE
        DO 25 j = 1,m
          read (10,*) x(j),delta(j)
   25   CONTINUE
*
* closing input file
        CLOSE(10)
*
* determining the appropriate look up table.
*
        CALL LUT
*
*  Obtaining C and M from the look up table.
*
        CALL CEEEMM(h,cee,emm)
*
```

```
      PRINT*, ''
      WRITE (11,51) cee,emm
   51 FORMAT (1X,D20.10,1X,D20.10)
*
*  D k Search
*
*  The program runs through a series of D seeds and finds the one which
*  gives the smallest finlQ. It then uses the result with the smallest
*  finlQ as Dseed(1). Dseeds(2) and (3) are Dseed(1) plus and minus
*  20% of Dseed(1). The kseeds are kept at 100, 1000, and 50. Another
*  search using plus and minus 10% of the D that provided the "best"
*  finl Q is performed. A final search using plus and minus 5% of the
*  D that provided the"best" final Q is performed.The extra Dksearches
*  are performed to reduce errors in backcalculated D and k values.
*  Once the D and k are obtained, they are used in conjunction with
*  the empirical equations to search for Es and Eb. Eb is given an
*  upper limit (designated Ebul) of 2000000 for a stabilized base a lower
*  limit (designated Ebll) of 100000 for a stabilized base.
*
* initial seeds for Dksearch
*
      Dseed(1) = 0.1D+07
      kseed(1) = 0.1D+03
      Dseed(2) = 0.5D+07
      kseed(2) = 0.1D+04
      Dseed(3) = 0.2D+07
      kseed(3) = 0.5D+02
*
*   notation for input parameters
*
*   l        = number of pavements in batch
*   n        = number of layers in the composite
*   m        = number of measured deflections from NDT
*   h(i)     = thickness of ith layer in inches
*   MU       = Poisson's ratio (assumed same for all layers)
*   k(i),D(i) = seed values for modulus of subgrade reaction (pci) and
*               Dbar values (lb-in)
*   P        = Load intensity in psi
*   (A,F)    = plate dimensions in inches
*   (C,G)    = load dimenesions in inches
*   (XO,YO)  = location of center of load
*   PERR     = percent error acceptable for calculated deflections
*   x(j)     = location of jth measured deflecton from center of load (inches)
*   delta(j) = depth of jth measured deflection (inches)
*   Ebseed(j) = seed values for Eb (psi)
*   Cee,emm  = coefficient and exponent for impirical equation for Es=f(Eb)
*
* calculating double precision PI (recall that PI/4 = arctan of 1.0)
      PI = (4.0d+00) * DATAN(1.0D+00)
*
*  searching for D and k (first round)
*
      DO 50 int1 = 1,7
```

```
*
   k(1) = kseed(1)
   D(2) = Dseed(2)
   k(2) = kseed(2)
   D(3) = Dseed(3)
   k(3) = kseed(3)
*
   CALL DKSEARCH(D,k,m,A,F,C,G,P,PI,YO,XO,
  c          X,PERR,DELTA,finlD,finlk,finlQ)
*
*  first round... run through a series of Dseeds and find smallest finlQ
*
     IF (int1.EQ.1) THEN
       fD = finlD
       fQ = finlQ
       nt = int1
     ENDIF
     IF (int1.GE.2) THEN
       IF (finlQ.LT.fQ) THEN
         fD = finlD
         fQ = finlQ
         nt = int1
       ENDIF
     ENDIF
*
     Dseed(1) = Dseed(1)*10D+00
     Dseed(2) = Dseed(2)*10D+00
     Dseed(3) = Dseed(3)*10D+00
*
  50 CONTINUE
*
*    second DKSEARCH uses best finlD (smallest finlQ) from first
*    DKSEARCH and + or - 20% of itself as Dseeds.
*
    D(1) = fD
    k(1) = kseed(1)
    D(2) = fD + fD*0.20D+00
    k(2) = kseed(2)
    D(3) = fD - fD*0.20D+00
    k(3) = kseed(3)
    write(*,77) fD
  77 format (1x,'fD = ',D20.10)
    CALL DKSEARCH(D,k,m,A,F,C,G,P,PI,YO,XO,
   c          X,PERR,DELTA,finlD,finlk,finlQ)
*
*  third DKSEARCH, uses plus and minus 10 percent of most recent D value
*  as seeds
*
    D(1) = finlD
    k(1) = kseed(1)
    D(2) = finlD + finlD*0.10D+00
    k(2) = kseed(2)
    D(3) = finlD - finlD*0.10D+00
```

```
      k(3) = kseed(3)
      CALL DKSEARCH(D,k,m,A,F,C,G,P,PI,YO,XO,
     c          X,PERR,DELTA,finlD,finlk,finlQ)
*
* fourth DKSEARCH, uses plus and minus 5 percent of most resent D value
* as seeds
*
      D(1) = finlD
      k(1) = kseed(1)
      D(2) = finlD + finlD*0.05D+00
      k(2) = kseed(2)
      D(3) = finlD - finlD*0.05D+00
      k(3) = kseed(3)
      CALL DKSEARCH(D,k,m,A,F,C,G,P,PI,YO,XO,
     c          X,PERR,DELTA,finlD,finlk,finlQ)
*
       Dbar = finlD
       kay  = finlk
* results are used to calculate Enb
*
      Enb = Dbar*12*(1-mu*mu)/(h(1)**3)
*
* searching for Eb and Es
*
* initial Eb seeds
*
      Ebseed(1) = 200000.0D+00
      Ebseed(2) = 500000.0D+00
*
* limits on Eb
*
       Ebul = 2000000.0D+00
       Ebll = 100000.0D+00
*
        CALL Ebase(Dbar,Cee,emm,Enb,Ebseed,mu,h,Ebul,Ebll,
     c          finlEs,finlEb,finlF)
*
       Es = finlEs
       Eb = finlEb
       fF = finlF
*
* writing to output file
*
      write(11,21) nt,Dbar,kay,finlQ,Es,Eb,fF
   21 format (1x,i3,1x,D20.10,1x,D10.5,1x,D10.5,1x,D20.10,1x,
     c      D20.10,1x,D10.5)
*
*   next data set in the batch
   30 CONTINUE
*
      write(*,99)
   99 format(1x,'Stop the world, I wanna get off!!!')
*
```

```
* closing batch file, output file, and look up table file.
*
      CLOSE(9)
      CLOSE(11)
      CLOSE(12)
*
      END
```

SUBROUTINES (presented in the order they are called):

LUT.FOR

```
* subroutine to determine the correct lookup table and open it.
*
      SUBROUTINE LUT
*
 40   PRINT*, 'input case: 1 = strong E > 1500000 psi'
      PRINT*, '            2 = average 700000 < E < 1500000 psi'
      PRINT*, '            3 = weak   E < 700000 psi'
      PRINT*,' '
      READ*, icase
*
* Opening the correct lookup table
*
      IF (icase.EQ.1) THEN
        OPEN (UNIT=12, FILE='lut/stacsbcm.dat', ACCESS = 'direct',
     c        STATUS='old', FORM = 'formatted', RECL = 42)
      ELSEIF (icase.EQ.2) THEN
        OPEN (UNIT=12, FILE='lut/avacsbcm.dat', ACCESS = 'direct',
     c        STATUS='old', FORM = 'formatted', RECL = 42)
      ELSEIF (icase.EQ.3) THEN
        OPEN (UNIT=12, FILE='lut/wkacsbcm.dat', ACCESS = 'direct',
     c        STATUS='old', FORM = 'formatted', RECL = 42)

      ELSE
        PRINT*, 'INVALID CASE NUMBER, SHOULD BE 1,2,OR 3'
        GOTO 40
      ENDIF
*
      RETURN
      END
```

CEEEMM.FOR

```
* Subroutine to determine which record to use to obtain cee and emm values
* and to determine if interpolation is necessary and if so, do it.
*
      SUBROUTINE CEEEMM(ZH,cee,emm)
*
      implicit real*8 (a-h,o-z)
      double precision zh(100),ache(100)
      logical nol1,nol2

*
```

```
      ache(1) = zh(1)
      ache(2) = zh(2)
*
      CALL record(ache,rcrd,nol1,nol2)
*
      IF (.NOT.nol1.AND..NOT.nol2) THEN
        WRITE (11,55) RCRD
 55     FORMAT (1X,'RCRD = ',I3)
        READ (12,50,REC = RCRD) CEE, EMM
        WRITE (11,50) CEE,EMM
 50     FORMAT (1X,D20.10,1X,D20.10)
*
      ELSEIF (nol1.AND..NOT.nol2) THEN
*
        ache(1) = zh(1)-1
        ache(2) = zh(2)
        CALL record(ache,rcrd,nol1,nol2)
        WRITE (11,55) RCRD
        READ (12,50,REC = RCRD) CEE, EMM
        WRITE (11,50) CEE,EMM
        CE1 = cee
        EM1 = emm
*
        ache(1) = zh(1)+1
        CALL record(ache,rcrd,nol1,nol2)
        WRITE (11,55) RCRD
        READ (12,50,REC = RCRD) CEE, EMM
        WRITE (11,50) CEE,EMM
        CE2 = cee
        EM2 = emm
        CALL INTERP2A(CE1,EM1,CE2,EM2,CEI,EMI)
        CEE = CEI
        EMM = EMI
*
      ELSEIF (.NOT.nol1.AND.nol2) THEN
*
        ache(1) = zh(1)
        CALL lowhi(zh(2),h2lo,h2hi)
        ache(2) = h2lo
        CALL record(ache,rcrd,nol1,nol2)
        WRITE (11,55) RCRD
        READ (12,50,REC = RCRD) CEE, EMM
        WRITE (11,50) CEE,EMM
        CE1 = cee
        EM1 = emm
*
        ache(2) = h2hi
        CALL record(ache,rcrd,nol1,nol2)
        WRITE (11,55) RCRD
        READ (12,50,REC = RCRD) CEE, EMM
        WRITE (11,50) CEE,EMM
        CE2 = cee
        EM2 = emm
```

```
*
      CALL INTERP2B(CE1,EM1,CE2,EM2,zh(2),H2LO,H2HI,CEI,EMI)
      CEE = CEI
      EMM = EMI
*
    ELSE
* assuming both nol1 and nol2 are true
*
      CALL lowhi(zh(2),h2lo,h2hi)
      ache(1) = zh(1)-1
      ache(2) = h2lo
      CALL record(ache,rcrd,nol1,nol2)
      WRITE (11,55) RCRD
      READ (12,50,REC = RCRD) CEE, EMM
      WRITE (11,50) CEE,EMM
      CE1 = cee
      EM1 = emm
*
      ache(1) = zh(1)+1
      CALL record(ache,rcrd,nol1,nol2)
      WRITE (11,55) RCRD
      READ (12,50,REC = RCRD) CEE, EMM
      WRITE (11,50) CEE,EMM
      CE2 = cee
      EM2 = emm
*
      ache(1) = zh(1)-1
      ache(2) = h2hi
      CALL record(ache,rcrd,nol1,nol2)
      WRITE (11,55) RCRD
      READ (12,50,REC = RCRD) CEE, EMM
      WRITE (11,50) CEE,EMM
      CE3 = cee
      EM3 = emm
*
      ache(1) = zh(1)+1
      CALL record(ache,rcrd,nol1,nol2)
      WRITE (11,55) RCRD
      READ (12,50,REC = RCRD) CEE, EMM
      WRITE (11,50) CEE,EMM
      CE4 = cee
      EM4 = emm
*
      CALL INTERP4(CE1,EM1,CE2,EM2,CE3,EM3,CE4,EM4,zh(2),
     C           H2LO,H2HI,CEI,EMI)
      CEE = CEI
      EMM = EMI
*
    ENDIF
*
    RETURN
    END
```

RECORD.FOR

```
* subroutine to determine the record number to be used in the
* look up table to obtain the cee and emm values.
*
      SUBROUTINE RECORD(zache,rcrd,nol1,nol2)
      implicit real*8 (a-h,o-z)
      double precision zache(100)
      logical nol1,nol2
*
*  determining the record number which locates the appropriate cee
*  and emm values.
*
      RCRD = 0
      nol1 = .false.
      nol2 = .false.
*
* looking at h(1)
*
      IF (zache(1).EQ.2) THEN
        RCRD = 0
      ELSEIF (zache(1).EQ.4) THEN
        RCRD = 10
      ELSEIF (zache(1).EQ.6) THEN
        RCRD = 20
      ELSEIF (zache(1).EQ.8) THEN
        RCRD = 30
      ELSEIF (zache(1).EQ.10) THEN
        RCRD = 40
      ELSE
        nol1 = .true.
      ENDIF
*
* looking at h(2)
*
      IF (zache(2).EQ.6) THEN
        RCRD = RCRD + 1
      ELSEIF (zache(2).EQ.10) THEN
        RCRD = RCRD + 2
      ELSEIF (zache(2).EQ.14) THEN
        RCRD = RCRD + 3
      ELSEIF (zache(2).EQ.18) THEN
        RCRD = RCRD + 4
      ELSEIF (zache(2).EQ.22) THEN
        RCRD = RCRD + 5
      ELSEIF (zache(2).EQ.26) THEN
        RCRD = RCRD + 6
      ELSEIF (zache(2).EQ.30) THEN
        RCRD = RCRD + 7
      ELSEIF (zache(2).EQ.34) THEN
        RCRD = RCRD + 8
      ELSEIF (zache(2).EQ.38) THEN
        RCRD = RCRD + 9
      ELSEIF (zache(2).EQ.42) THEN
```

```
  RCRD = RCRD + 10
ELSE
  nol2 = .true.
ENDIF
*
RETURN
END
```

LOHI.FOR

```
* This subroutine is to take the value of h(2) and determine the upper and
* lower h(2) values to be used in the interpolation of cee and emm values.
*
   SUBROUTINE LOWHI(zh2,h2lo,h2hi)
*
   implicit real*8 (a-h,o-z)
*
   IF (6.LT.ZH2.AND.ZH2.LT.10) THEN
     h2lo = 6
     h2hi = 10
   ELSEIF (10.LT.ZH2.AND.ZH2.LT.14) THEN
     h2lo = 10
     h2hi = 14
   ELSEIF (14.LT.ZH2.AND.ZH2.LT.18) THEN
     h2lo = 14
     h2hi = 18
   ELSEIF (18.LT.ZH2.AND.ZH2.LT.22) THEN
     h2lo = 18
     h2hi = 22
   ELSEIF (22.LT.ZH2.AND.ZH2.LT.26) THEN
     h2lo = 22
     h2hi = 26
   ELSEIF (26.LT.ZH2.AND.ZH2.LT.30) THEN
     h2lo = 26
     h2hi = 30
   ELSEIF (30.LT.ZH2.AND.ZH2.LT.34) THEN
     h2lo = 30
     h2hi = 34
   ELSEIF (34.LT.ZH2.AND.ZH2.LT.38) THEN
     h2lo = 34
     h2hi = 38
   ELSEIF (38.LT.ZH2.AND.ZH2.LT.42) THEN
     h2lo = 38
     h2hi = 42
   ELSE
     PRINT*,'THE SKY IS FALLING'
   ENDIF
*
   RETURN
   END
```

INTERP2A.FOR

```
* this subroutine is to interpolate between two values to obtain the
```

```
* cee and emm values needed. For the case when h1 is NOT on the list
* and h2 is.
*
      SUBROUTINE INTERP2A(zCE1,zEM1,zCE2,zEM2,CEI,EMI)
*
      implicit real*8 (a-h,o-z)
*
      CEI = ((zCE1-zCE2)/(-2))+zCE1
*
      EMI = ((zEM1-zEM2)/(-2))+zEM1
*
      RETURN
      END
```

INTERP2B.FOR

```
* this subroutine is to interpolate between two values to obtain the
* cee and emm values needed. For the case when h1 is on the list
* and h2 is NOT.
*
      SUBROUTINE INTERP2B(zCE1,zEM1,zCE2,zEM2,ZH,ZH2LO,ZH2HI,CEI,EMI)
*
      implicit real*8 (a-h,o-z)
*
      CEI = ((zCE1-zCE2)/(zH2LO-zH2HI))*(ZH-ZH2LO)+zCE1
*
      EMI = ((zEM1-zEM2)/(zH2LO-zH2HI))*(ZH-ZH2LO)+zEM1
*
      RETURN
      END
```

INTERP4.FOR

```
* this subroutine is to interpolate between two values to obtain the
* cee and emm values needed. For the case when h1 is not on the list
* and h2 is.
*
      SUBROUTINE INTERP4(zCE1,zEM1,zCE2,zEM2,zCE3,zEM3,zCE4,zEM4,zH,
     C                   zH2LO,zH2HI,CEI,EMI)
*
      implicit real*8 (a-h,o-z)
      double precision M1,M2
*
      C1 = ((zCE1-zCE2)/(-2))+zCE1
*
      C2 = ((zCE3-zCE4)/(-2))+zCE3
*
      M1 = ((zEM1-zEM2)/(-2))+zEM1
      M2 = ((zEM3-zEM4)/(-2))+zEM3
*
      CEI = ((C1-C2)/(zH2LO-zH2HI))*(ZH-ZH2LO)+C1
*
      EMI = ((M1-M2)/(zH2LO-zH2HI))*(ZH-ZH2LO)+M1
*
```

```
      RETURN
      END
```

DKSRCHR.FOR

```
* multi-directional search for solution to Q
      SUBROUTINE DKSEARCH(zD,zk,m,zA,zF,zC,zG,zP,zPI,zYO,zXO,
     c         zX,zPERR,ZDELTA,finlD,finlk,finlQ)
      implicit real*8 (a-h,o-z)
      double precision kay
      double precision zQ(100),zdelta(100),rD(100),
     c         rk(100),eD(100),ek(100),cD(100),
     c         ck(100),zD(100),zk(100),rQ(100),
     c         eQ(100),cQ(100)
      logical REPLACED, NEG(100)
*
* expansion factor
      znu = 2.0D+00
*
* contraction factor
      theta = 0.5D+00
*
* start with initial Simplex of three (D,k) pairs (input for now)
*
* initialization loop
*
* L = loop counter
      L = 0
      DO 10 i = 1,3
        Dbr = zD(i)
        kay = zk(i)
        zQ(i) = Q(m,Dbr,zA,zF,zC,zG,kay,zP,zPI,zYO,zXO,zX,
     c        zPERR,zDELTA)
   10 CONTINUE
*
* outer while loop
*
   20 IF (zQ(1).GT.(1.0D-10)) THEN
       REPLACED = .FALSE.
*
* find new best vertex
*
         IF (zQ(1).GT.zQ(2)) THEN
           TEMP1 = zQ(1)
           TEMP2 = zD(1)
           TEMP3 = zk(1)
           zQ(1) = zQ(2)
           zD(1) = zD(2)
           zk(1) = zk(2)
           zQ(2) = TEMP1
           zD(2) = TEMP2
           zk(2) = TEMP3
         ENDIF
```

```
      IF (zQ(1).GT.zQ(3)) THEN
        TEMP1 = zQ(1)
        TEMP2 = zD(1)
        TEMP3 = zk(1)
        zQ(1) = zQ(3)
        zD(1) = zD(3)
        zk(1) = zk(3)
        zQ(3) = TEMP1
        zD(3) = TEMP2
        zk(3) = TEMP3
      ENDIF
      Do 90 i = 1,3
 90   continue
    ENDIF
*
* check stopping criteria
*
    IF (zQ(1).LT.(1.0D-10)) THEN
      GOTO 100
    ENDIF
*
*  check if all values are same (to avoid endless loop)
*
 25 z1 = ABS(zD(1)-zD(2))
    z2 = ABS(zD(1)-zD(3))
*
    IF ((z1.LT.(0.1D+00)).AND.
   C   (z2.LT.(0.1D+00))) THEN
      GOTO 100
    ENDIF
*
*  inner repeat loop
*
*   rotation step
*
      DO 30 i = 2,3
        NEG(i) = .FALSE.
        rD(i) = zD(1) - (zD(i)-zD(1))
        rk(i) = zk(1) - (zk(i)-zk(1))
*
*  in the event that either D or k becomes negative
*
        IF (rD(i).LE.(0.0D+00).OR.rk(i).LE.(0.0D+00)) THEN
          NEG(i) = .TRUE.
*  in the event of a vertical shift (slope undefined)
          IF (zD(i).EQ.zD(1)) THEN
            rD(i) = zD(1)
            rk(i) = 1.0D+00
*  in the event of a horizontal shift (slope is zero)
          ELSEIF (zk(i).EQ.zk(1)) THEN
            rD(i) = 1.0D+00
            rk(i) = zk(1)
*  if shift is nieither horizontal nor vertical
```

```
          ELSE
            SLOPE = (zk(i)-zk(1))/(zD(i)-zD(1))
            b = zk(1)-SLOPE*zD(1)
            IF (b.GE.(0.0D+00)) THEN
              rD(i) = 1.0D+00
              rk(i) = b+SLOPE
            ELSE
              rD(i) = (1.0D+00-b)/SLOPE
              rk(i) = 1.0D+00
            ENDIF
          ENDIF
        ENDIF
*
*  calculation of Q(rD(i),rk(i))
*
        Dbr = rD(i)
        kay = rk(i)
        rQ(i) = Q(m,Dbr,zA,zF,zC,zG,kay,zP,zP1,zYO,zXO,zX,
     c         zPERR,zDELTA)
*
 30   CONTINUE
*
        IF (DMIN1(rQ(2),rQ(3)).LT.zQ(1)) THEN
          REPLACED = .TRUE.
        ENDIF
*
*  if rotation is a success, then expansion step is performed
*  unless we are on the boundary
*
       IF (REPLACED) THEN
          DO 40 i = 2,3
            IF (.NOT.NEG(i)) THEN
*             do expansion if neg(i) = false
              eD(i) = zD(1) - znu*(zD(i)-zD(1))
              ek(i) = zk(1) - znu*(zk(i)-zk(1))
*
*  again we must check that D or k has not become negative
*
              IF ((eD(i).LE.(0.0D+00)).OR.(ek(i).LE.(0.0D+00)))
     c         THEN
*  in the event of a vertical shift (slope undefined)
                IF (zD(i).EQ.zD(1)) THEN
                  eD(i) = zD(1)
                  ek(i) = 1.0D+00
*  in the event of a horizontal shift (slope is zero)
                ELSEIF (zk(i).EQ.zk(1)) THEN
                  eD(i) = 1.0D+00
                  ek(i) = zk(1)
*  if shift is nieither horizontal nor vertical
                ELSE
                  SLOPE = (zk(i)-zk(1))/(zD(i)-zD(1))
                  b = zk(1)-SLOPE*zD(1)
                  IF (b.GE.(0.0D+00)) THEN
```

```
                  eD(i) = 1.0D+00
                  ek(i) = b+SLOPE
                ELSE
                  eD(i) = (1.0D+00-b)/SLOPE
                  ek(i) = 1.0D+00
                ENDIF
              ENDIF
            ENDIF
*
*  calculation of Q(eD(i),ek(i))
*
            Dbr = eD(i)
            kay = ek(i)
            eQ(i) = Q(m,Dbr,zA,zF,zC,zG,kay,zP,zPI,zYO,zXO,
  c                 zx,zPERR,zDELTA)
*
            IF ((eQ(i)).LT.zQ(1)) THEN
*           accept expansion
              zD(i) = eD(i)
              zk(i) = ek(i)
              zQ(i) = eQ(i)
            ELSE
*           accept rotation
              zD(i) = rD(i)
              zk(i) = rk(i)
              zQ(i) = rQ(i)
            ENDIF
          ELSE
*       if neg(i) = true, then eccept rotation without doing expansion
*
            zD(i) = rD(i)
            zk(i) = rk(i)
            zQ(i) = rQ(i)
          ENDIF
 40       CONTINUE
*
        Do 91 i = 1,3
 91       Continue
        L = L+1
        GOTO 20
      ELSEIF (.NOT.REPLACED) THEN
*       contraction step, is automatically accepted wether replaced
*       is true or not. Contractions will never be negative, so there
*       is no need to check.
*
        DO 50 i = 2,3
          cD(i) = zD(1) + theta*(zD(i)-zD(1))
          ck(i) = zk(1) + theta*(zk(i)-zk(1))
*
*         calculating cQ(cD(i),ck(i))
*
          Dbr = cD(i)
          kay = ck(i)
```

```
      cQ(i) = Q(m,Dbr,zA,zF,zC,zG,kay,zP,zPI,zYO,zXO,zX,
 c           zPERR,zDELTA)
      zD(i) = cD(i)
      zk(i) = ck(i)
      zQ(i) = cQ(i)
 50   CONTINUE
*
      IF (DMIN1(cQ(2),cQ(3)).LT.zQ(1)) THEN
        REPLACED = .TRUE.
        L = L+1
        GOTO 20
      ELSE
        GOTO 25
      ENDIF
    ENDIF
*
 100 finlD = zD(1)
   finlk = zk(1)
   finlQ = zQ(1)
   RETURN
   END
```

QUE.FOR

```
* to calculate the function (Q) to be optimized in DkSEARCH
   FUNCTION Q(m,zDbr,zA,zF,zC,zG,zkay,zP,zPI,zYO,zXO,zX,
  c       zPERR,zdelta)
   implicit real*8 (a-h,o-z)
   double precision zdelta(100),zX(100),w(100)
*
* to generate deflection bowl
*
   CALL dBOWL(m,zDbr,zA,zF,zC,zG,zkay,zP,zPI,zYO,zXO,zX,
  c       zPERR,w,j2max,j3max)
*
* to calculate the equation to be optimized
*
   term = 0.0D+00
   Q    = 0.0D+00
   DO 10 j = 1, m
     term = (w(j) - zdelta(j))*(w(j) - zdelta(j))
     Q    = Q+term
     term = 0.0D+00
 10 CONTINUE
*
   RETURN
   END
```

DBOWL.FOR

```
* subroutine for calculating the deflections.
   SUBROUTINE DBOWL(m,zDbr,zA,zF,zC,zG,zkay,zP,zPI,zyo,zxo,zx,
  c       zperr,w,j2max,j3max)
   implicit real*8 (a-h,o-z)
```

```
      double precision w(100),zx(100)
      logical even
*
*  checking for convergence to allowable error (PERR)
*
      DCOEF11 = DCOEF(1.0D+00,1.0D+00,zA,zF,zKay,zDbr,zP,zPI)
      CONV = zPERR*DCOEF11/100
      S = 1.0D+00
      U = 1.0D+00
      L = 1
      EVEN = .FALSE.
      DCOEFMN = DCOEF(S,U,zA,zF,zKay,zDbr,zP,zPI)
    2 IF (DCOEFMN.GT.CONV) THEN
        IF (l .eq. 1) THEN
          EVEN = .TRUE.
          l = 2
          U = 2.0D+00
        ELSEIF (EVEN) THEN
          EVEN = .FALSE.
          S = S + 1.0D+00
          l = l + 1
        ELSE
          EVEN = .TRUE.
          U = U + (1.0D+00)
          l = l + 1
        ENDIF
        DCOEFMN = DCOEF(S,U,zA,zF,zKay,zDbr,zP,zPI)
        GOTO 2
      ENDIF
      SMAX = S
      UMAX = U
*
*    deflection calculations
*
* Notation:  j1 = "subscript" for the deflection array
*            S, U = variables to sum over like m and n
*            j2, j3, = integers for do loops
*            l = counter of loops
*
*    (we are going to calculate deflections at the given locations
*     x(i) along the line the line parallell to the x axis, where
*     y is fixed at YO, the location of the center of the load)
*
      j2max = IDINT(SMAX)
      j3max = IDINT(UMAX)
      Y    = zYO
*
      DO 60 j1 = 1, m
*
*  (initializing values)
*
      w(j1) = 0.0D+00
      S    = 1.0D+00
```

```
      U    = 1.0D+00
*
*   X in global coordinates
      EX    = zXO + zX(j1)
*
      DO 50 j2 = 1,j2max
        U = 1.0D+00
        DO 40 j3 = 1, j3max
          w(j1)  = w(j1) + DDEFL(S,U,zP,zPI,zA,zF,zC,zG,zXO,zYO,
     +                  zKay,zDbr,EX,Y)
          U = U + 1.0D+00
   40   CONTINUE
        S = S + 1.0D+00
   50 CONTINUE
*   (writing result to output file)
      ix = IDINT(zX(j1))
*   the above gives location of the deflection as measured
*   from center of load as an integer
*
*    WRITE (11,80) ix,J1,w(J1)
*  80 FORMAT (1X,3x,I3,5x,I3,5x,D20.10)
*
   60 CONTINUE
*
*    write (11,90) j2max,j3max
*  90 format (1x,'SUMMED TO m = ',I4,'; n = ',I4)
*
      RETURN
      END
```

DCOEFF.FOR

```
* coefficient to check for convergence
      FUNCTION dcoef(zs,zu,za,zf,zkay,zDbr,zp,zpi)
      implicit real*8 (a-h,o-z)
*
      zdden  = dden(zs,zu,za,zf,zkay,zDbr,zpi)
      dcoef1 = 16.0*zp/(zs*zu*zpi*zpi)
      dcoef  = dcoef1/zdden
*
      RETURN
      END
```

DDENOM.FOR

```
* messy denominator
      FUNCTION dden(zs,zu,za,zf,zkay,zDbr,zpi)
      implicit real*8 (a-h,o-z)
*
      dden1 = (((zs*zs)/(za*za))+((zu*zu)/(zf*zf)))**2
      dden = ((zpi**4)*dden1*zdbr)+zkay
*
      RETURN
      END
```

DDEFL.FOR

```
* DEFLECTION TERMS
    FUNCTION DDEFL(zs,zu,zp,zpi,za,zf,zc,zg,zxo,zyo,
   +          zkay,zDbr,zx,zy)
    implicit real*8 (a-h,o-z)
*
    ZAMN = AMN(zs,zu,zp,zpi,za,zf,zc,zg,zxo,zyo)
    zdden = dden(zs,zu,za,zf,zkay,zDbr,zpi)
    ddefl = (zamn/zdden)*DSIN(zs*zpi*zx/za)*DSIN(zu*zpi*zy/zf)
*
    RETURN
    END
```

AMN.FOR

```
* Amn = load expressed as a double fourier sine series
    FUNCTION Amn(zs,zu,zp,zpi,za,zf,zc,zg,zxo,zyo)
    implicit real*8 (a-h,o-z)
*
    AMN1 = 16.0*ZP/(ZS*Zu*ZPI*ZPI)
    AMN2 = DSIN(ZS*ZPI*ZXO/ZA)*DSIN(ZS*ZPI*ZC/(2.0*ZA))
    AMN3 = DSIN(ZU*ZPI*ZYO/ZF)*DSIN(ZU*ZPI*ZG/(2.0*ZF))
    AMN  = AMN1*AMN2*AMN3
*
    RETURN
    END
```

EBSRCH.FOR

```
* one-dimensional search for solution to F (to find Eb)
    SUBROUTINE Ebase(zD,zCee,zemm,zEnb,zEbseed,zmu,zh,
   c          zebul,zebll,finlEs,finlEb,finlF)
    implicit real*8 (a-h,o-z)
    double precision zh(100), zF(100), zE(100), zEbseed(100),
   c          zEb(100),zEs(100)
    logical REPLACED, NEG
*
* expansion factor
    znu = 2.0D+00
*
* contraction factor
    theta = 0.5D+00
*
* start with initial Simplex of two Eb seed values
*
* initialization loop
*
* L = loop counter
    L = 0
    DO 10 i = 1,2
      zEb(i) = zEbseed(i)
      zEs(i) = zEnb/(zCee*(zEb(i))**zemm)
      zE(1) = zEs(i)
```

```
      zE(2) = zEb(i)
      zF(i) = F(zD,zh,zE,zmu)
   10 CONTINUE
*
*  outer while loop
*
   20 IF (zF(1).GT.(1.0D-10)) THEN
     REPLACED = .FALSE.
*
*  find new best vertex
*
       IF (zF(1).GT.zF(2)) THEN
         TEMP1 = zF(1)
         TEMP2 = zEb(1)
         TEMP3 = zEs(1)
         zF(1) = zF(2)
         zEb(1) = zEb(2)
         zEs(1) = zEs(2)
         zF(2) = TEMP1
         zEb(2) = TEMP2
         zEs(2) = TEMP3
       ENDIF
       Do 90 i = 1,2
   90  continue
     ENDIF
*
*  check stopping criteria
*
     IF (zF(1).LT.(1.0D-10)) THEN
       GOTO 100
     ENDIF
*
*   check if all values are same (to avoid endless loop)
*
   25 z1 = ABS(zEb(1)-zEb(2))
*
     IF (z1.LT.(0.1D-02)) THEN
       GOTO 100
     ENDIF
*
*  inner repeat loop
*
*    rotation step
*
         NEG = .FALSE.
         rEb = zEb(1) - (zEb(2)-zEb(1))
*
*  in the event that Eb becomes negative set to 1, cannot be 0
*  due to equation for Es
*
         IF (rEb.LE.(0.0D+00)) THEN
           NEG = .TRUE.
           rEb = 1.0D+00
```

```
      ENDIF
*
* checking that Eb limits are not exceeded
*
      IF (rEb.GT.zEbul) THEN
        rEb = zEbul
      ENDIF
      IF (rEb.LT.zEbll) THEN
        rEb = zEbll
      ENDIF
*
*  calculation of rEs
*
      rEs = zEnb/(zCee*(rEb)**zemm)
*
*  calculation of F(rEb)
      zE(1) = rEs
      zE(2) = rEb
      rF = F(zD,zh,zE,zmu)
*
      IF (rF.LT.zF(1)) THEN
        REPLACED = .TRUE.
      ENDIF
*
*  if rotation is a success, then expansion step is performed
*  unless we are on the boundary
*
      IF (REPLACED) THEN
         IF (.NOT.NEG) THEN
*           do expansion if neg = false
           eEb = zEb(1) - znu*(zEb(2)-zEb(1))
*
*    we must check that eEb is not negative or zero
*
           IF (eEb.LE.(0.0D+00)) THEN
             eEb = 1.0D+00
           ENDIF
*
* checking that Eb limits are not exceeded
*
           IF (eEb.GT.zEbul) THEN
             eEb = zEbul
           ENDIF
           IF (eEb.LT.zEbll) THEN
             eEb = zEbll
           ENDIF
*
*  calculation of eEs
*
           eEs = zEnb/(zCee*(eEb)**zemm)
*
*  calculation of F(eEb)
*
```

```
            zE(1) = eEs
            zE(2) = eEb
            eF = F(zD,zh,zE,zmu)
*
            IF ((eF).LT.zF(1)) THEN
*          accept expansion
              zF(2) = zF(1)
              zEb(2) = zEb(1)
              zEs(2) = zEs(1)
              zF(1) = eF
              zEb(1) = eEb
              zEs(1) = eEs
            ELSE
*          accept rotation
              zF(2) = zF(1)
              zEb(2) = zEb(1)
              zEs(2) = zEs(1)
              zF(1) = rF
              zEb(1) = rEb
              zEs(1) = rEs
            ENDIF
          ELSE
*      if neg = true, then eccept rotation without doing expansion
*
              zF(2) = zF(1)
              zEb(2) = zEb(1)
              zEs(2) = zEs(1)
              zF(1) = rF
              zEb(1) = rEb
              zEs(1) = rEs
          ENDIF
*
        L = L+1
        GOTO 20
*
      ELSEIF (.NOT.REPLACED) THEN
*       contraction step, is automatically accepted wether replaced
*       is true or not.
        cEb = zEb(1) + theta*(zEb(2)-zEb(1))
*
        IF (cEb.GT.zEbul) THEN
          cEb = zEbul
        ENDIF
        IF (cEb.LT.zEbll) THEN
          cEb = zEbll
        ENDIF
*
*        calculating cEs
*
        cEs = zEnb/(zCee*(cEb)**zemm)
*
*        calculating cF(cEb)
*
```

```
        zE(1) = cEs
        zE(2) = cEb
        cF = F(zD,zh,zE,zmu)
        zEs(2) = cEs
        zEb(2) = cEb
        zF(2) = cF
*
       IF (cF.LT.zF(1)) THEN
         L = L+1
         GOTO 20
       ELSE
         GOTO 25
       ENDIF
     ENDIF
*
 100 finlEs = zEs(1)
     finlEb = zEb(1)
     finlF = zF(1)
*
     RETURN
     END
```

EFF.FOR

```
* to calculate the function (F) to be optimized
      FUNCTION F(zD,zh,zE,zmu)
      implicit real*8 (a-h,o-z)
      double precision zh(100),zE(100)
*
* to generate calculated D value (Dcal)
*
      Dcal = Dee(zE,zh,zmu)
*
* to calculate the equation to be optimized
*
      F   = (zD - Dcal)*(zD - Dcal)
*
      RETURN
      END
```

DEE.FOR

```
* to generate equation for Dbar as a function of Eb only.  to be used in
* backcalculation of Eb.
      FUNCTION Dee(zE,zh,zmu)
      implicit real*8 (a-h,o-z)
      double precision zE(100),zh(100)
*
      AH1 = (zh(1)**5)*(zE(1)**3)
      AH2 = (5*(zh(1)**4)*zh(2)+6*(zh(1)**3)*(zh(2)**2)+4*(zh(1)**2)*
     c      (zh(2)**3))*zE(1)*zE(1)*zE(2)
      AH3 = (4*(zh(1)**3)*(zh(2)**2)+6*(zh(1)**2)*(zh(2)**3)+5*zh(1)*
     c      (zh(2)**4))*zE(1)*zE(2)*zE(2)
      AH4 = (zh(2)**5)*(zE(2)**3)
```

```
AH = AH1+AH2+AH3+AH4
dn = (zE(1)*zh(1)+zE(2)*zh(2))*(zE(1)*zh(1)+zE(2)*zh(2))
Dee = AH/(dn*12*(1-zmu*zmu))
RETURN
END
```

Appendix 4A

Deflection Verification Results

This Appendix presents the results from cases 4 through 8 as described in Chapter 4 of this paper.

Table 4A-1: Case 4

Location	deflection (in)			% Errors	
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	Elsym5	Sap w/s.s.
0	0.0148	0.0465	0.014	214.19	-5.41
6	0.0146	0.0454	0.0137	210.96	-6.16
12	0.013	0.0432	0.0131	232.31	0.77
24	0.0102	0.0382	0.0114	274.51	11.76
36	0.0069	0.0329	0.0096	376.81	39.13
54	0.0039	0.0258	0.007	561.54	79.49

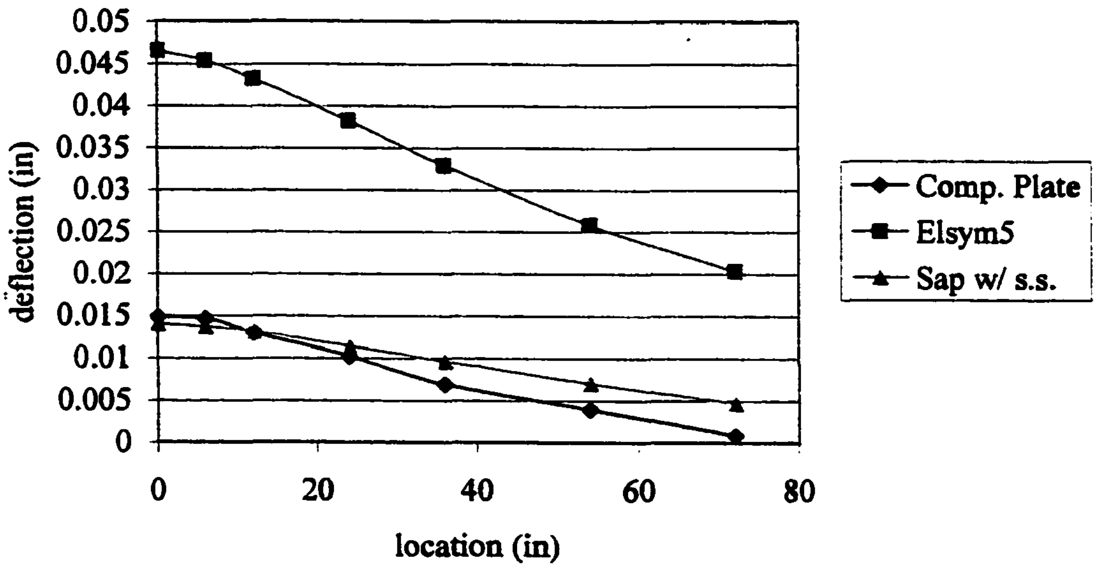


Figure 4A-1: Case 4

Table 4A-2: Case 5

Location	deflection (in)			% Errors	
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	Elsym5	Sap w/s.s.
0	0.0112	0.0384	0.0096	242.86	-14.29
6	0.0111	0.0377	0.0094	239.64	-15.32
12	0.0101	0.0349	0.0091	245.54	-9.90
24	0.0083	0.0321	0.0081	286.75	-2.41
36	0.0061	0.0287	0.007	370.49	14.75
54	0.004	0.0239	0.0054	497.50	35.00
72	0.0017	0.0197	0.0038	1058.82	123.53

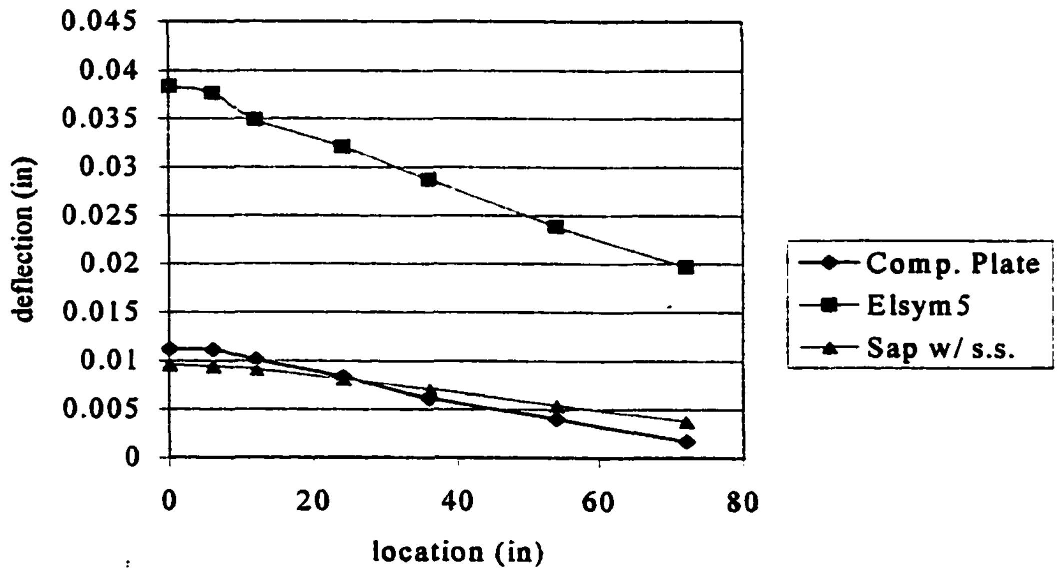


Figure 4A-2: Case 5

Table 4A-3: Case 6

Location	deflection (in)			% Errors	
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	Elsym5	Sap w/s.s.
0	0.0081	0.0322	0.0061	297.53	-24.69
6	0.008	0.0317	0.0059	296.25	-26.25
12	0.0074	0.0276	0.0057	272.97	-22.97
24	0.0063	0.0261	0.0051	314.29	-19.05
36	0.0051	0.0241	0.0045	372.55	-11.76
54	0.0038	0.021	0.0036	452.63	-5.26
72	0.0022	0.0182	0.0026	727.27	18.18

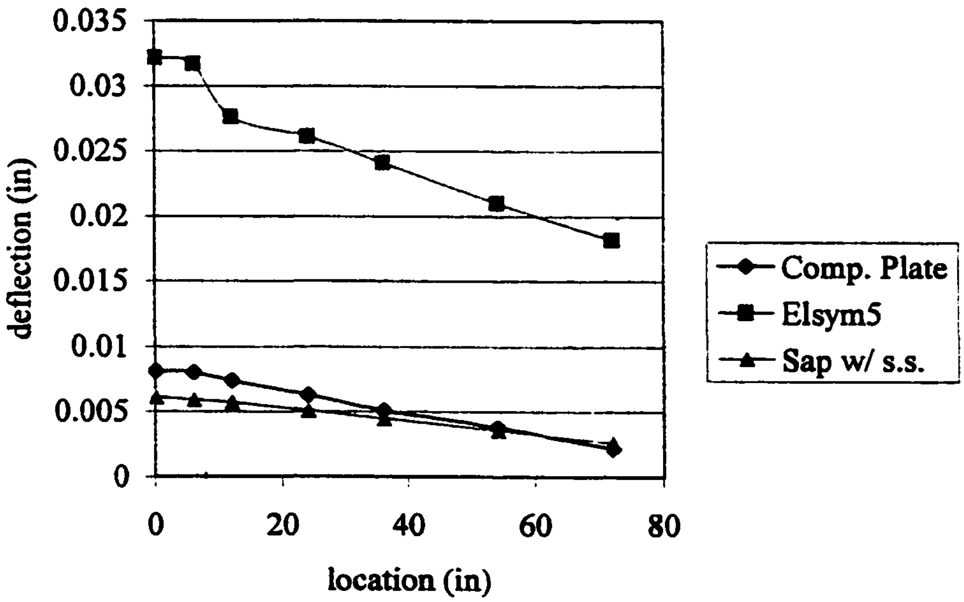


Figure 4A-3: Case 6

Table 4A-4: Case 7

Location	deflection (in)			% Errors	
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	Elsym5	Sap w/s.s.
0	0.00419	0.0215	0.00229	413.13	-45.35
6	0.00418	0.02113	0.00219	405.50	-47.61
12	0.00403	0.01928	0.00203	378.41	-49.63
24	0.00375	0.01716	0.00183	357.60	-51.20
36	0.00336	0.01664	0.00163	395.24	-51.49
54	0.0029	0.01543	0.00132	432.07	-54.48
72	0.00215	0.01419	0.001	560.00	-53.49

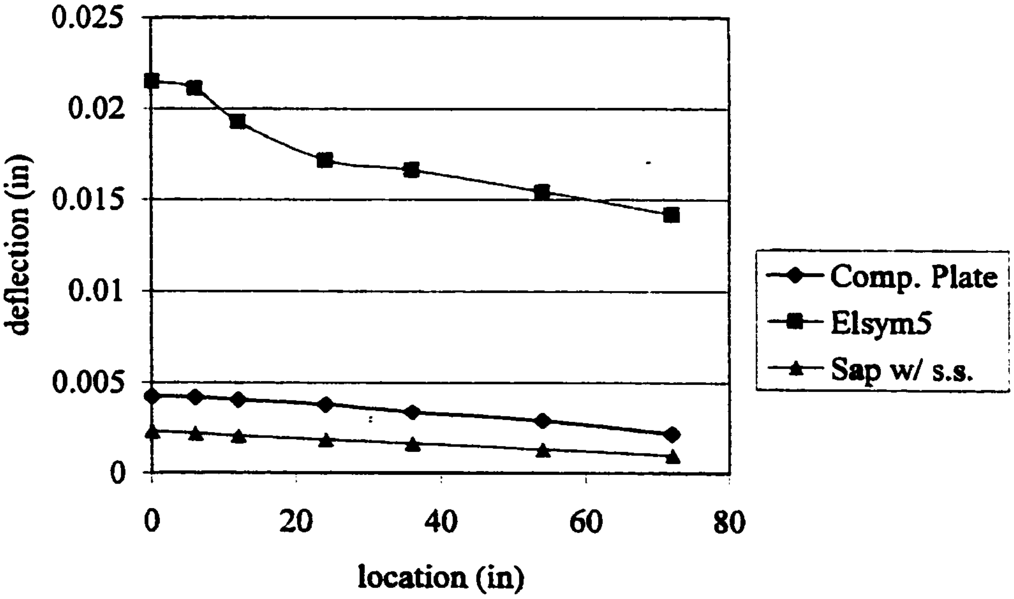


Figure 4A-4: Case 7

Table 4A-5: Case 8

Location	deflection (in)			% Errors	
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	Elsym5	Sap w/s.s.
0	0.0115	0.0432	0.0118	275.65	2.61
6	0.0114	0.042	0.0116	268.42	1.75
12	0.0111	0.0386	0.0111	247.75	0.00
24	0.0098	0.0349	0.0099	256.12	1.02
36	0.0078	0.0307	0.0085	293.59	8.97
54	0.0045	0.0249	0.0065	453.33	44.44
72	0.0016	0.02	0.0047	1150.00	193.75

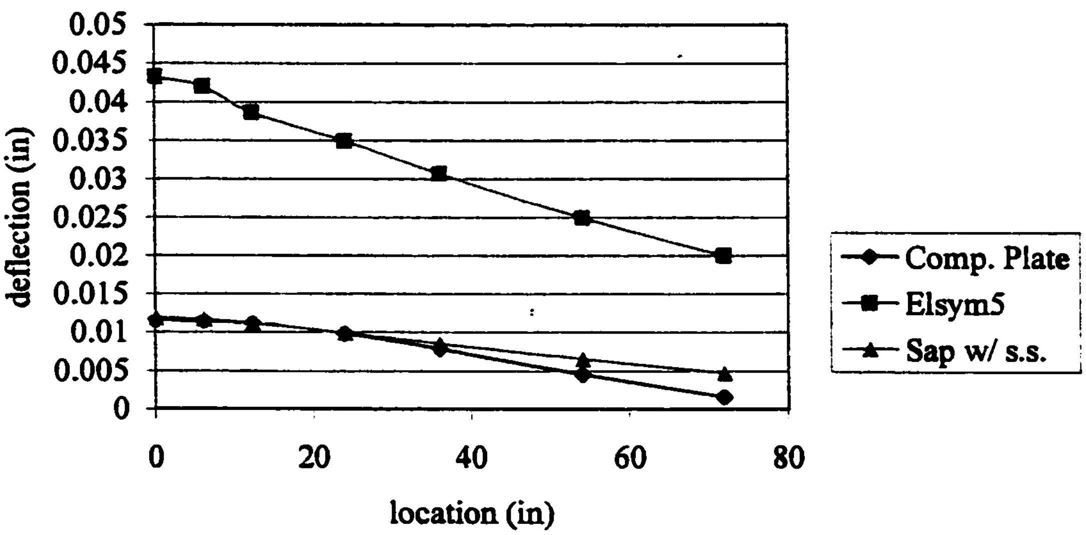


Figure 4A-5: Case 8

Appendix 5A
Sensitivity Analysis

Table 5A-1: Model 1 Concrete with an Asphalt Overlay

parameter varied	% change in parameter	RMS%	% change in $\overline{D}$
hl	-33.33	11.48	26.20
hl	-16.67	5.48	14.02
hl	33.33	30.39	-33.95
hl	16.67	4.89	-15.96
hl	66.67	21.46	-76.37
hl	50.00	24.85	-54.06
h2	-33.33	32.42	54.35
h2	-16.67	14.19	30.91
h2	16.67	28.38	-39.53
h2	33.33	20.40	-88.84
h2	66.67	23.22	-221.40
El	-75.00	21.27	41.44
El	-50.00	11.46	26.17
El	-25.00	4.79	12.43
El	25.00	3.58	-11.31
E2	-87.50	50.04	68.91
E2	-75.00	30.21	52.07
E2	-62.50	20.13	39.90
E2	-50.00	13.68	30.05
E2	-25.)0	5.41	13.86
E2	25.00	3.98	-12.69
k	-75.00	249.99	NA
k	-50.00	82.56	NA
k	-25.00	36.21	NA
k	200.00	71.00	NA
k	400.00	81.53	NA
k	700.00	94.57	NA
k	900.00	95.76	NA
μ	33.33	0.62	-1.82
μ	66.67	1.43	-4.27
μ	100.00	2.42	-7.42
μ	133.33	3.61	-11.40
μ	166.67	5.00	-16.37
μ	200.00	6.58	-22.57

** negative means a decrease in value by the given percentage

NA means not applicable because k has no affect on $\overline{D}$ refer to equation 5-2.

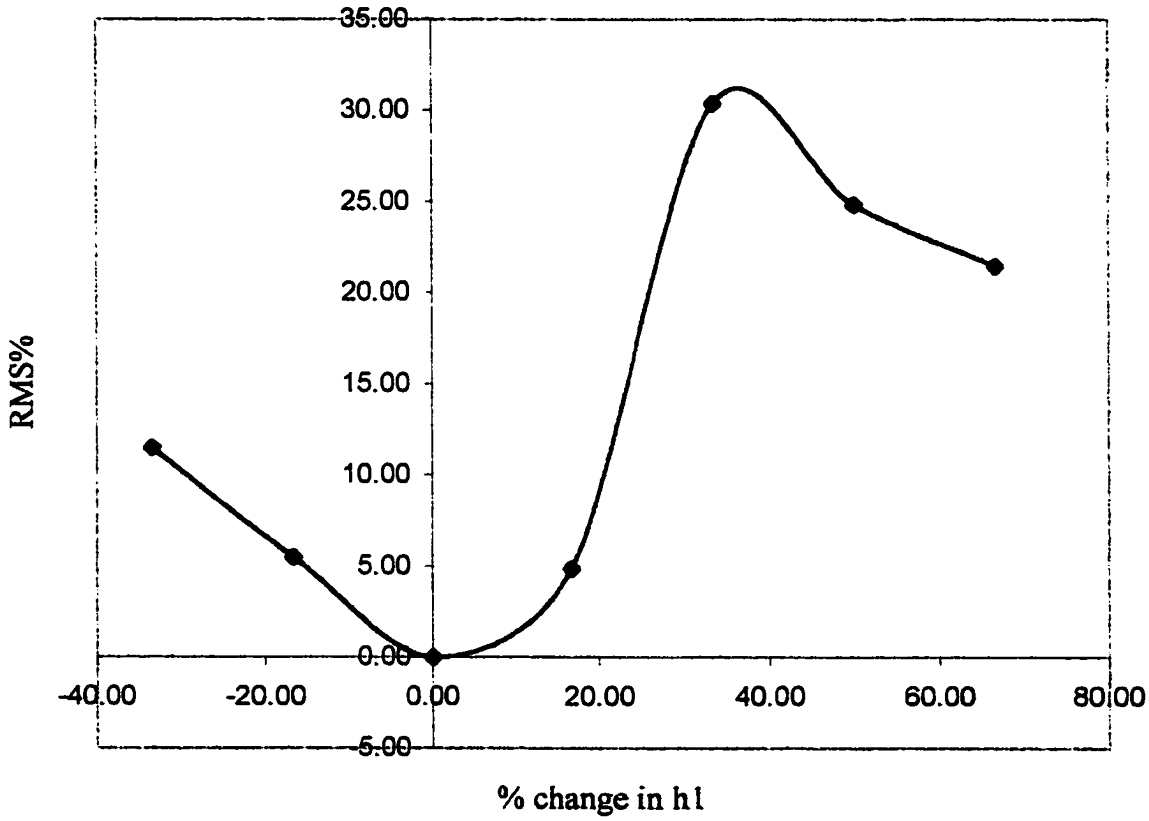


Figure 5A-1: Model 1, h1

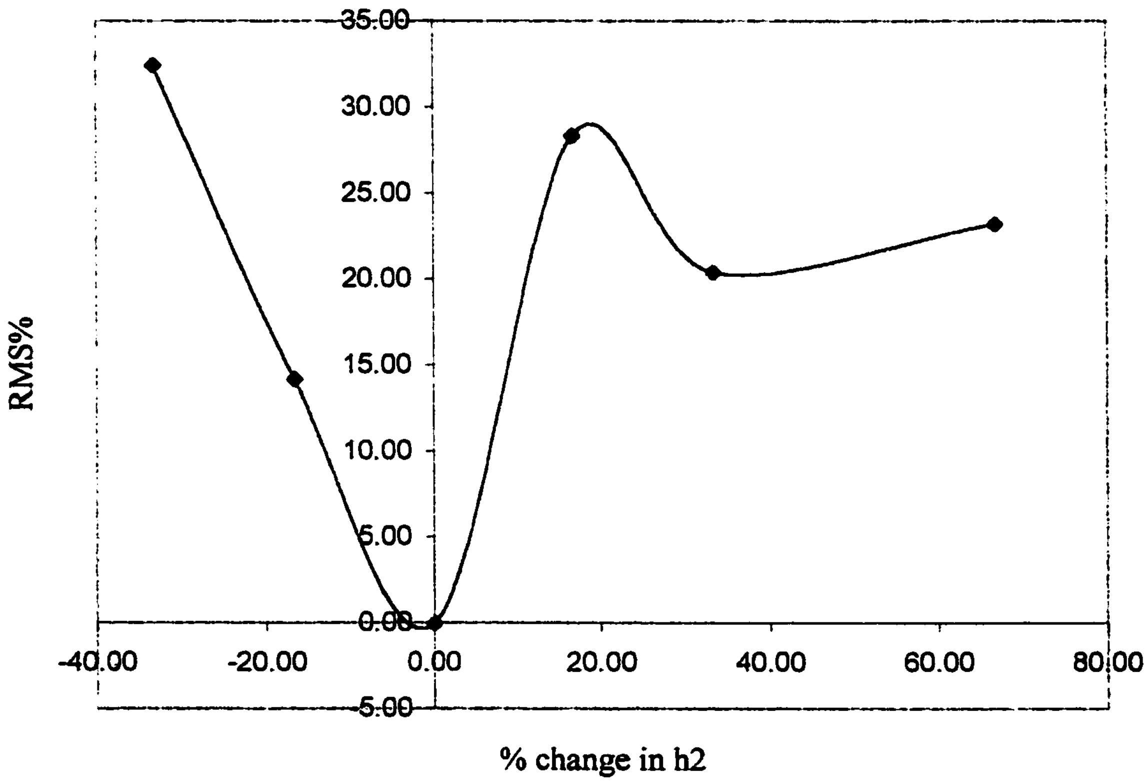


Figure 5A-2: Model 1, h2

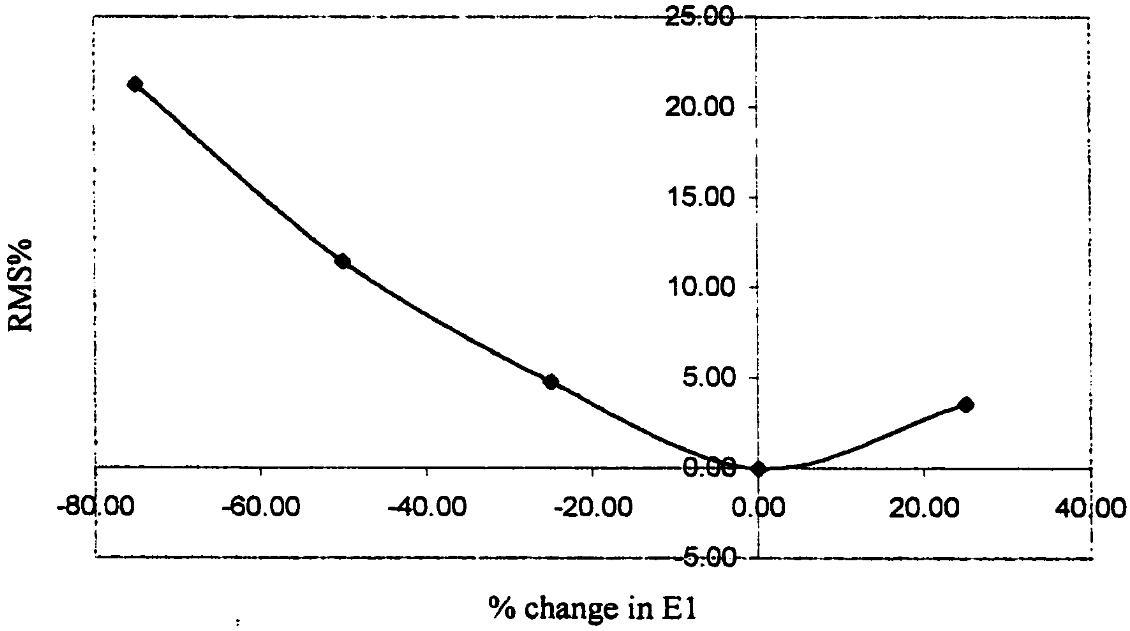


Figure 5A-3: Model 1, E1

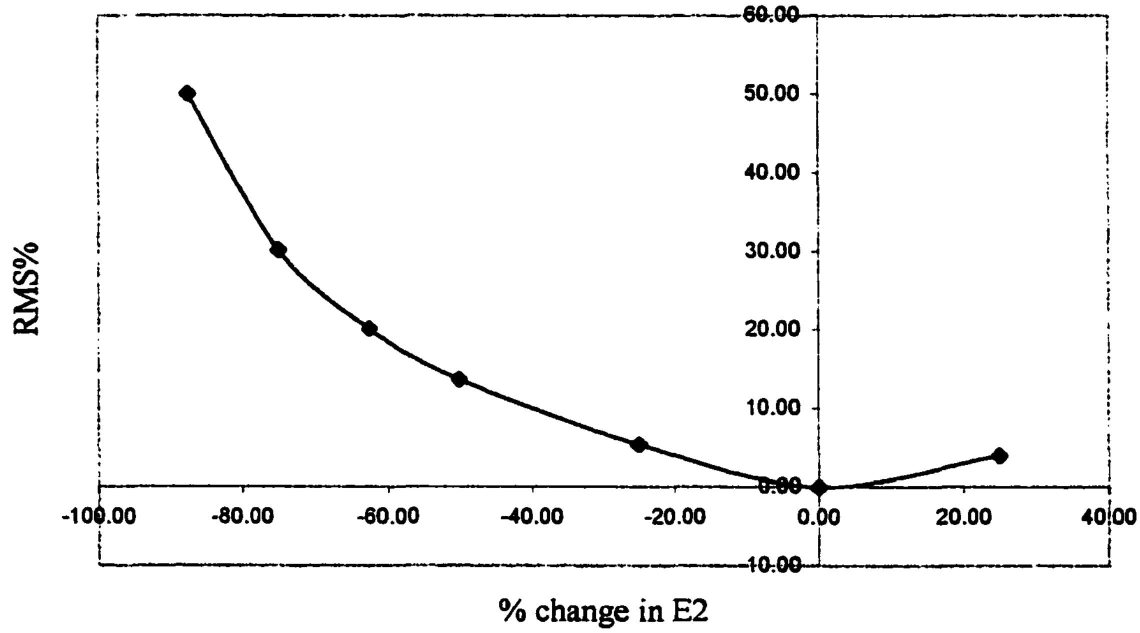


Figure 5A-4: Model 1, E2

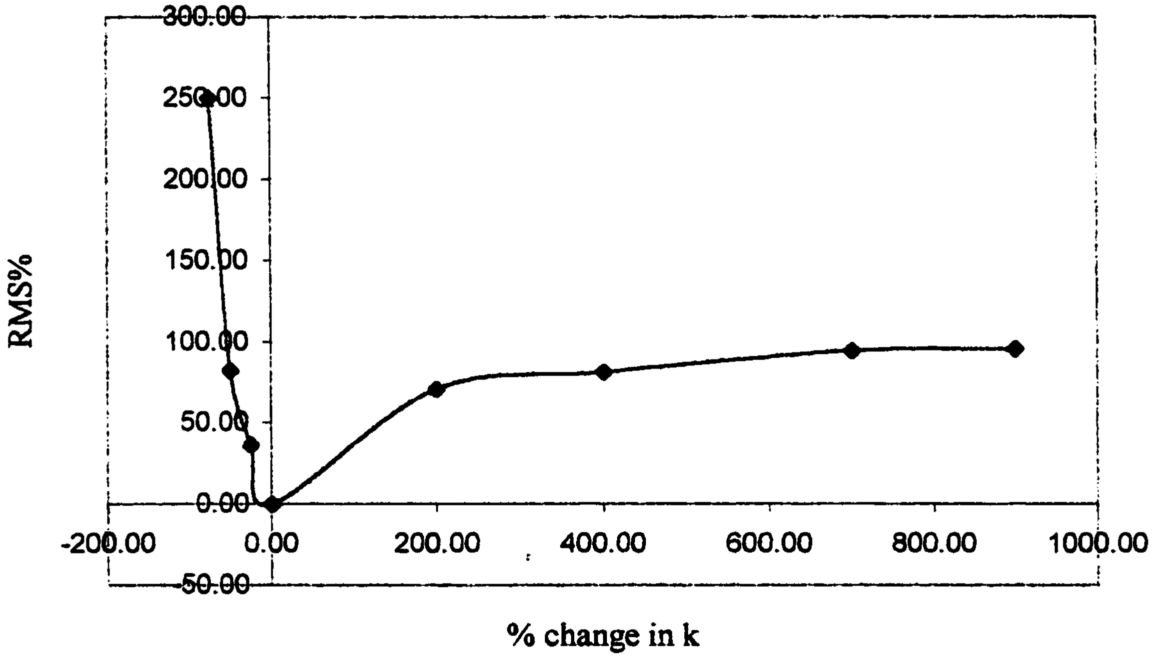


Figure 5A-5: Model 1, k

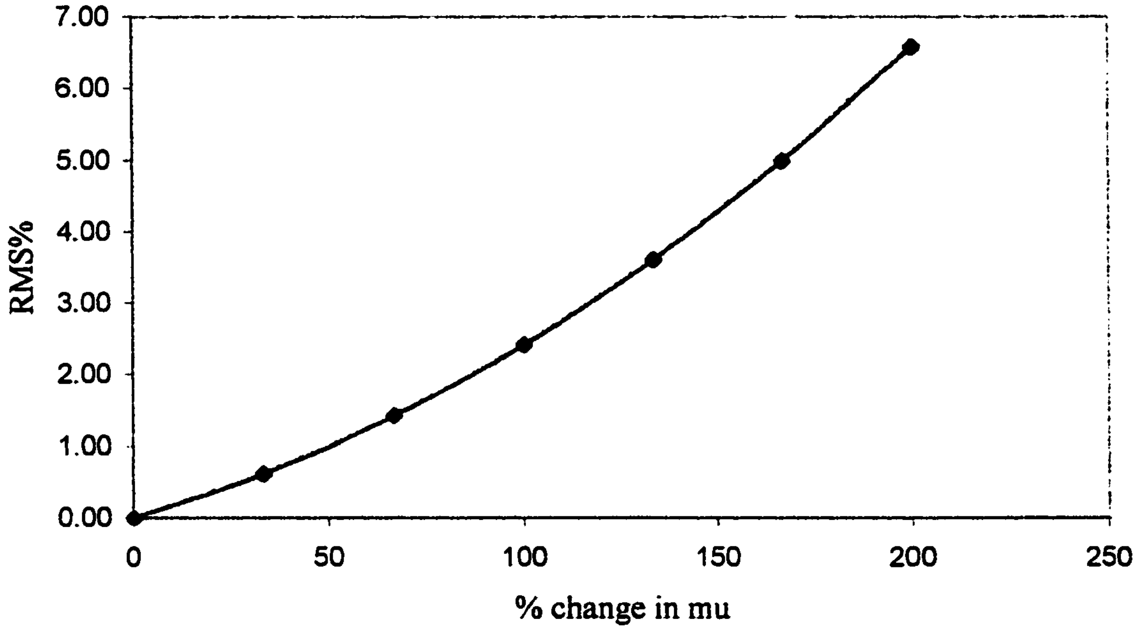


Figure 5A-6: Model 1, mu

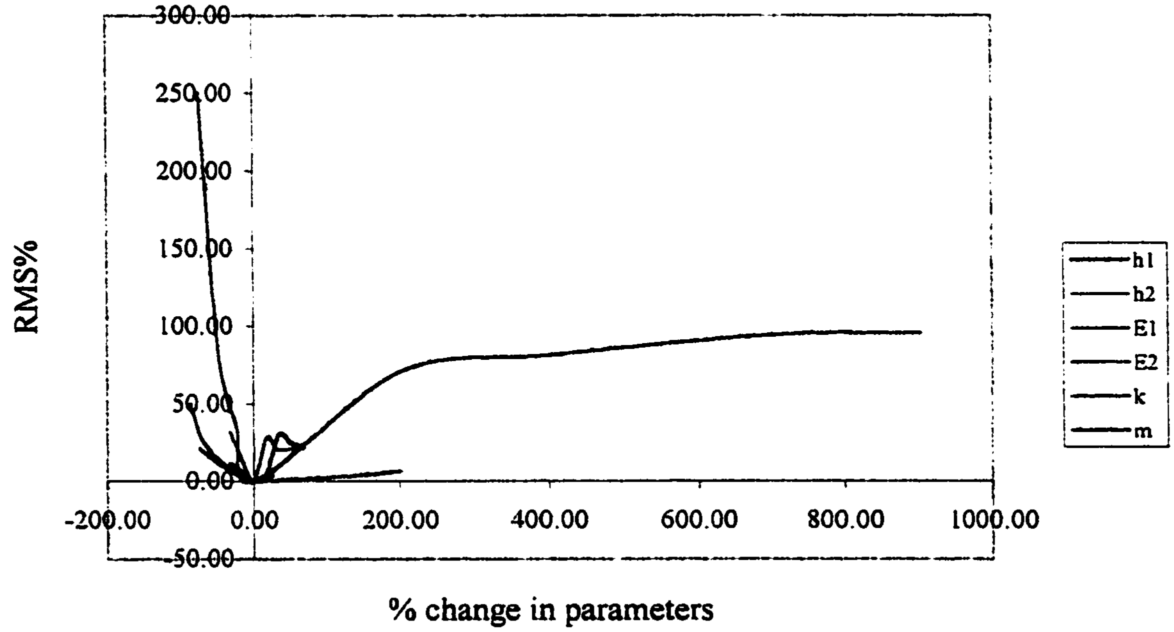


Figure 5A-7: Model1, All Parameters

Table 5A-2: Model 2 Concrete over a Stabilized Base

parameter varied	% change in parameter	RMS%	% change in $\overline{D}$
h1	-33.33	46.06	43.52
h1	-16.67	54.18	24.17
h1	16.67	19.92	-29.79
h1	33.33	33.40	-66.00
h1	66.67	49.10	-160.75
h2	-33.33	48.43	37.96
h2	100.00	51.17	-228.77
h2	150.00	50.73	-421.67
h2	200.00	50.12	-675.93
h2	300.00	56.92	-1393.15
E1	-87.5	42.66	62.33
E1	-75.00	45.37	45.21
E1	-62.5	50.33	33.56
E1	-50.00	53.99	24.66
E1	-25.00	10.88	10.96
E1	25.00	7.69	-9.59
E2	-50.00	50.72	32.65
E2	50.00	18.66	-27.33
E2	100.00	28.67	-50.68
E2	150.00	34.70	-70.97
k	-75.00	358.20	0.00
k	-50.00	142.23	0.00
k	-25.00	50.30	0.00
k	200.00	62.10	0.00
k	400.00	77.77	0.00
k	700.00	83.97	0.00
k	900.00	85.95	0.00
μ	33.33	1.58	-1.82
μ	66.67	3.61	-4.27
μ	100.00	6.08	-7.42
μ	133.33	8.98	-11.40
μ	166.67	12.32	-16.37
μ	200.00	16.06	-22.57

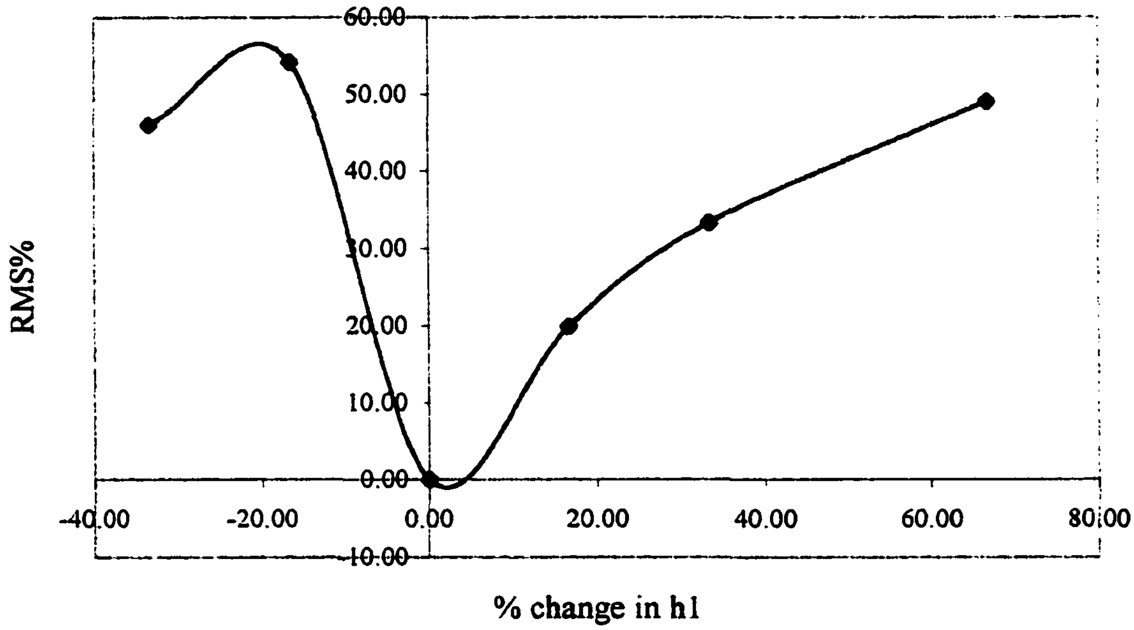


Figure 5A-8: Model 2, h1

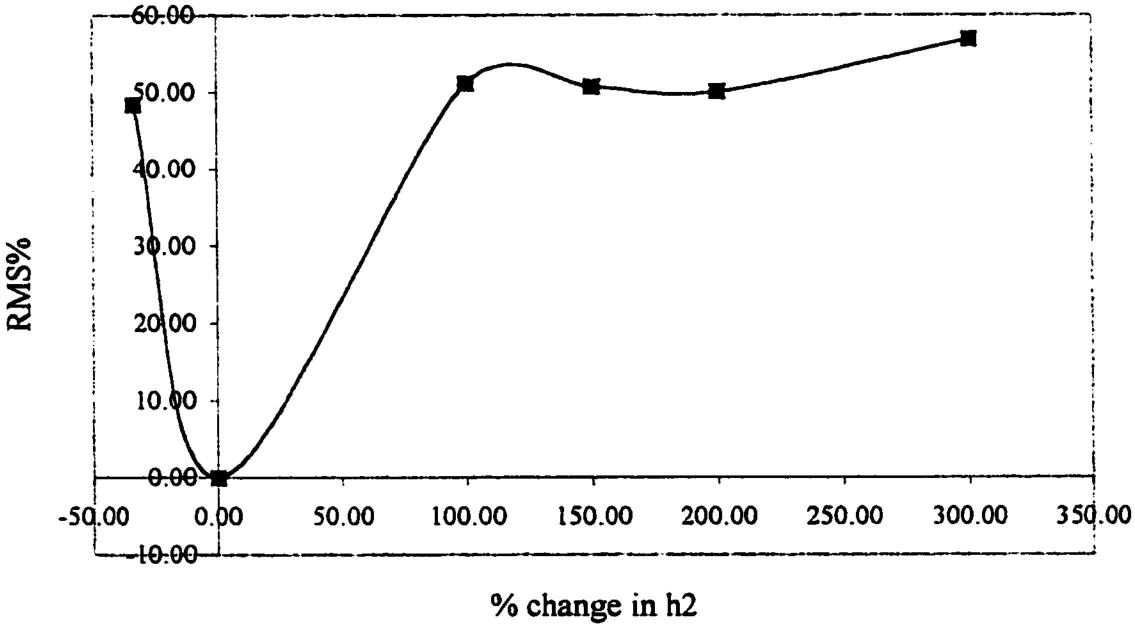


Figure 5A-9: Model 2, h2

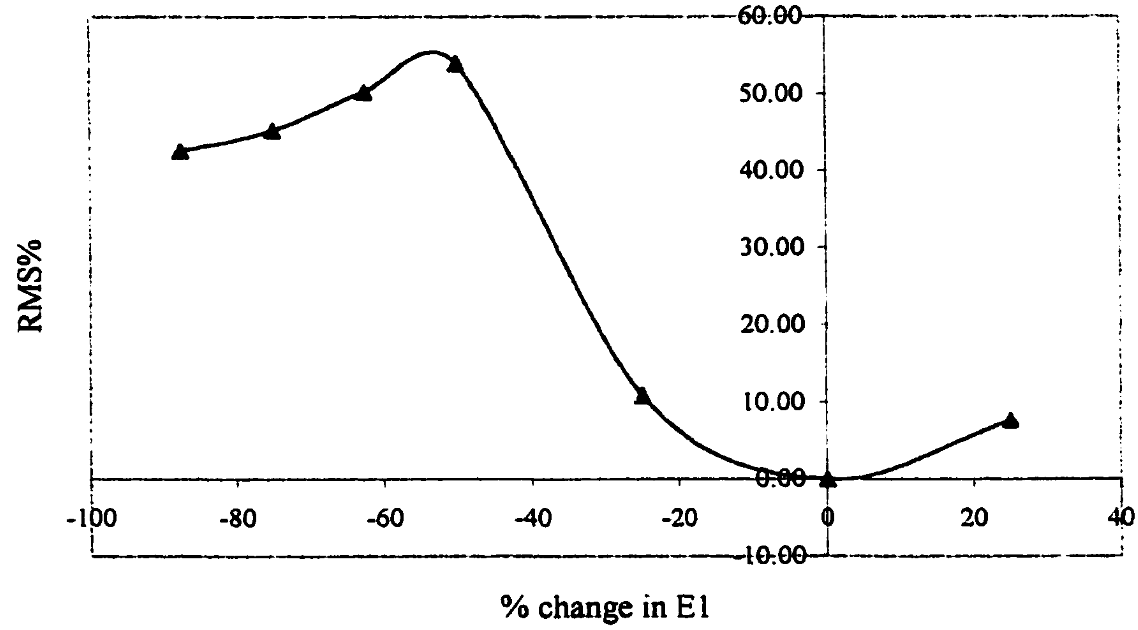


Figure 5A-10: Model 2, E1

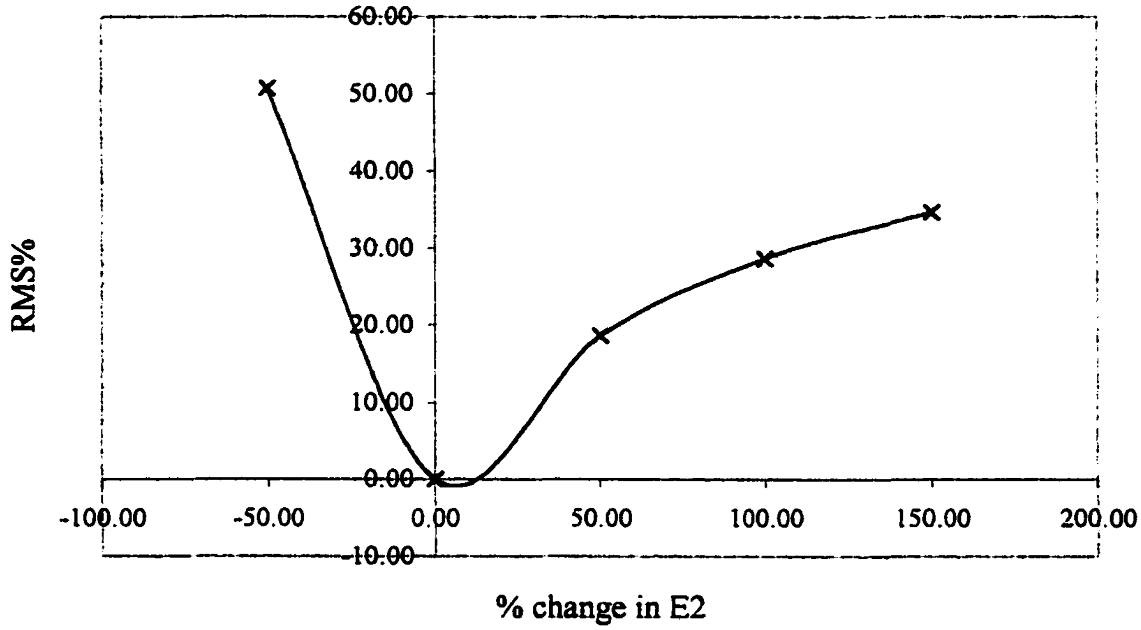


Figure 5A-11: Model 2, E2

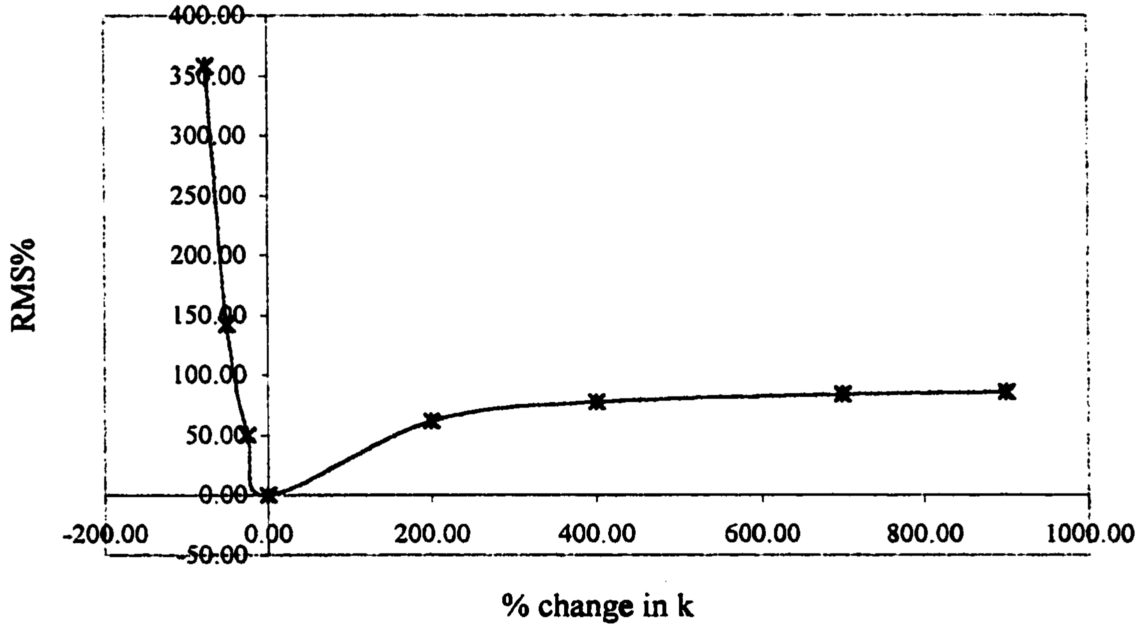


Figure 5A-12: Model 2, k

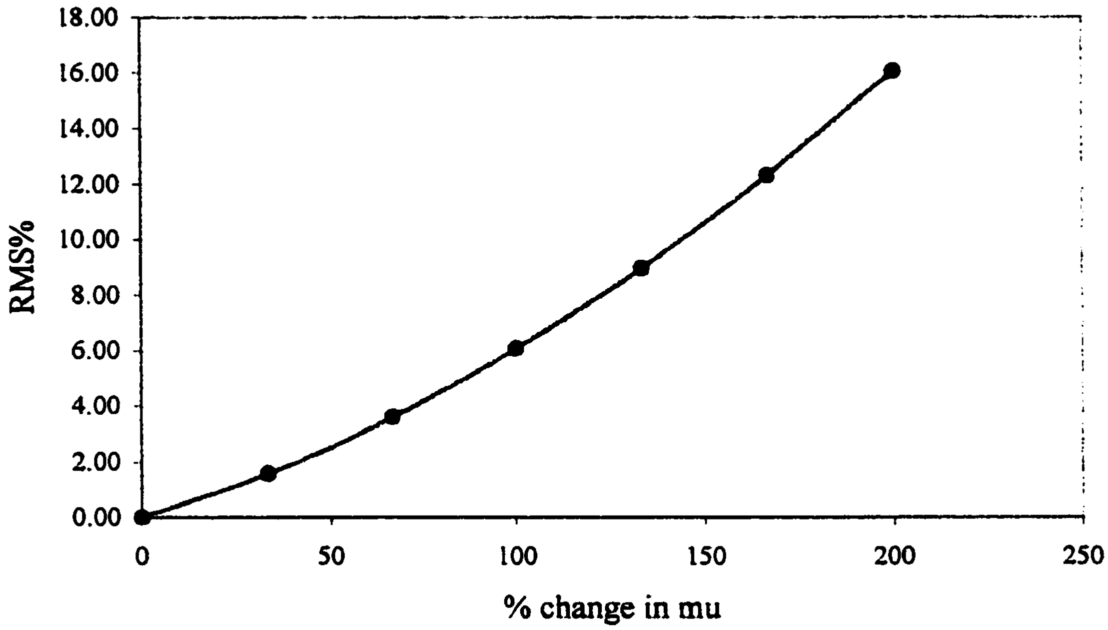


Figure 5A-13: Model 2, mu

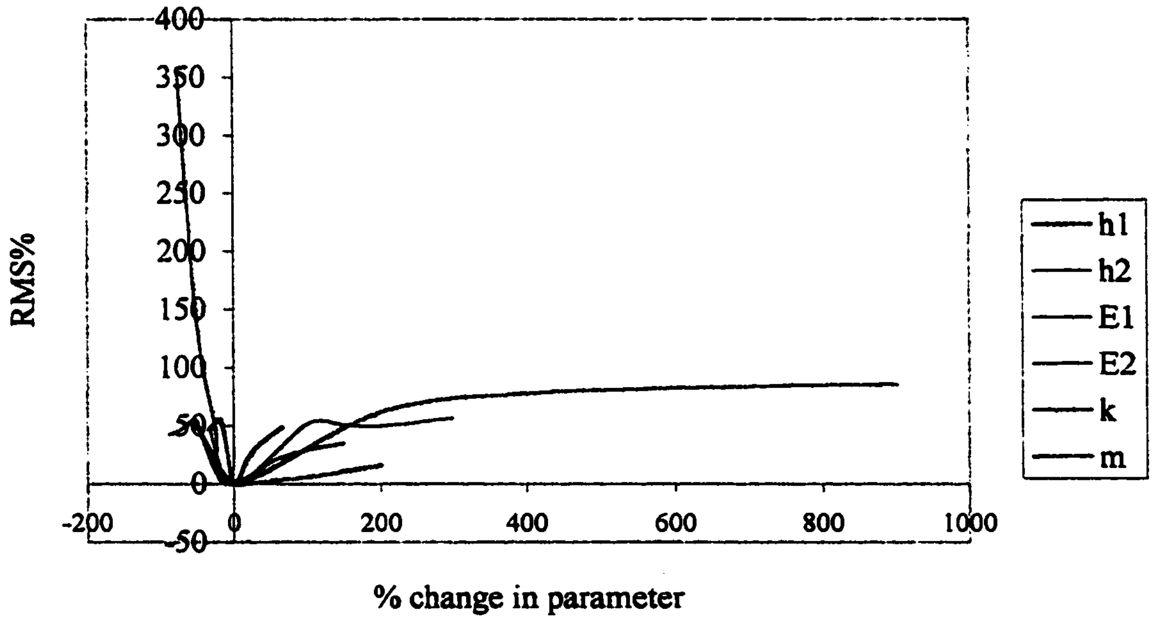


Figure 5A-14: Model 2, All Parameters

Table 5A-3: Model 3 Concrete over a Granular Base

parameter varied	% change in parameter	RMS%	% change in $\overline{D}$
h1	-33.33	81.31	66.12
h1	-16.67	33.47	39.37
h1	16.67	26.99	-54.47
h1	33.33	46.60	-126.54
h1	66.67	50.06	-333.49
h2	-33.33	3.76	5.56
h2	100.00	20.30	-37.84
h2	150.00	33.25	-72.84
h2	200.00	45.57	-121.42
h2	300.00	46.68	-268.09
E1	-87.5	173.74	79.16
E1	-75.00	85.77	67.63
E1	-62.5	59.05	56.30
E1	-50.00	40.13	45.01
E1	-25.00	16.92	22.49
E1	25.00	13.02	-22.48
E2	-28.57	1.92	2.88
E2	-14.29	0.95	1.44
E2	14.29	0.93	-1.44
E2	28.57	1.85	-2.88
k	-75.00	381.22	0.00
k	-50.00	154.54	0.00
k	-25.00	49.24	0.00
k	200.00	74.44	0.00
k	400.00	101.73	0.00
k	700.00	102.02	0.00
k	900.00	101.99	0.00
μ	-25.00	1.18	1.79
μ	25.00	1.55	-2.40
μ	50.00	3.49	-5.49
μ	75.00	5.84	-9.40
μ	100.00	8.64	-14.29
μ	125.00	11.93	-20.38

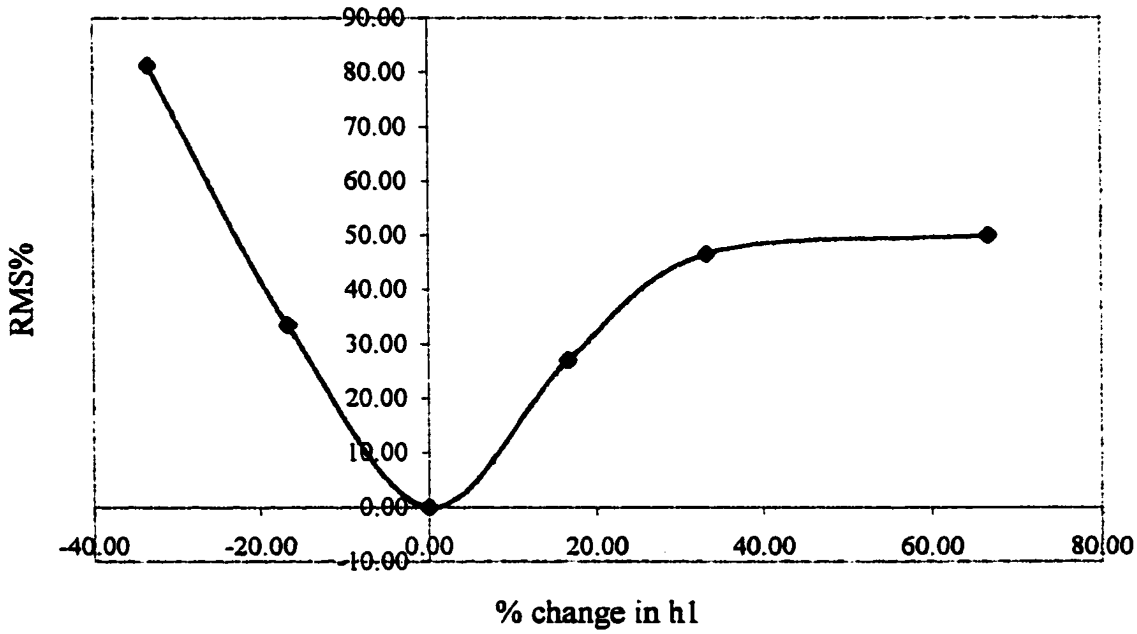


Figure 5A-15: Model 3, h1

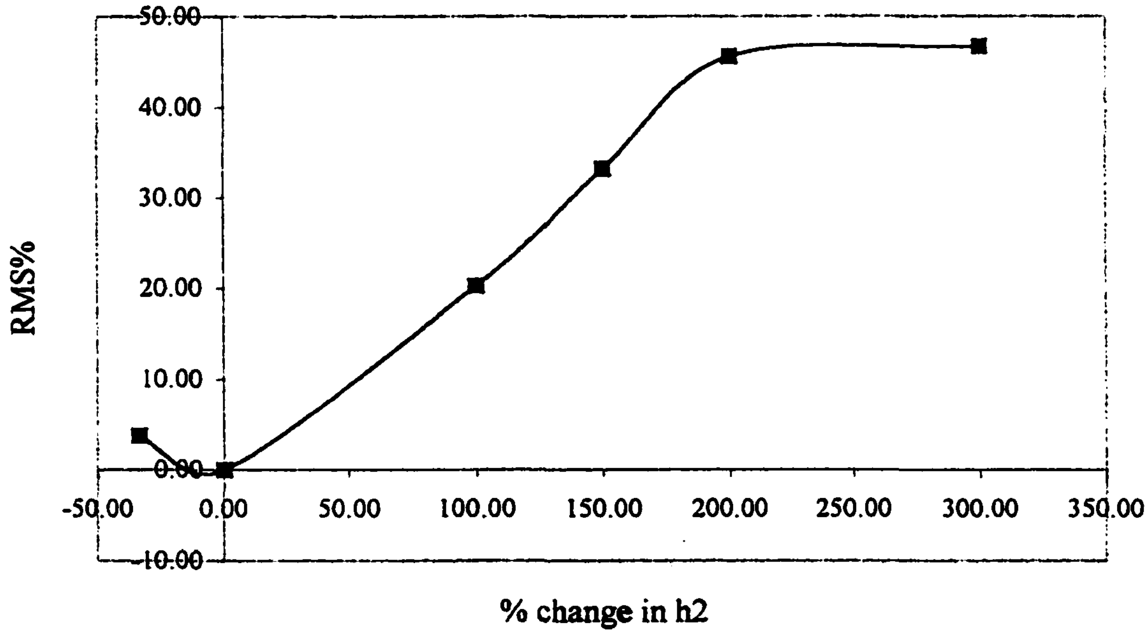


Figure 5A-16: Model 3, h2

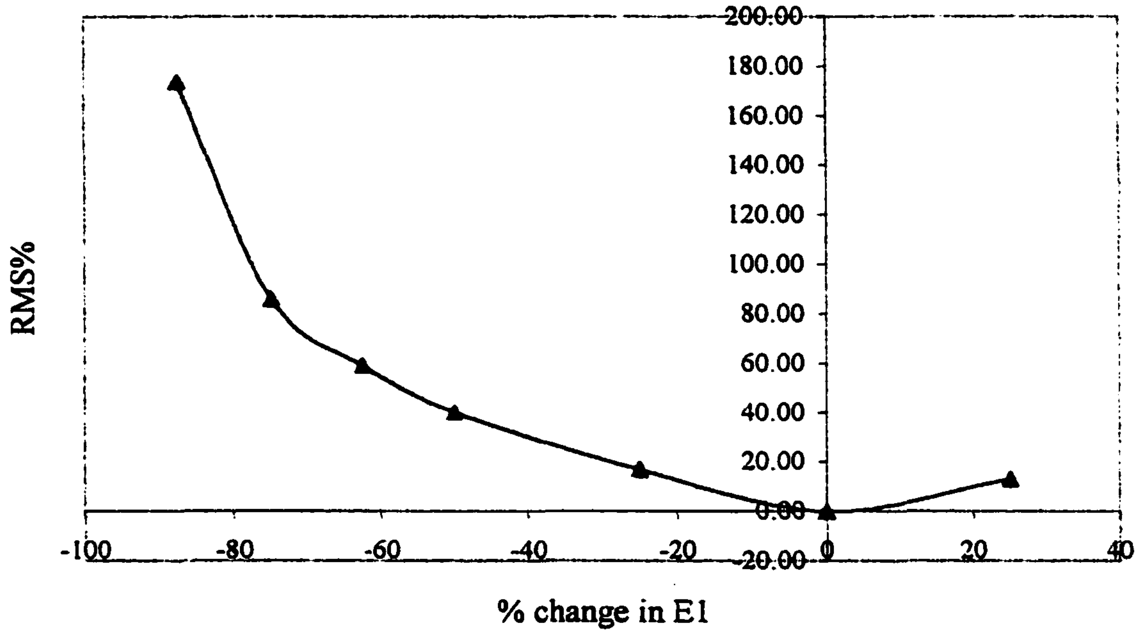
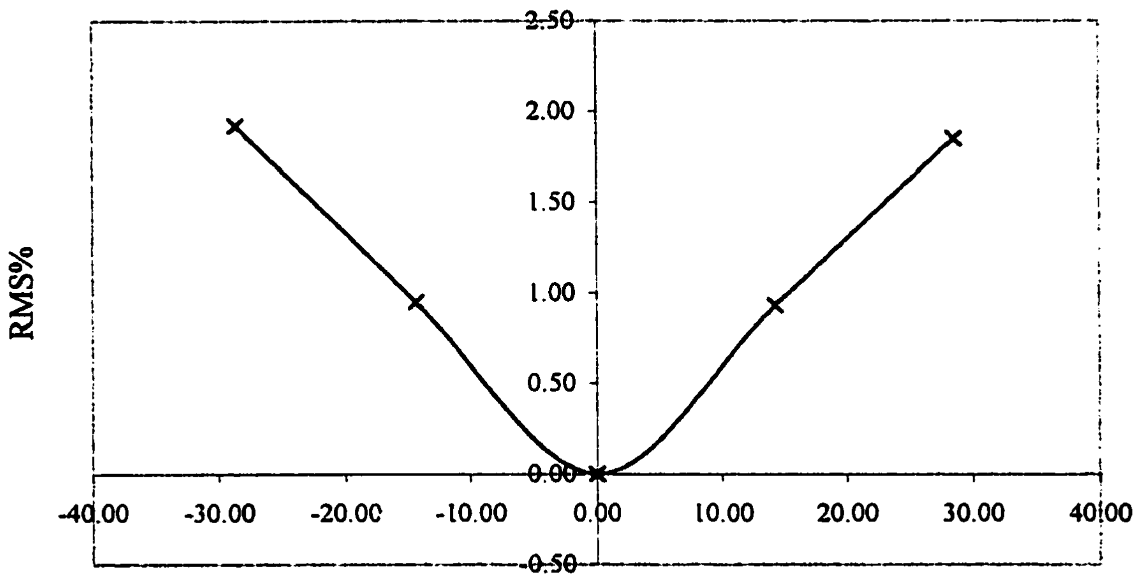


Figure 5A-17: Model 3, E1



% change in E2

Figure 5A-18: Model 3, E2

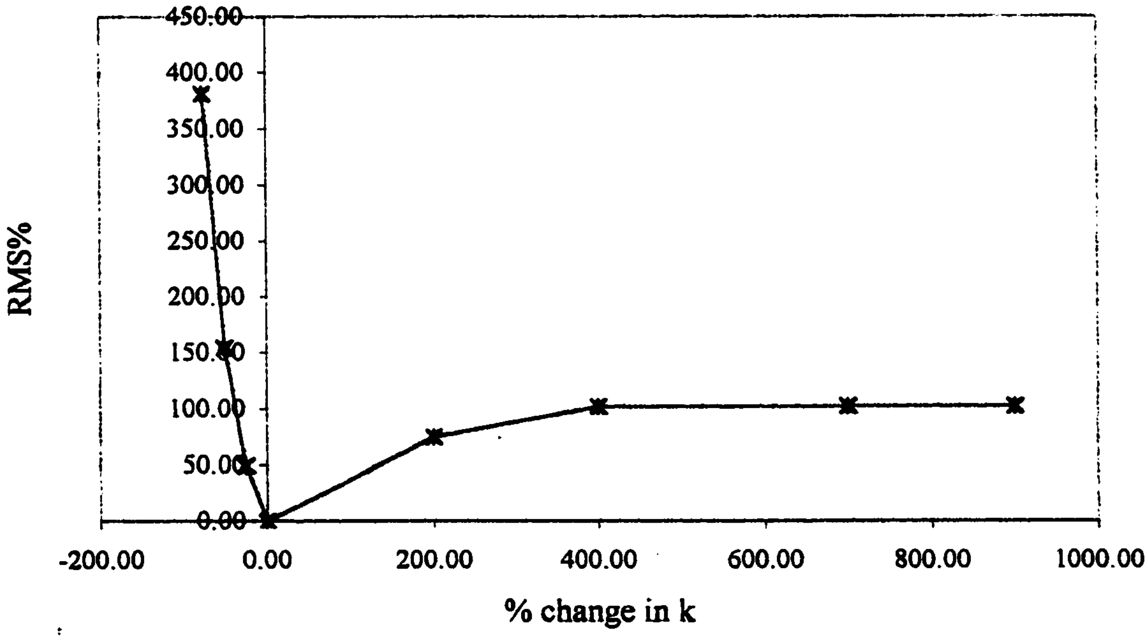


Figure 5A-19: Model 3, k

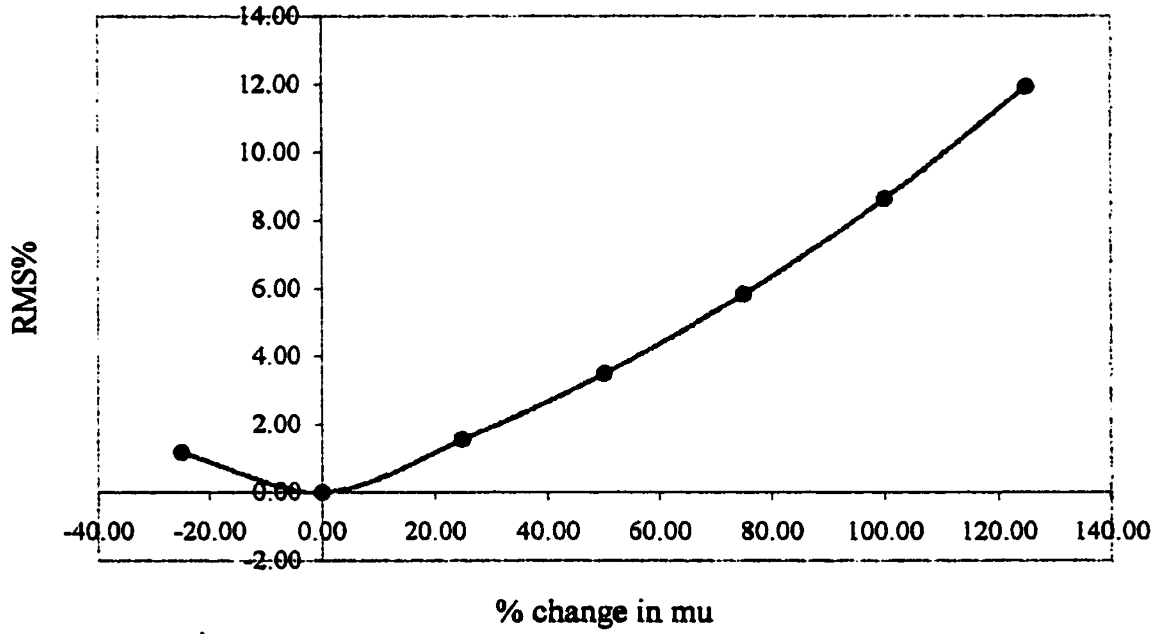


Figure 5A-20: Model 3, mu

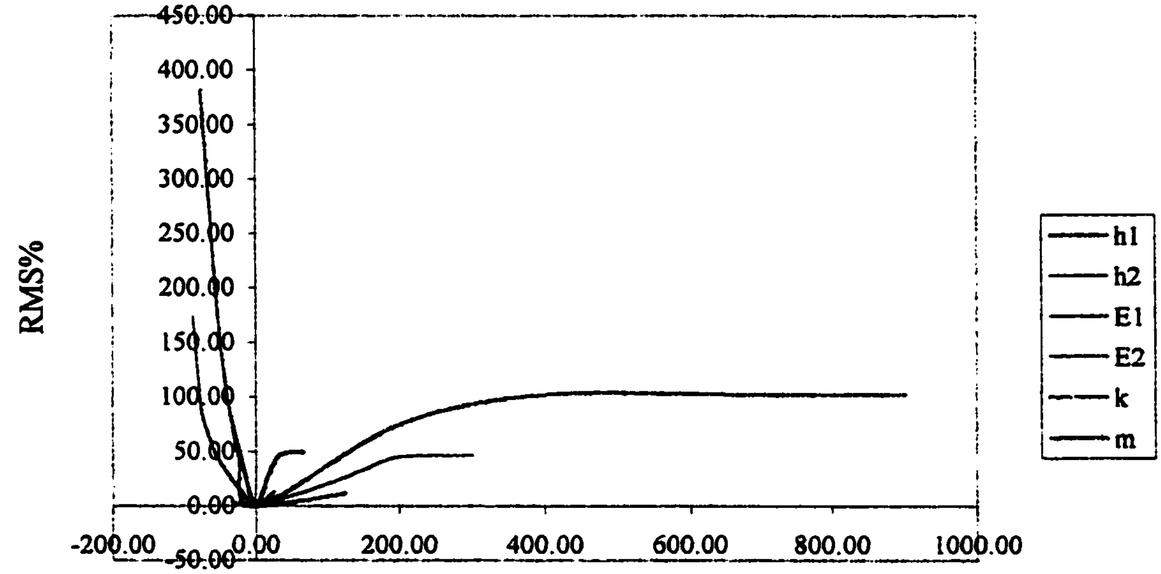


Figure 5A-21: Model 3, All Parameters

Table 5A-4: Model 5 Asphalt over a Granular Base

parameter varied	% change in parameter	RMS%	% change in $\overline{D}$
h1	-33.33	157.79	25.96
h1	-16.67	185.55	14.45
h1	16.67	77.64	-18.05
h1	33.33	151.96	-40.32
h1	50.00	231.56	-67.43
h1	66.67	292.43	-99.97
h2	-60.00	269.68	70.25
h2	20.00	173.07	-47.83
h2	50.00	360.15	-149.88
h2	80.00	469.92	-294.44
h2	140.00	571.48	-737.39
E1	-75.00	159.82	25.05
E1	-50.00	186.86	13.93
E1	-25.00	205.75	6.44
E1	25.00	27.92	-5.99
E2	-28.57	167.34	21.81
E2	-14.29	194.69	10.84
E2	14.29	48.28	-10.70
E2	28.57	89.60	-21.26
k	-75.00	1813.96	0.00
k	-50.00	558.89	0.00
k	-25.00	166.07	0.00
k	200.00	120.57	0.00
k	400.00	128.71	0.00
k	700.00	100.52	0.00
k	900.00	103.40	0.00
μ	-50.00	204.58	6.91
μ	-33.33	208.82	5.21
μ	-16.67	14.59	2.93
μ	16.67	17.55	-3.70
μ	33.33	38.23	-8.33
μ	50.00	62.25	-14.11

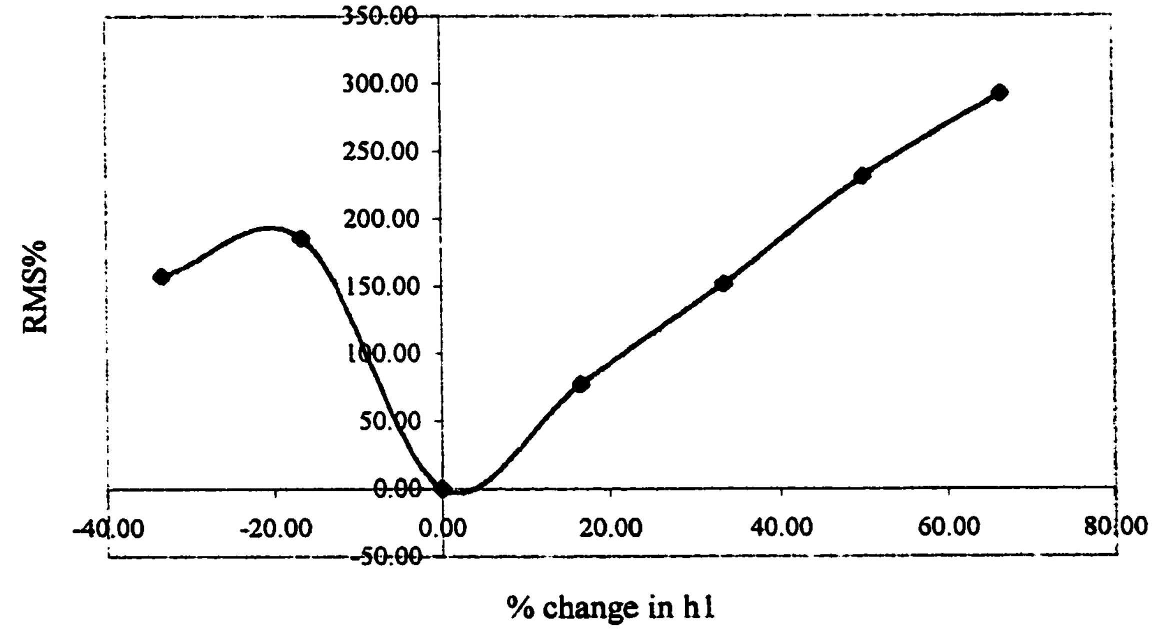


Figure 5A-22: Model 5, h1

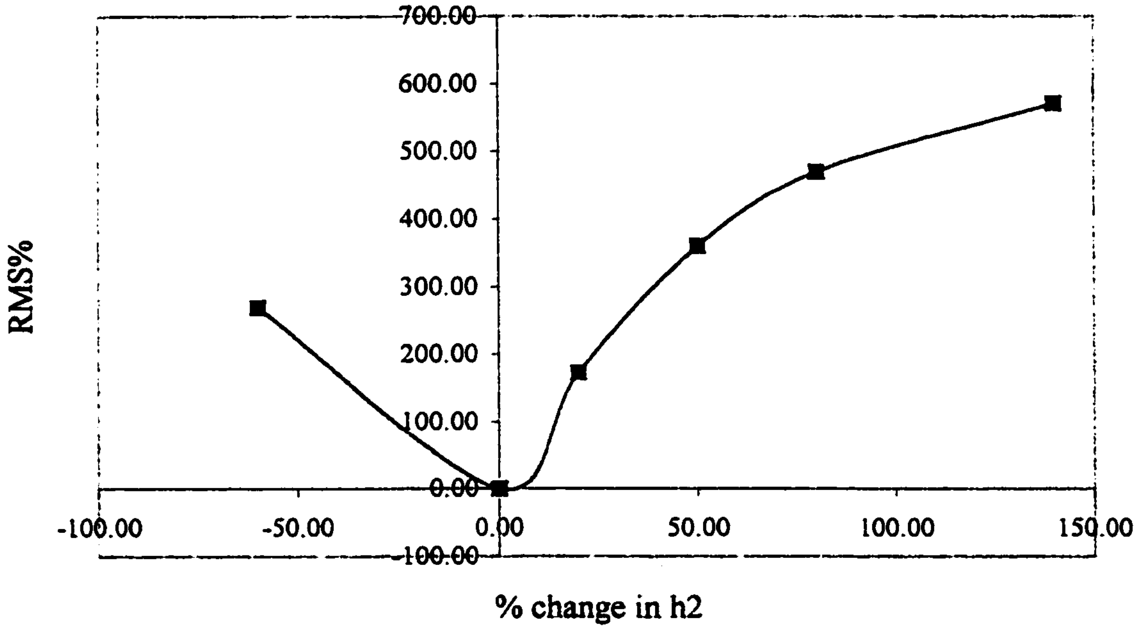


Figure 5A-23: Model 5, h2

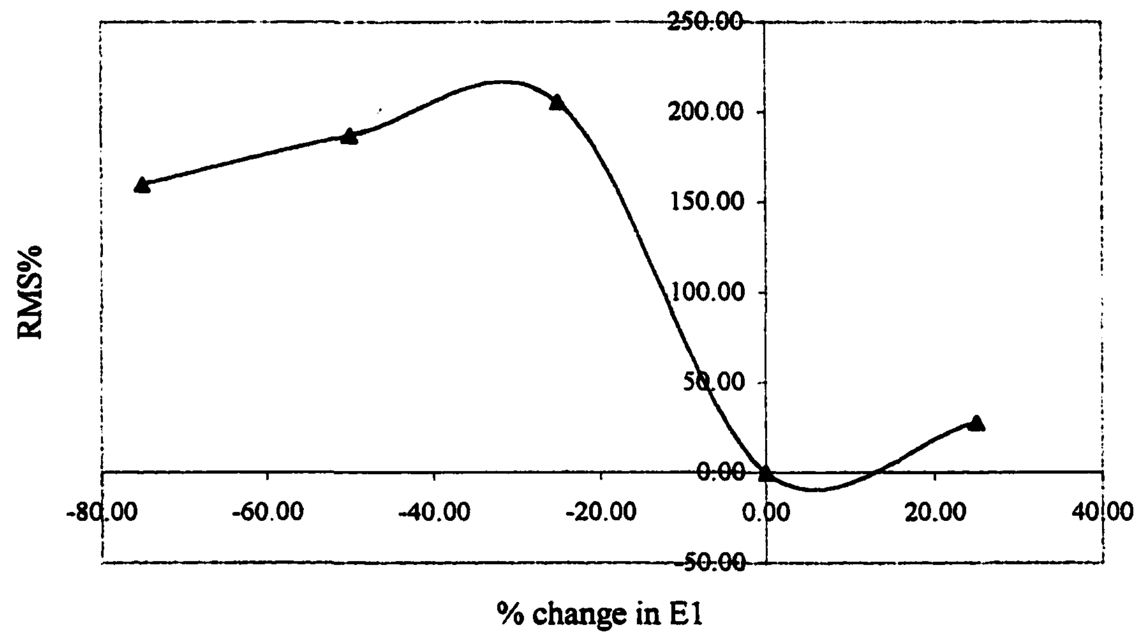


Figure 5A-24: Model 5, E1

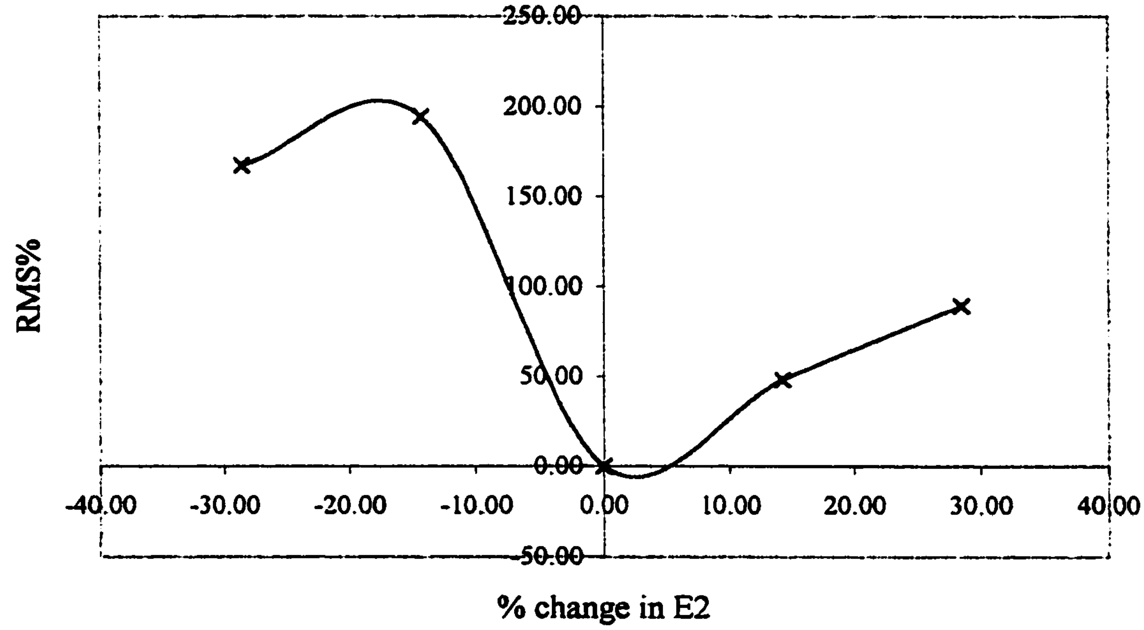


Figure 5A-25: Model 5, E2

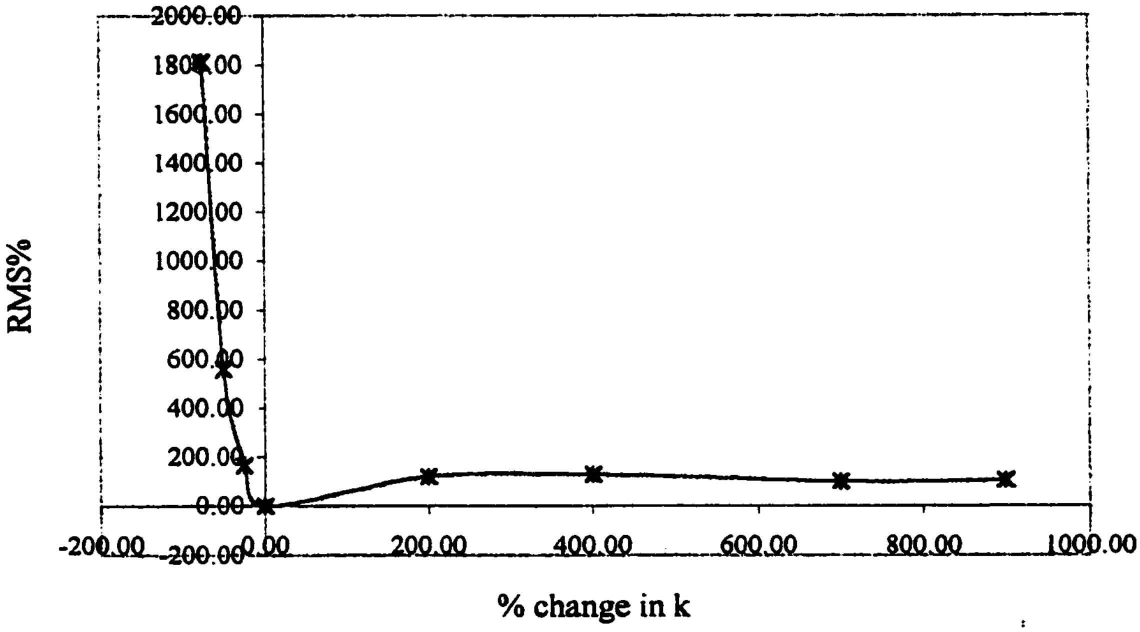


Figure 5A-26: Model 5, k

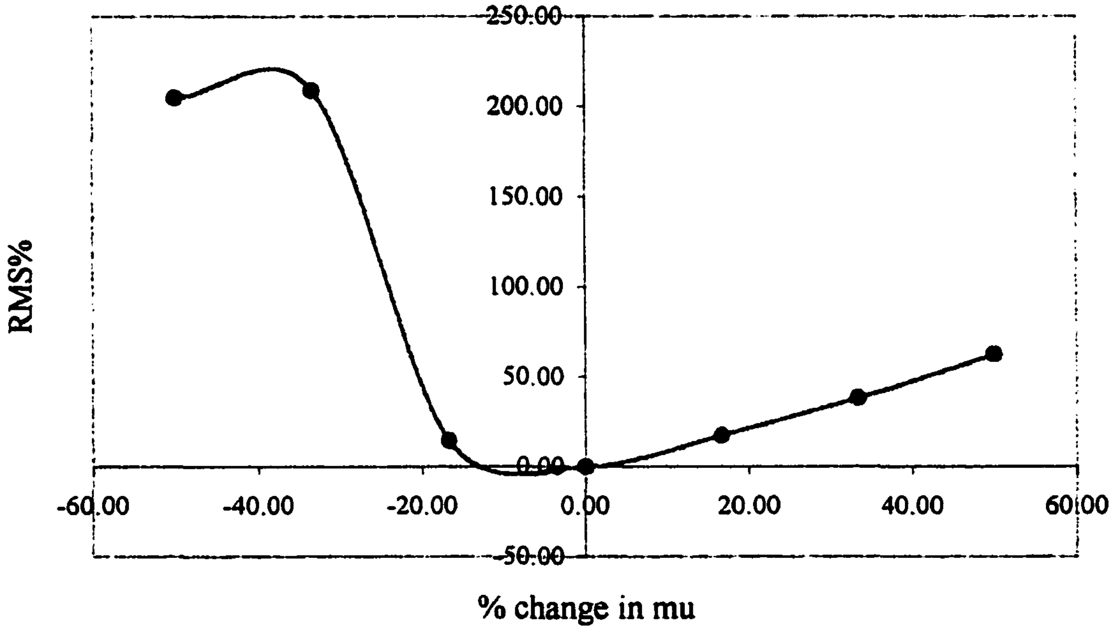


Figure 5A-27: Model 5, mu

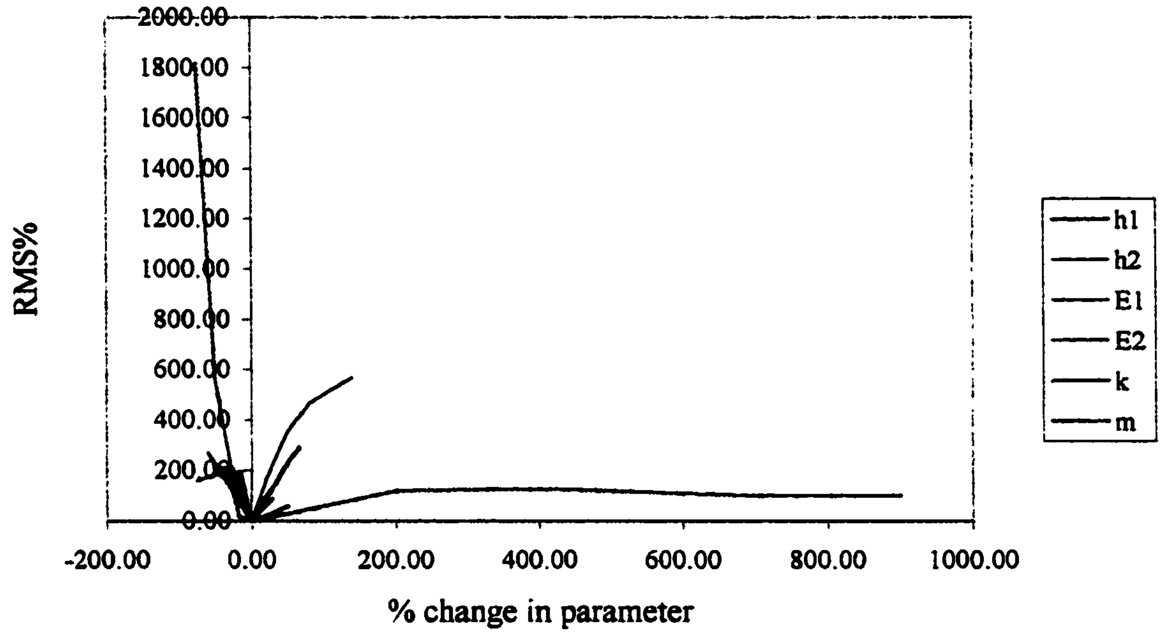


Figure 5A-28: Model 5, All Parameters

Table 5A-5: Model 3-1 Concrete over a Stabilized Base with an AC Overlay

parameter varied	% change in parameter	RMS%	% change in $\overline{D}$
h1	-33.33	6.91	16.33
h1	-16.67	3.33	8.62
h1	16.67	3.06	-9.57
h1	33.33	5.85	-20.12
h1	50.00	10.42	-31.71
h1	66.67	12.56	-44.34
h2	-33.33	22.33	38.47
h2	-16.67	9.48	21.09
h2	16.67	7.04	-25.26
h2	33.33	14.14	-55.17
h2	66.67	22.09	-130.78
h3	-33.33	15.09	29.75
h3	100.00	25.26	-169.41
h3	150.00	34.95	-308.27
h3	200.00	44.81	-489.14
h3	300.00	61.65	-992.84
E1	-75.00	13.70	27.79
E1	-50.00	7.62	17.71
E1	-25.00	3.27	8.48
E1	25.00	2.54	-7.82
E2	-87.50	6.44	15.39
E2	-75.00	5.32	13.09
E2	-62.50	4.29	10.85
E2	-50.00	3.33	8.64
E2	-25.00	1.58	4.29
E2	25.00	1.43	-4.26
E3	-50.00	13.34	27.27
E3	50.00	6.62	-23.40
E3	100.00	12.47	-43.75
E3	150.00	15.01	-61.68
k	-75.00	195.15	0.00
k	-50.00	86.43	0.00
k	-25.00	32.68	0.00
k	200.00	61.34	0.00
k	400.00	77.10	0.00
k	700.00	85.43	0.00
k	900.00	87.98	0.00
μ	33.33	0.63	-1.82
μ	66.67	1.43	-4.27
μ	100.00	2.41	-7.42
μ	133.33	3.58	-11.40
μ	166.67	4.91	-16.37
μ	200.00	6.43	-22.57

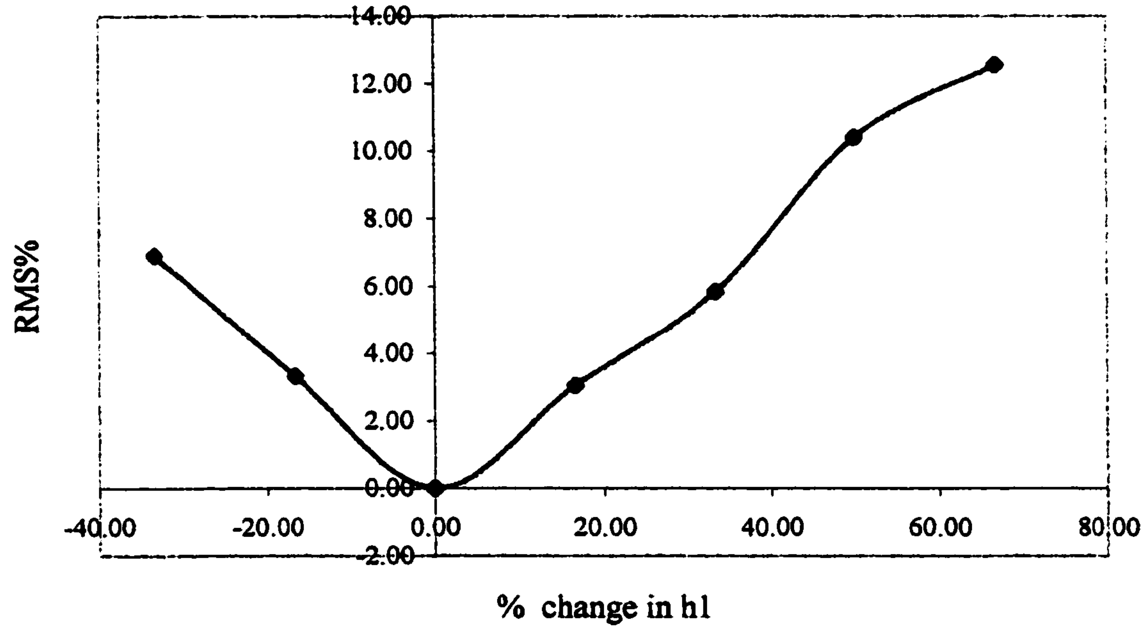


Figure 5A-29: Model 3-1, hl

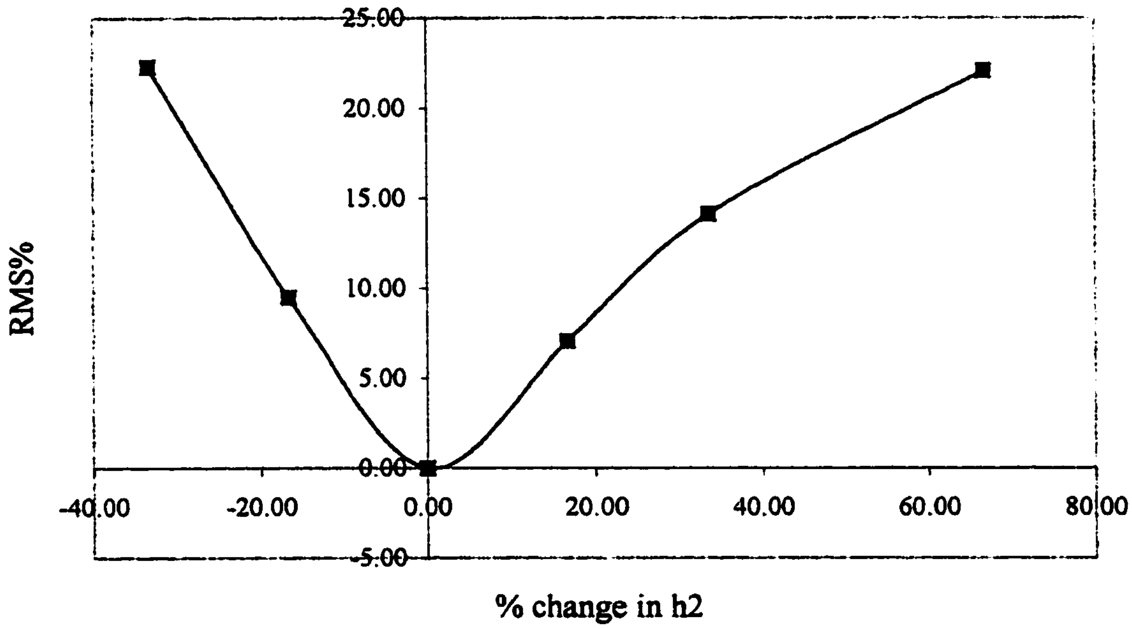


Figure 5A-30: Model 3-1, h2

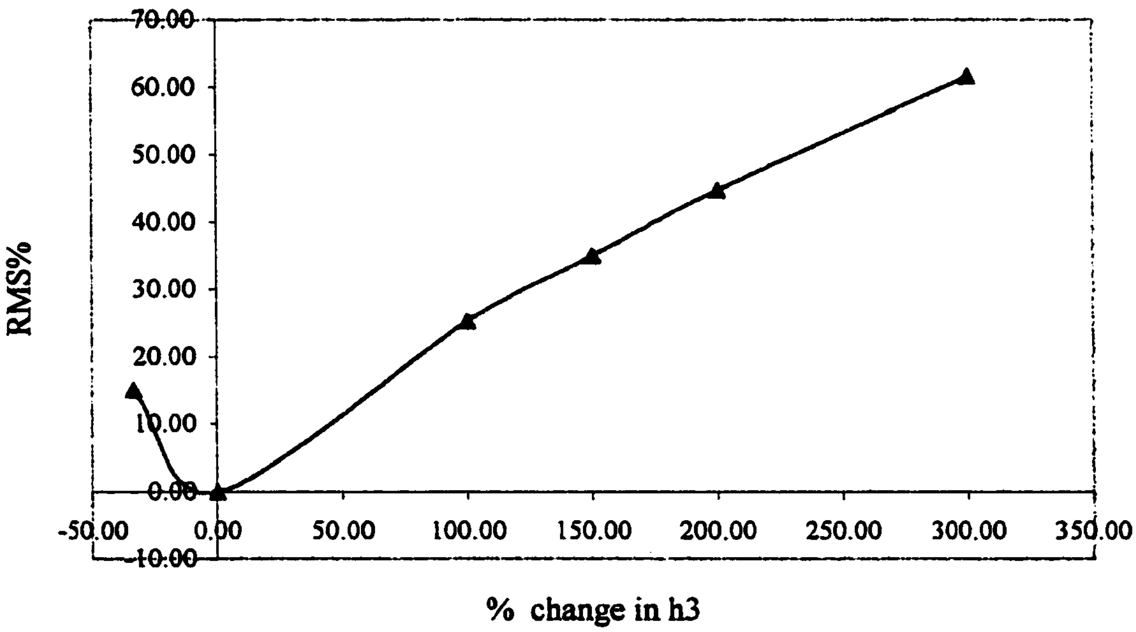


Figure 5A-31: Model 3-1, h3

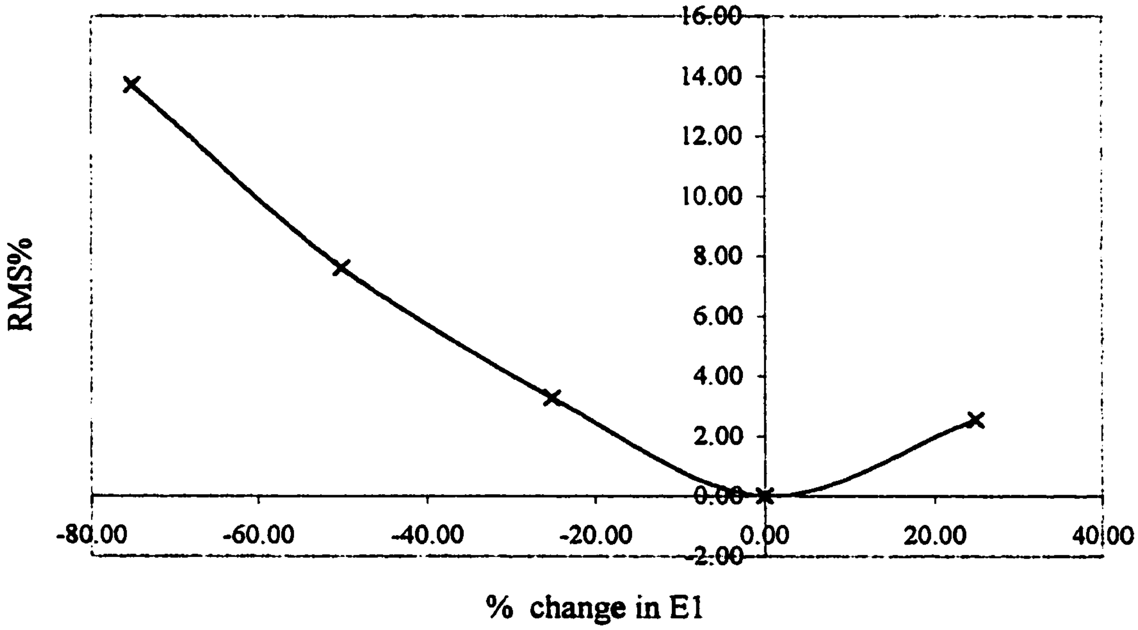


Figure 5A-32: Model 3-1, E1

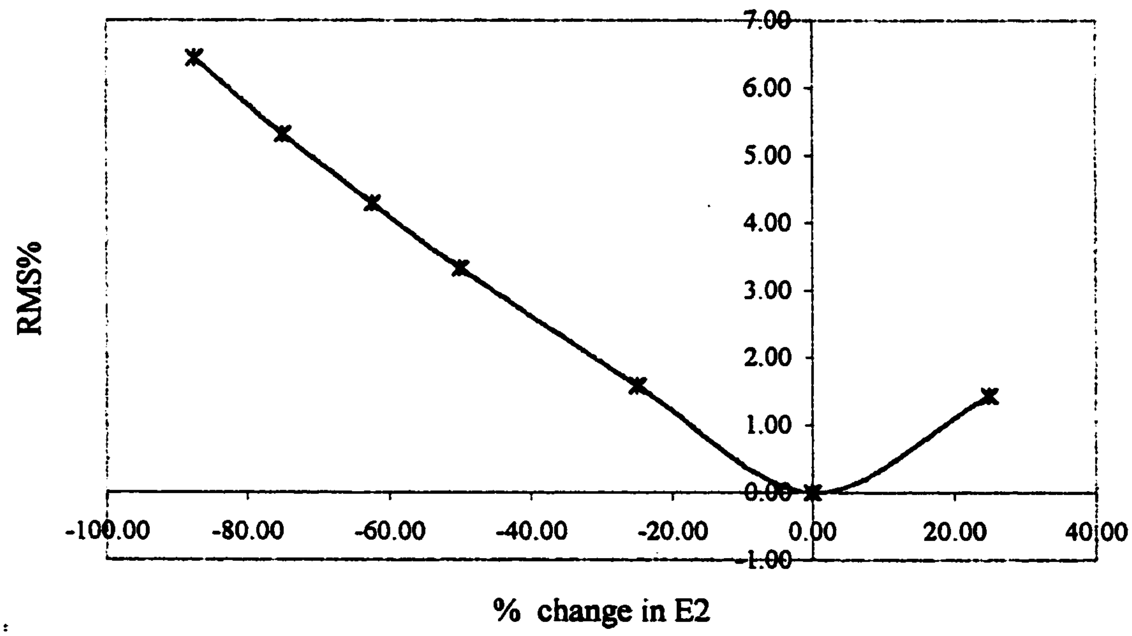


Figure 5A-33: Model 3-1, E2

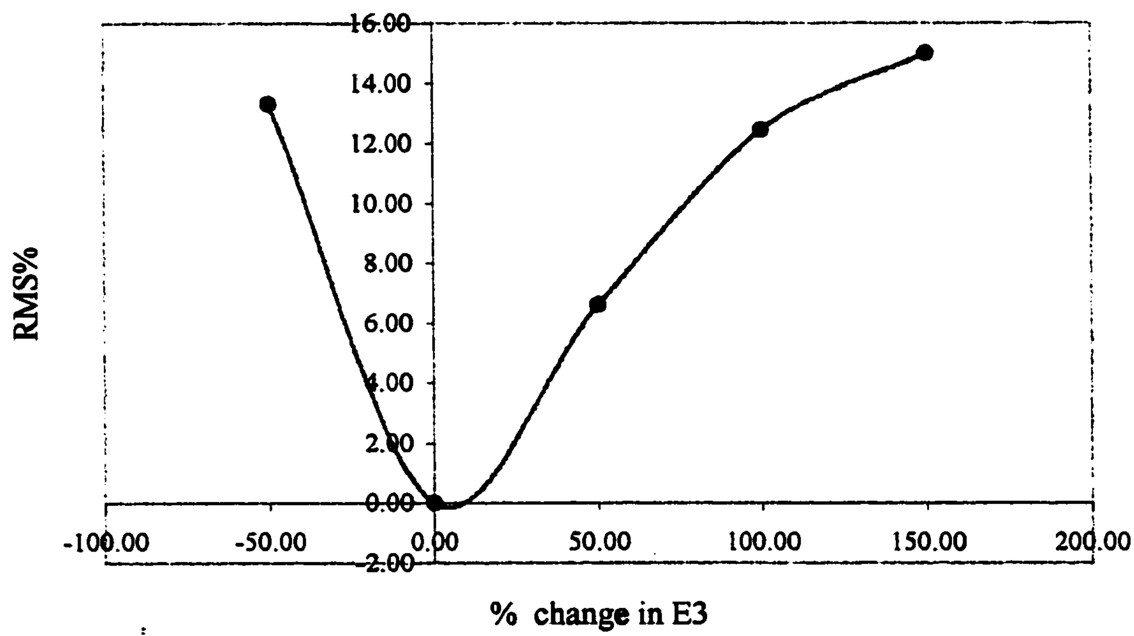


Figure 5A-34: Model 3-1, E3

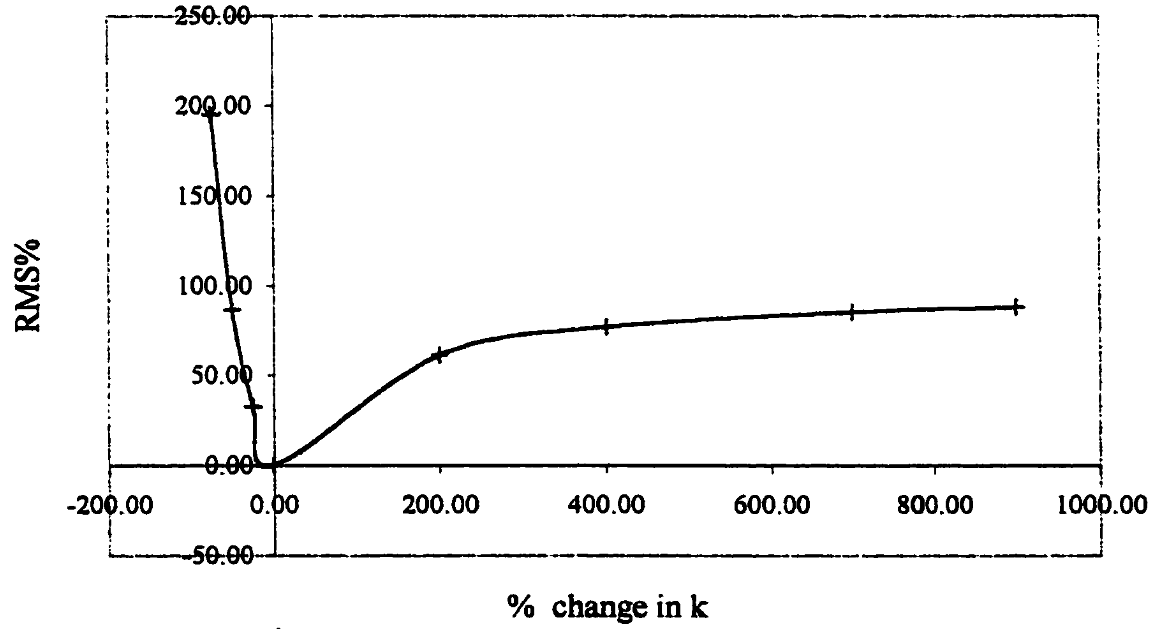


Figure 5A-35: Model 3-1, k

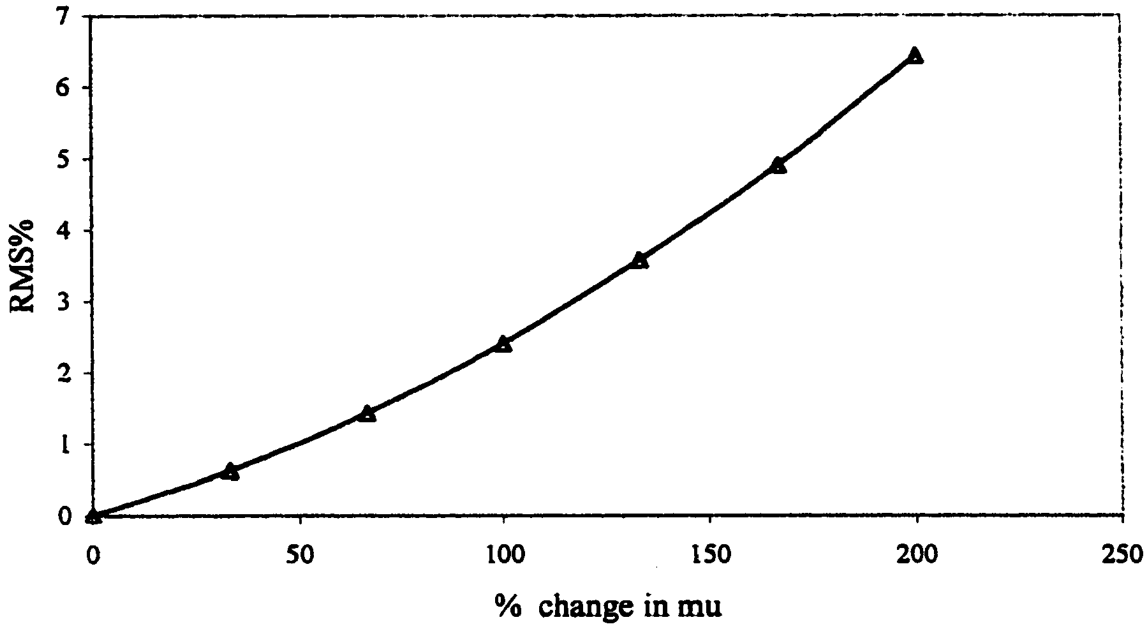


Figure 5A-36: Model 3-1, mu

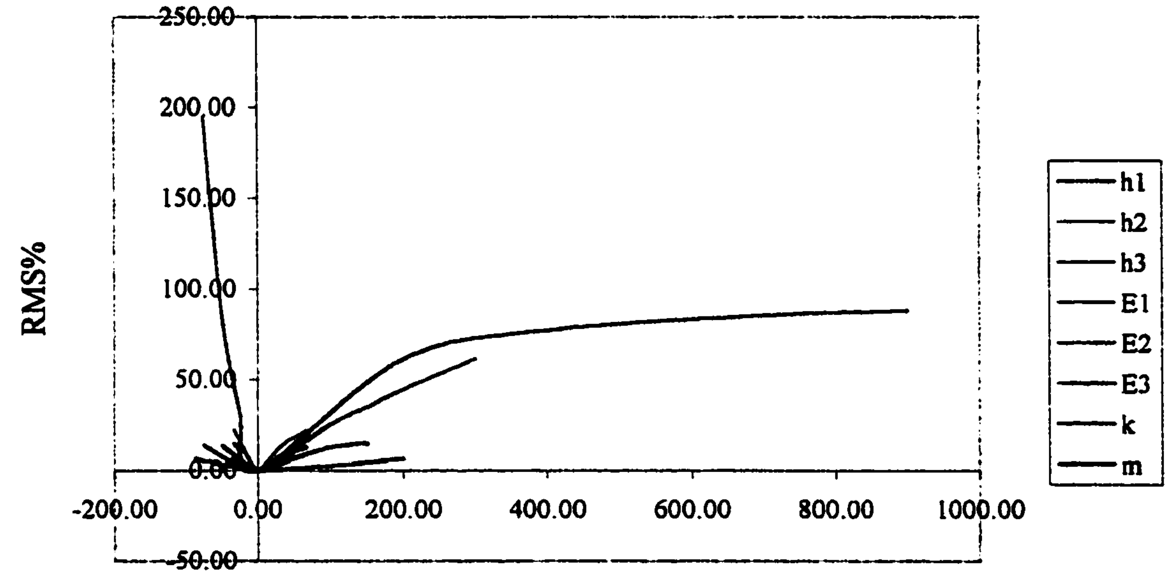


Figure 5A-37: Model 3-1, All Parameters

Table 5A-2: Model 3-6 Concrete over a Granular Base with an AC Overlay

parameter varied	% change in parameter	RMS%	% change in $\bar{D}$
h1	-33.33	10.05	25.20
h1	-16.67	4.79	13.48
h1	16.67	4.26	-15.32
h1	33.33	25.88	-32.57
h1	50.00	21.81	-51.83
h1	66.67	19.36	-73.18
h2	-33.33	28.82	52.62
h2	-16.67	12.50	29.86
h2	16.67	24.50	-38.05
h2	33.33	18.67	-85.37
h2	66.67	22.69	-212.20
h3	-33.33	0.97	3.00
h3	100.00	30.14	-19.04
h3	150.00	24.99	-36.00
h3	200.00	20.75	-59.23
h3	300.00	18.60	-128.26
E1	-75.00	18.76	40.02
E1	-50.00	10.09	25.28
E1	-25.00	4.21	12.01
E1	25.00	3.15	-10.93
E2	-87.50	37.88	61.33
E2	-75.00	23.84	46.85
E2	-62.50	16.22	36.16
E2	-50.00	11.17	27.38
E2	-25.00	4.49	12.72
E2	25.00	3.36	-11.74
E3	-28.57	0.52	1.63
E3	-14.29	0.26	0.81
E3	14.29	0.25	-0.81
E3	28.57	0.50	-1.62
k	-75.00	233.69	0.00
k	-50.00	79.99	0.00
k	-25.00	31.82	0.00
k	200.00	70.14	0.00
k	400.00	82.08	0.00
k	700.00	86.95	0.00
k	900.00	94.96	0.00
μ	-50.00	2.31	6.91
μ	-33.33	1.72	5.21
μ	-16.67	0.95	2.93
μ	16.67	1.13	-3.70
μ	33.33	2.45	-8.33
μ	50.00	3.96	-17.11

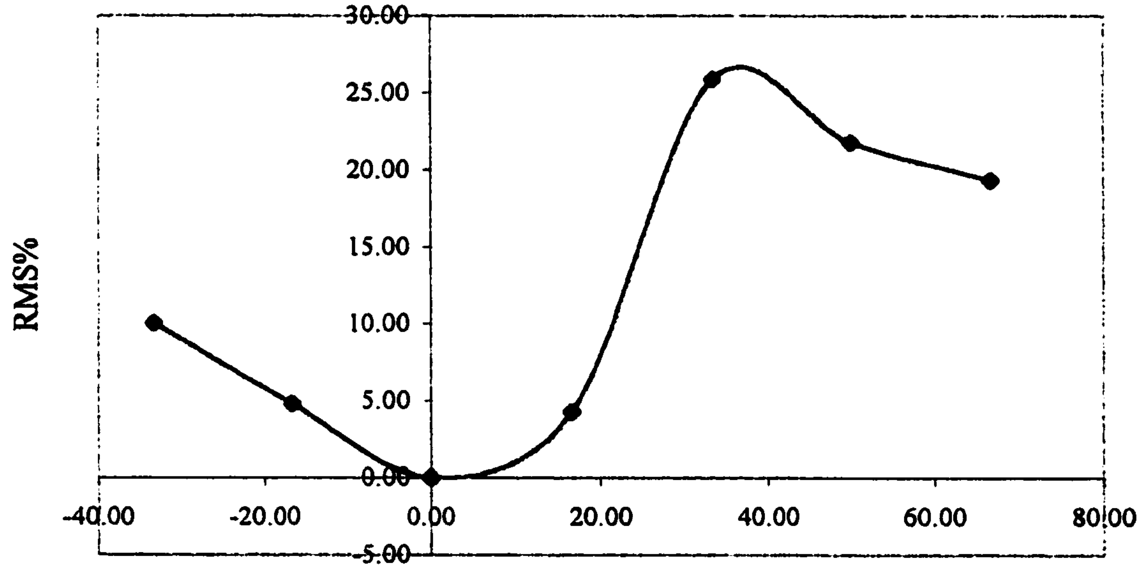


Figure 5A-38: Model 3-2, h1

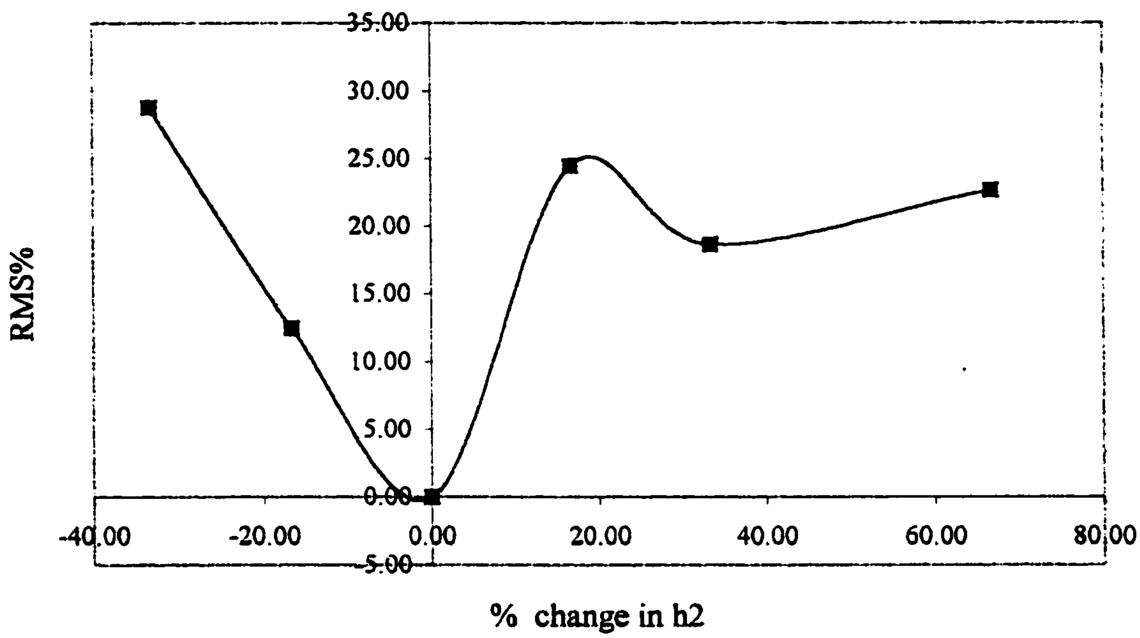


Figure 5A-39: Model 3-2, h2

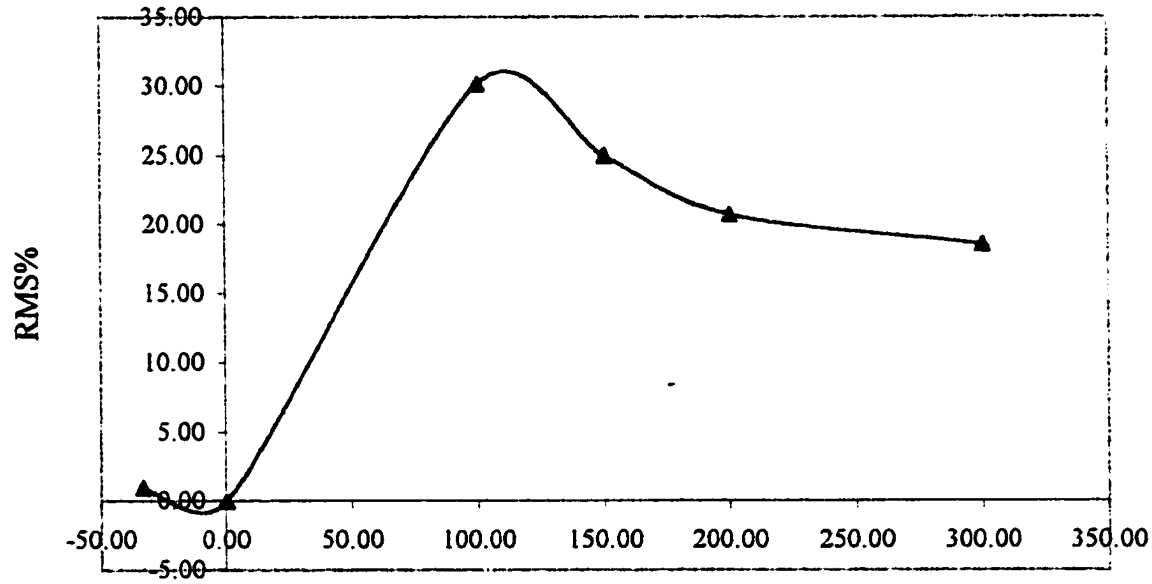


Figure 5A-40: Model 3-2, h3

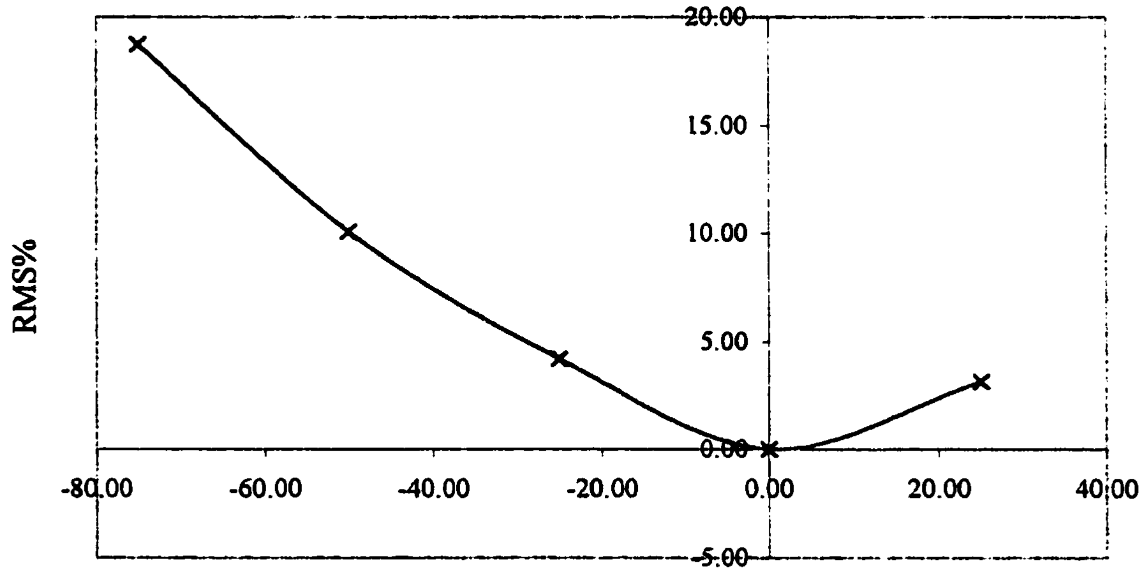


Figure 5A-41: Model 3-2, E1

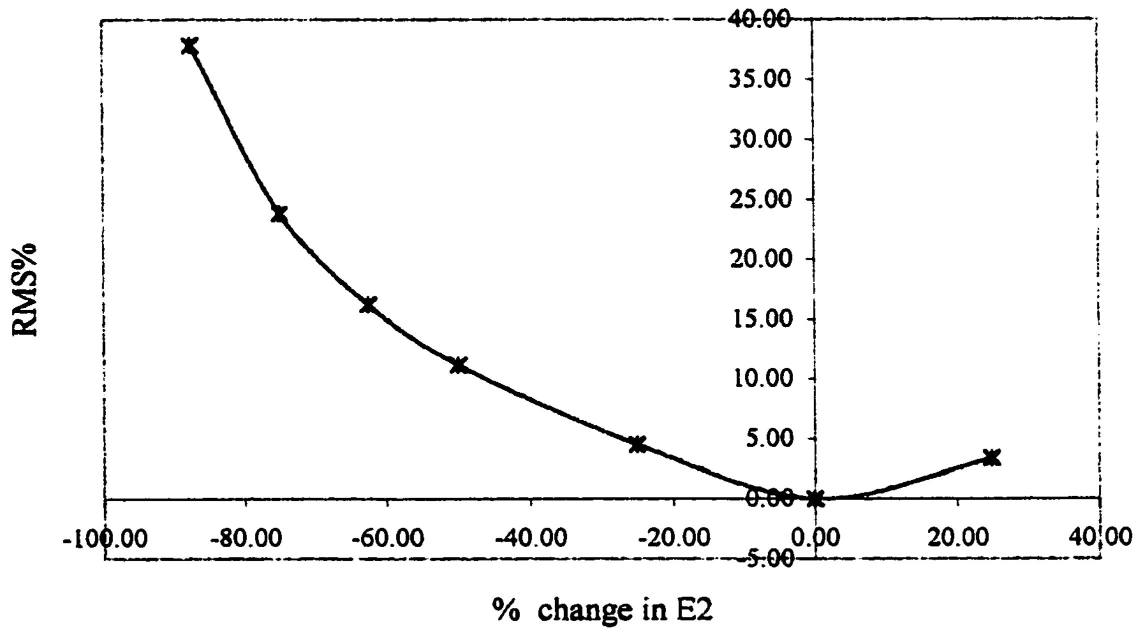


Figure 5A-42: Model 3-2, E2

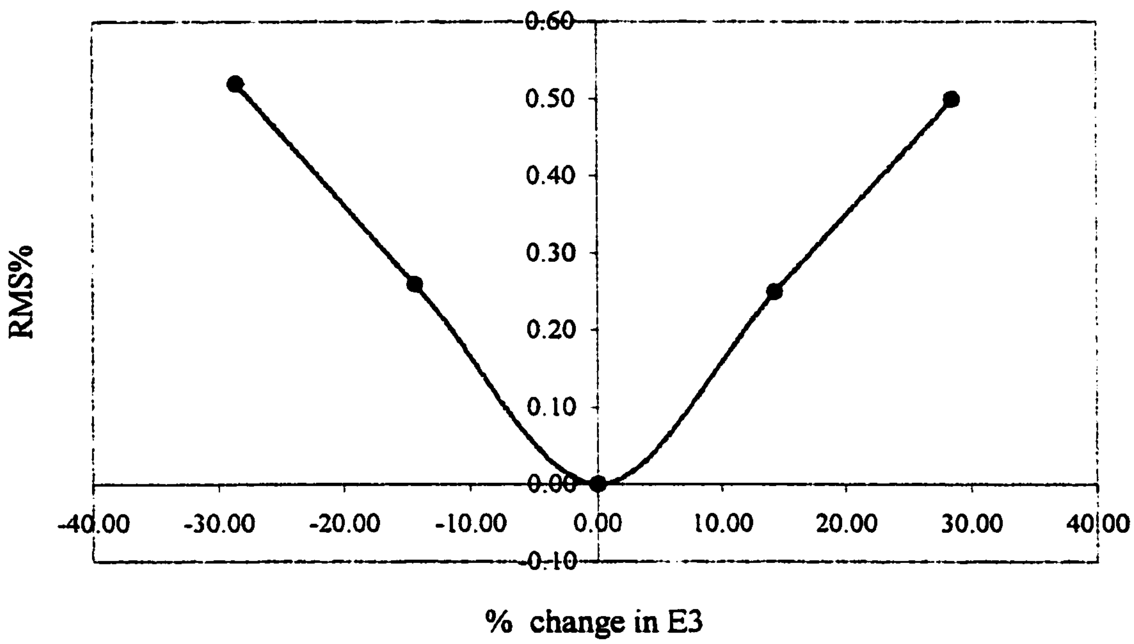


Figure 5A-43: Model 3-2, E3

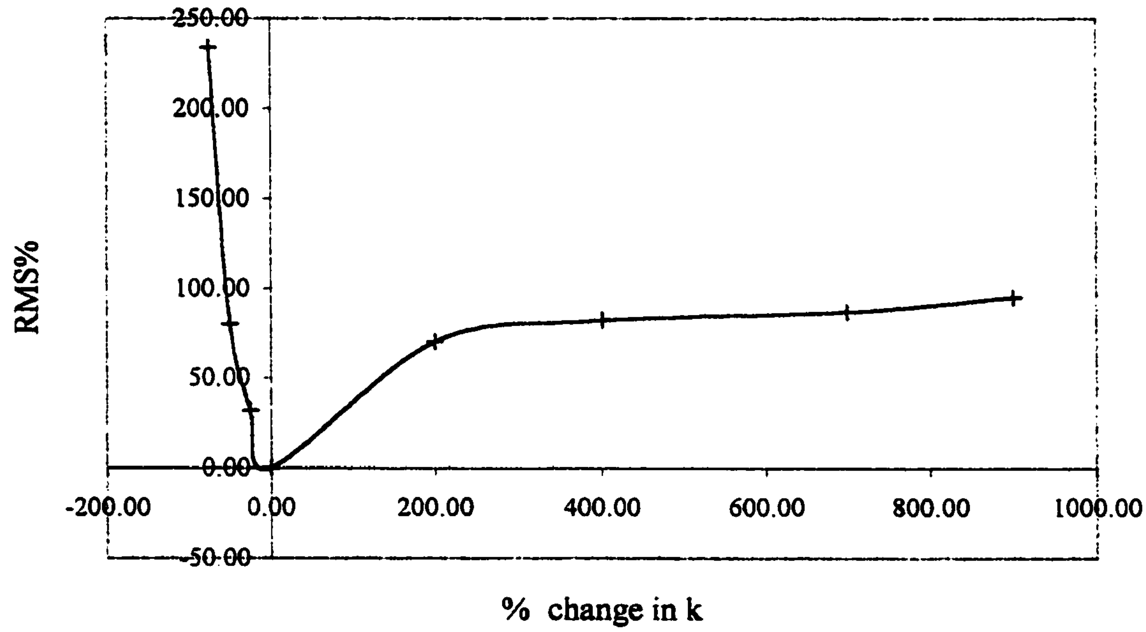


Figure 5A-44: Model 3-2, k

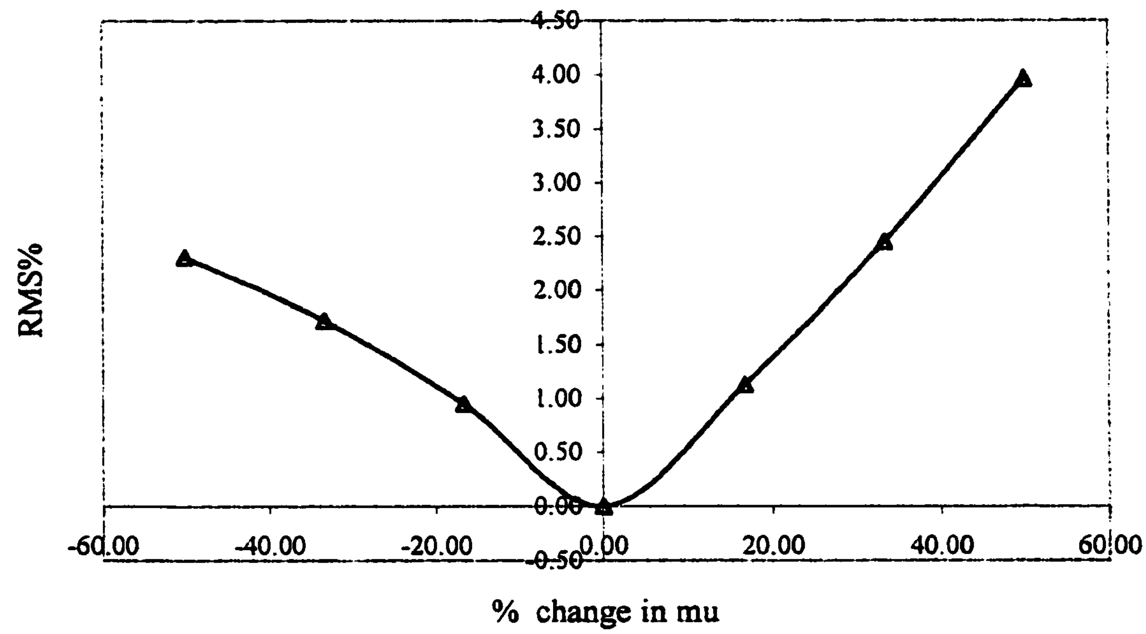


Figure 5A-45: Model 3-2, mu

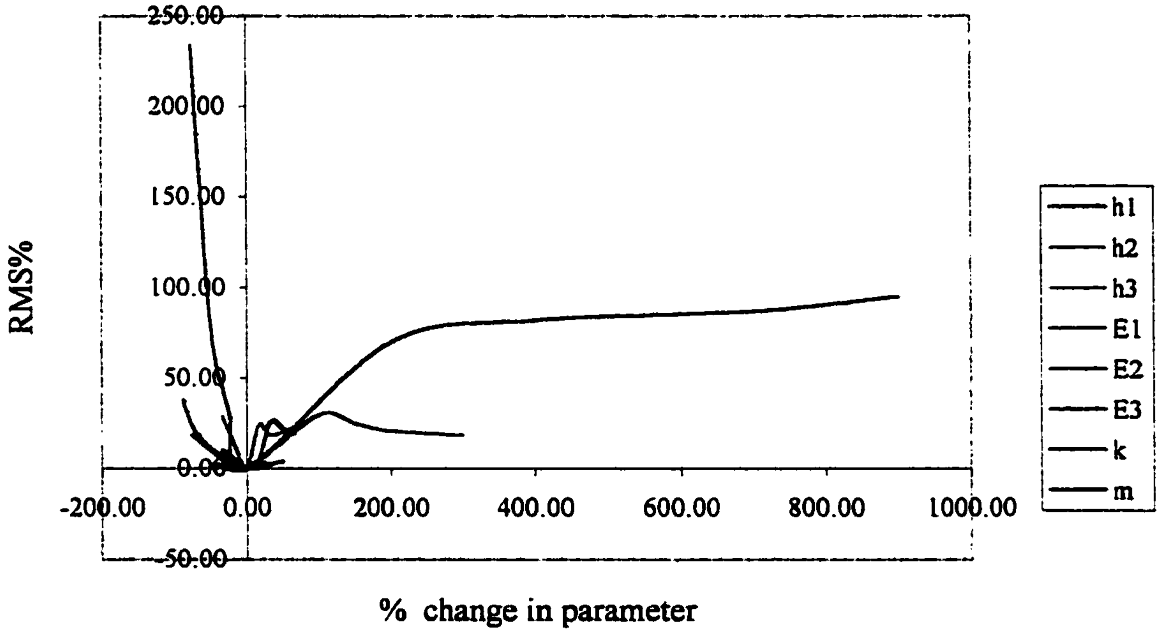


Figure 5A-46: Model 3-2, All Parameters

APPENDIX 6A
LTPP Cases Used For Comparison

Nineteen cases were chosen from the LTPP Database using the DataPave 2.0 software. Nine of these were asphalt concrete (AC) over a stabilized base, three were AC over a granular (untreated) base, three were for an asphalt wearing surface over an asphalt binder course, three cases were 1-layer cases of AC laid directly over the subgrade and one case was for a Portland cement concrete (PCC) over an untreated base.

The cases were chosen to be comparable, that is locations were chosen representative from a wide spread of regions available but testing date and time was chosen to be comparable. The attempt was made to obtain data from tests that were taken during the Fall of the year in the geographic location, usually August or September depending on the region. In some cases data was not available for Fall season for the specific case chosen so the nearest date was used. Also test data was chosen which was taken at a similar time of day, somewhere near 10:00 a.m.. Both the choice of time and season of the testing was done to eliminate the chances that tests were taken under extreme conditions such as extremely hot or cold temperatures. The most recent test data available that came nearest to fitting these desired conditions were used. The following pages describe each case in detail. Note that case numbers 5 and 13 are not presented here. These two cases were thrown out due to inappropriate conditions.

Case 1 (LTPP case no. 1015)
Location: Oklahoma, Seminole County, approximately 2.4 miles South of Seminole City Limit.
Description: Rural minor arterial, state road, two 12 foot lanes, 500 ft section.
Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Geological class: limestone
Asphalt source: Kerr-McGee Refining co., Wynnewood, OK
AC specific Gravity: 1.01, Asphalt Cements 85-100 pen
Thickness: 1.5 in.
Gradation: 100% passing ½" sieve
65% passing no. 4
46% passing no. 10
26% passing no. 40
16% passing no. 80
8% passing no. 200

Base Layer: Dense Graded, Hot laid, Central Plant Mix.
Geological class: limestone
Asphalt source: Kerr-McGee Refining co., Wynnewood, OK
AC specific Gravity: 1.01, asphalt cements 85-100 pen
Thickness: 8.8 in.
Gradation: 100% passing 1½" sieve
93% passing 1" sieve
66% passing ½" sieve
43% passing no. 4
33% passing no. 10
24% passing no. 40
13% passing no. 80
6% passing no. 200

Subgrade Layer: Sand, AASHTO soil classification A-1-b
Gradation: 41% passing no. 40
12% passing no. 200

Test Data: Plate radius = 150 mm (5.91 in) Test Date: February 11, 1999 at 11:25 am
Pavement surface temp 13 °C (55 °F); Air temp 12 °C (54 °F)

Table 6A-1: Case 1 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.0041	0.00632	0.00862	0.01104
8	0.00378	0.00577	0.00784	0.01002
12	0.00347	0.0053	0.00718	0.00924
18	0.00304	0.00468	0.00636	0.00815
24	0.00261	0.00402	0.00546	0.00698
26	0.00191	0.00296	0.00398	0.00515
60	0.00094	0.00144	0.00195	0.00254

Case 2 (LTPP case no. 1017)
Location: Oklahoma, Muskogee County, approximately 1580 ft. South of Arkansas River.
Description: Rural principal arterial, two 12 foot lanes, 500 ft section, constructed 1981.
Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Geological class: limestone
Asphalt source: Sun Co., Tulsa, OK
AC specific Gravity: 0.999, asphalt cement 85-100 pen, Mean AC content 5.2%
Thickness: 2.0 in.
Gradation: 100% passing ¾" sieve
93% passing ½" sieve
83% passing 3/8" sieve
63% passing no. 4
46% passing no. 10
26% passing no. 40
13% passing no. 80
5% passing no. 200

Base Layer: Dense Graded, Hot laid, Central Plant Mix.
Geological class: limestone
Asphalt source: Sun Co., Tulsa, OK
AC specific Gravity: 0.999, asphalt cement 85-100 pen., mean AC content 4.1%
Thickness: 8.0 in.
Gradation: 100% passing 1½" sieve
93% passing 1" sieve
73% passing ½" sieve
51% passing no. 4
40% passing no. 10
26% passing no. 40
12% passing no. 80
4% passing no. 200

Subgrade Layer: Clayey Silt, AASHTO soil classification A-4
Gradation: 99% passing no. 40
42% passing no. 200

Case 2 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: May 13, 1993 at 8:32 am
Pavement surface temp 22 °C (72 °F); Air temp 18 °C (64 °F)

Table 6A-2: Case 2 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00819	0.01248	0.0163	0.02106
8	0.00683	0.01045	0.01373	0.01775
12	0.00585	0.00901	0.0119	0.01541
18	0.00460	0.00714	0.0094	0.01225
24	0.00359	0.00566	0.00749	0.00975
26	0.00226	0.00355	0.00464	0.00605
60	0.00113	0.00183	0.00238	0.00308

Case 3 (LTPP case no. 1034)
Location: New Jersey, Rockingham County, Begins 0.5 mi. south of city. MP58.5 (ST-55 SB) Deptford, NJ
Description: Urban principal arterial, two 12 foot lanes, 500 ft section, constructed 1985.
Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Atlantic Refining & Marketing Corp., Philadelphia, PA
AC specific Gravity: 1.02, AC-20, Mean AC content 4.4%
Mineral filler = stone dust
Thickness: 2.0 in.
Gradation: 100% passing 1" sieve
98% passing ¾" sieve
82% passing ½" sieve
71% passing 3/8" sieve
46% passing no. 4
40% passing no. 8
16% passing no. 50
5% passing no. 200

Base Layer: Dense Graded, Hot laid, Central Plant Mix.
Asphalt source: Atlantic Refining & Marketing Corp., Philadelphia, PA
AC specific Gravity: 1.02, AC-20, Mean AC content 4.9%
Mineral filler = stone dust
Thickness: 10.0 in.
Gradation: 100% passing 1" sieve
74% passing ½" sieve
45% passing no. 4
38% passing no. 8
16% passing no. 50
6% passing no. 200

Subgrade Layer: Silt, AASHTO soil classification A-4
No info available

Case 3 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: July 30, 1997 at 10:01 am
Pavement surface temp 39 °C (102 °F); Air temp 26 °C (79 °F)

Table 6A-3: Case 3 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00741	0.01147	0.01486	0.01923
8	0.00569	0.0087	0.01147	0.0149
12	0.00488	0.00764	0.0101	0.01318
18	0.00390	0.00655	0.00862	0.01131
24	0.00347	0.00542	0.00725	0.00948
26	0.00230	0.00371	0.00495	0.00655
60	0.00117	0.00191	0.00254	0.00324

Case 4 (LTPP case no. 1048)

Location: Texas, Ector County, Station 134+00 to 129+00, 134+00 is 75' south of 45th st., 1200' north of US 385 SB, is in fron of Ector Co. Convention Center.

Description: Urban minor arterial,constructed in 1974.

Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.

Asphalt source: Fina Oil & Chemical Co., Big Spring, TX
AC specific Gravity: 1.03, AC-5, Mean AC content 5.9%
Thickness: 1.5 in.
Crushed stones, crushed gravel, and gravel
Gradation: 100% passing ½" sieve
95% passing 3/8" sieve
52% passing no. 4
32% passing no. 10
20% passing no. 40
12% passing no. 80
3% passing no. 200

Base Layer: Dense Graded, Hot laid, Central Plant Mix.

Asphalt source: Fina Oil & Chemical Co., Big Spring, TX
AC specific Gravity: 1.03, AC-5, Mean AC content 9%
Geological class: limestone
Thickness: 9.0 in.
Gradation: 100% passing 1½" sieve
83% passing 7/8" sieve
70% passing 5/8" sieve
55% passing 3/8" sieve
40% passing no. 4
30% passing no. 10
21% passing no. 40
12% passing no. 100
8% passing no. 200

Subgrade Layer: Clayey sand, AASHTO soil classification A-2-4

96% passing no. 40
6% passing no. 200

Case 4 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: November 14, 1994 at 10:00 am Pavement surface temp 16 °C (61 °F); Air temp 10 °C (50 °F)

Table 6A-4: Case 4 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.01076	0.01529	0.01973	0.02555
8	0.00811	0.0117	0.01509	0.01966
12	0.00718	0.01037	0.01353	0.01751
18	0.00597	0.00874	0.01139	0.01482
24	0.00503	0.00741	0.00971	0.01268
26	0.00355	0.00523	0.00686	0.00909
60	0.00183	0.00281	0.00374	0.00491

Case 6 (LTPP case no. 2015)
Location: Wyoming, Laramie county, sstart of section is 3.5 miles north of Exit 39 "Little Bear Community"
Description: Rural principal arterial, Interstate, two 12 foot lanes, 500 ft section, constructed 1978.
Pavement Type: AC over a non-bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Texaco, Casper WY
AC specific Gravity: 1.023, AC-10
Thickness: 4.0 in.
Gradation: 100% passing ½" sieve
94% passing 3/8" sieve
68% passing no. 4
47% passing no. 8
22% passing no. 30
5% passing no. 200

Base Layer: Cement-Aggregate Mixture
Geological class: Conglomerate
Thickness: 6.0 in.
Gradation: 100% passing 1½" sieve
99% passing the 1" sieve
93% passing ¾" sieve
86% passing ½" sieve
80% passing 3/8" sieve
66% passing no. 4
51% passing no. 8
27% passing no. 30
22% passing no. 40
7% passing no. 200

Subgrade Layer: Silt, AASHTO soil classification A-4
61% passing no. 40
44.2% passing no. 200

Case 6 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: August 19, 1998 at 11:25 am
Pavement surface temp 44 °C (111 °F); Air temp 31 °C (88 °F)

Table 6A-5: Case 6 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00308	0.00413	0.0055	0.00733
8	0.00238	0.00332	0.00437	0.00577
12	0.00222	0.00316	0.00413	0.00546
18	0.00203	0.00289	0.00374	0.00499
24	0.00179	0.00257	0.00335	0.00445
26	0.00137	0.00191	0.00254	0.00332
60	0.00066	0.00094	0.00121	0.0016

Case 7 (LTPP case no. 2812)
Location: Alberta Canada, highway district no. 6, start of section is approximately 1.5 miles north of Junction SR 583, Highway 21, SB, tThree Hills, ALTA.
Description: Rural principal arterial, one 11 foot lane, 500 ft section, constructed 1978.
Pavement Type: AC over a non-bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Imperial Oil Ed
AC specific Gravity: 1.036, Asphalt grade 200-300
Mineral filler: silt & clay
Thickness: 6.0 in.
Gradation: 99% passing 5/8" sieve
92% passing ½" sieve
79% passing 3/8" sieve
57% passing no. 4
44% passing no. 8
42% passing no. 10
34% passing no. 16
24% passing no. 30
20% passing no. 40
17% passing no. 50
12% passing no. 100
9% passing no. 200

Base Layer: Soil Cement
Thickness: 7.2 in.
Gradation: 99% passing no. 30
81% passing no. 50
22% passing no. 100
6% passing no. 200

Subgrade Layer: Sandy Clay, AASHTO soil classification A-6
95% passing no. 40
75% passing no. 200

Case 7 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: July 10, 1997 at 11:27 am
Pavement surface temp 34 °C (93 °F); Air temp 20 °C (68 °F)

Table 6A-6: Case 7 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00683	0.01026	0.01346	0.01798
8	0.00608	0.0092	0.01209	0.01615
12	0.00550	0.00831	0.01096	0.01463
18	0.00480	0.00725	0.00959	0.01283
24	0.00413	0.00628	0.00835	0.01108
26	0.00304	0.0046	0.00608	0.00819
60	0.00226	0.00339	0.00445	0.00601

Case 8 (LTPP case no. 4161)
Location: Oklahoma, Carter County, US 70 WB, Sta 380-375, approx. 1.09 mi W of SH 775 and 6.2 mi E of US 77
Description: Rural principal arterial, one 12 foot lane, 500 ft section, constructed 1982
Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Kerr-McGee Refining Co., Wynnewood, OK
Geological Class = granite
AC specific Gravity: 1.009, AC-20
Thickness: 2.0 in.
Gradation: 100% passing ¾" sieve
96% passing ½" sieve
88% passing 3/8" sieve
68% passing no. 4
50% passing no. 10
27% passing no. 40
13% passing no. 80
8% passing no. 200

Base Layer: Dense graded, hot laid, central plant mix
Asphalt source: Kerr-McGee Refining Co., Wynnewood, OK
Geological Class = granite
AC specific Gravity: 1.009, AC-20
Thickness: 8.0 in.
Gradation: 100% passing 1" sieve
76% passing ½" sieve
46% passing no. 4
35% passing no. 10
24% passing no. 40
12% passing no. 80
5% passing no. 200

Subgrade Layer: Sandy Clay,
No info available

Case 8 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: March 14, 1997 at 10:02 am
Pavement surface temp 8 °C (46 °F); Air temp 6 °C (43 °F)

Table 6A-7: Case 8 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00507	0.0085	0.01193	0.01494
8	0.00468	0.0078	0.01096	0.01365
12	0.00437	0.00729	0.01026	0.01283
18	0.00382	0.00644	0.00913	0.01154
24	0.00343	0.00569	0.00796	0.00995
26	0.00254	0.00425	0.00593	0.00745
60	0.00129	0.00215	0.00304	0.0039

Case 9 (LTPP case no. 4165)
Location: Oklahoma, Major County, approx. 2.2 mi W of SH 58 and 8.4 mi E of SH 8.
NOTE: Water Table at 11 ft.
Description: Rural principal arterial, one 12 foot lane, 500 ft section, constructed 1984
Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
(47% coarse, 34% fine) crushed stone, 66% crushed slag a(screening), 53% recycled concrete (mine chat)
Asphalt source: Muskogee Bulk, Muskogee, OK
Geological Class = limestone
AC specific Gravity: 1.029, AC-20
Thickness: 2.0 in.
Gradation: 100% passing ½" sieve
93% passing 3/8" sieve
68% passing no. 4
44% passing no. 10
26% passing no. 40
14% passing no. 80
6% passing no. 200

Base Layer: Dense graded, hot laid, central plant mix
Asphalt source: Muskogee Bulk, Muskogee, OK
AC specific Gravity: 1.029, AC-20
Thickness: 8.0 in.
Gradation: 100% passing 1½" sieve
93% passing 1" sieve
67% passing ½" sieve
48% passing no. 4
34% passing no. 10
20% passing no. 40
12% passing no. 80
4% passing no. 200

Subgrade Layer: Silty Sand, AASHTO soil class = A-2-4
9% passing no. 40 (THIS MUST BE AN ERROR BUT IT IS THE NUMBER IN THE LTPP DATA BASE)
87% passing no. 200

Case 9 (Continued)

<u>Test Data:</u> Plate radius = 150 mm (5.91 in) Test Date: September 11, 1997 at 10:20 am
Pavement surface temp 29 °C (84 °F); Air temp 24 °C (75 °F)

Table 6A-8: Case 9 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00374	0.00585	0.00815	0.01041
8	0.00324	0.00503	0.00698	0.00893
12	0.00293	0.00456	0.00632	0.00807
18	0.00246	0.00386	0.00534	0.00683
24	0.00203	0.0032	0.00441	0.00562
26	0.00137	0.00215	0.00296	0.00378
60	0.00062	0.00105	0.0014	0.00183

Case 10 (LTPP case no. 6454)
Location: Manitoba Canada, Portage (county?), 15.9 mi W of Hwy 16, 2.7 mi. W of Hwy 350, W. of Portage La Prairie, MT, Transcanaada 1, 1 mi W of MacGregor, MB (76 mi W of Winepeg, MB)
Description: Rural principal arterial, two 12 foot lanes, 500 ft section, constructed 1976
Pavement Type: AC over a bituminous treated base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Husky Oil SC 3000
Geological Class = limestone
AC specific Gravity: 1.008, Cutback Asphalts (RC, MC, SC) 3000
Thickness: 3.0 in.
Gradation: 100% passing ¾" sieve
98% passing 5/8" sieve
83% passing 3/8" sieve
61% passing no. 4
45% passing no. 10
22% passing no. 40
5% passing no. 200

Base Layer: Sand Asphalt
Thickness: 8.0 in.
Gradation: 100% passing 1½" sieve
99% passing 1" sieve
96% passing 3/4 " sieve
95% passing 5/8" sieve
94% passing ½" sieve
92% passing 3/8" sieve
87% passing no. 4
79% passing no. 10
69% passing no. 40
26% passing no. 80
16% passing no. 100
8% passing no. 200

Subgrade Layer: Sandy Silt, AASHTO soil class = A-4
100% passing no. 40
41% passing no. 200

Case 10 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: September 11, 1997 at 10:20 am
Pavement surface temp 29 °C (84 °F); Air temp 23 °C (73 °F)

Table 6A-9: Case 10 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00608	0.00991	0.01318	0.01798
8	0.00476	0.0078	0.01045	0.01424
12	0.00425	0.00671	0.00897	0.01225
18	0.00328	0.0053	0.0071	0.00967
24	0.00261	0.00421	0.00562	0.00764
26	0.00168	0.00277	0.00367	0.00491
60	0.00101	0.00152	0.00199	0.00261

Case 11 (LTPP case no. 1017)
Location: Minnessota, Wright County, I-94 WB, 2 mi NW of Albertville, MN, Sta 120590 to 121140
Description: Rural principal arterial, interstate, two 12 foot lanes, 500 ft section, constructed 1993
Pavement Type: AC over a granular base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Crushed stones, gravel, crushed slag
Asphalt source: Koch Refining Co., Rosemount, MN
Geological Class = granite
AC specific Gravity: 1.034, AC-20
Thickness: 7.8 in.
Gradation: 100% passing ¾" sieve
92% passing ½" sieve
82% passing 3/8" sieve
67% passing no. 4
57% passing no. 10
27% passing no. 40
4% passing no. 200

Base Layer: Crushed Stone, Gravel or Slag, AASHHTO soil class A-1-b
Thickness: 28.0 in.
Gradation: 100% passing ¾" sieve
99% passing 3/8" sieve
92% passing no. 4
82% passing no. 10
41% passing no. 40
11% passing no. 200

Subgrade Layer: Silty Clay, AASHTO soil class = A-6
88% passing no. 40
57.8% passing no. 200

Case 11(Continued)

<u>Test Data:</u> **Plate radius = 150 mm (5.91 in) Test Date: August 16, 1998 at 3:24 pm Pavement surface temp 51 °C (124 °F); Air temp 30 °C (86 °F)**

Table 6A-10: Case 11 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.01045	0.01548	0.0195	0.02594
8	0.00729	0.01096	0.01381	0.01837
12	0.00577	0.00878	0.01104	0.0147
18	0.00421	0.0064	0.00811	0.01088
24	0.00296	0.0046	0.00589	0.00796
26	0.00164	0.00257	0.00339	0.00468
60	0.0009	0.00144	0.00187	0.00261

Case 12 (LTPP case no. 1020)
Location: Minnessota, Wright County, I-94 WB, 2 mi NW of Albertville, MN, station 122280 to 122830
Description: Rural principal arterial, interstate, two 12 foot lanes, 500 ft section, constructed 1993
Pavement Type: AC over a granular base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Crushed stones, gravel, crushed slag
Asphalt source: Koch Refining Co., Rosemount, MN
Geological Class = granite
AC specific Gravity: 1.032, Asphaalt cements 120-150 pen
Thickness: 7.8 in.
Gradation: 100% passing ¾" sieve
92% passing ½" sieve
82% passing 3/8" sieve
67% passing no. 4
57% passing no. 10
27% passing no. 40
4% passing no. 200

Base Layer: Crushed Stone, Gravel or Slag, AASHHTO soil class A-1-b
Thickness: 28.0 in.
Gradation: 100% passing ¾" sieve
99% passing 3/8" sieve
92% passing no. 4
81% passing no. 10
39% passing no. 40
11% passing no. 200

Subgrade Layer: Silty Clay, AASHTO soil class = A-6
86.4% passing no. 40
59.8% passing no. 200

Case 12 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: August 16, 1998 at 11:02 am
Pavement surface temp 26 °C (79 °F); Air temp 23 °C (73 °F)

Table 6A-11: Case 12 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00784	0.0119	0.01517	0.01966
8	0.00585	0.00893	0.01135	0.01478
12	0.00484	0.00729	0.00928	0.01213
18	0.00328	0.00527	0.00686	0.00909
24	0.0025	0.00386	0.00503	0.00671
26	0.00172	0.00222	0.00289	0.00386
60	0.0007	0.00113	0.00148	0.00203

Case 14 (LTPP case no. 7780)
Location: Colorado, El Paso County, 5.5 mi W of Gravel pit, 1.5 mi W of Rampart-Tier St., 2.33 mi W of Pikes Peak North Pole Rd., US24 WB MP 291.36.
Description: two 12 foot lanes, 500 ft section, constructed 1972
Pavement Type: AC over a granular base.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Mineral Filler = Hydrated lime
Asphalt source: Conoco Inc., Commerce City, CO
Geological Class = granite
AC specific Gravity: 1.032, AC-10
Crushed stones
Thickness: 3.5 in.
Gradation: 100% passing ¾" sieve
96% passing ½" sieve
90% passing 3/8" sieve
70% passing no. 4
50% passing no. 8
35% passing no. 16
16% passing no. 50
11% passing no. 100
8% passing no. 200

Base Layer: UnCrushed gravel, AASHHTO soil class A-1a
Thickness: 6.0 in.
Gradation: 100% passing ¾" sieve
96% passing ½" sieve
89% passing 3/8" sieve
66% passing no. 4
46% passing no. 8
32% passing no. 30
14% passing no. 50
10% passing no. 100
8% passing no. 200

Subgrade Layer: Poorly graded sand, , AASHTO soil class = A-1a
20% passing no. 40
10% passing no. 200

Case 14 (Continued)

<u>Test Data:</u> Plate radius = 150 mm (5.91 in) Test Date: October 16, 1998 at 10:00 am
Pavement surface temp 10 °C (50 °F); Air temp 12 °C (54 °F)

Table 6A-12: Case 14 Deflection Data (in)

Location (in	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00624	0.00909	0.01186	0.01509
8	0.00519	0.00749	0.00975	0.01232
12	0.00445	0.00644	0.00839	0.01057
18	0.00347	0.00507	0.00663	0.00831
24	0.00277	0.00402	0.0053	0.00675
26	0.00179	0.00269	0.00355	0.00452
60	0.00094	0.0014	0.00191	0.00246

Case 15 (LTPP case no. 1002)
Location: Illinois, Stephenson county, 1.25 mi. E. of SR-26, 2.09 mi. W. of SR-75, N. of Freeport Il., US-20 EB lanes.
Description: one 12 foot lane, 500 ft section, constructed 1986
Pavement Type: AC wearing course over AC binder course.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Mineral Filler = Stone dust
Asphalt source: No info on source
Geological Class = dolomite
AC specific Gravity: no info on SG or Grade.
Crushed stones Thickness: 1.5 in.
Gradation: 100% passing 5/8" sieve
99% passing ½" sieve
89% passing 3/8" sieve
53% passing no. 4
33% passing no. 8
26% passing no. 16
13% passing no. 50
8% passing no. 100
4% passing no. 200

Base Layer: Dense Graded, hot laid, Central Plant Mix
Thickness: 11.5 in. (No info on source, grade, or Specific Gravity)
Geological Class = dolomite
Mineral filler = stone dust
Gradation: 100% passing 1" sieve
98% passing ¾" sieve
73% passing ½" sieve
56% passing 3/8" sieve
40% passing no. 4
30% passing no. 8
18% passing no. 30
11% passing no. 50
7% passing no. 100
4% passing no. 200

Subgrade Layer: Silty Clay (No info on gradation)

Case 15 (Continued)

<u>Test Data:</u> **Plate radius = 150 mm (5.91 in) Test Date: October 16, 1998 at 10:00 am Pavement surface temp 38 °C (100 °F); Air temp 27 °C (81 °F)**

Table 6A-13: Case 15 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00437	0.0064	0.00854	0.01131
8	0.00359	0.00534	0.0071	0.00944
12	0.00328	0.00488	0.00647	0.00866
18	0.00289	0.00429	0.00569	0.00757
24	0.00246	0.00363	0.00484	0.00644
26	0.00176	0.00265	0.00351	0.00468
60	0.00094	0.00148	0.00191	0.0025

Case 16 (LTPP case no. 1009)
Location: Kansas, Stafford County, 23 mi S of US-56, 4.5 mi N of US-50, US-281 SB just S of St. John, KS
Description: one 12 foot lane, 500 ft section, constructed 1985
Pavement Type: AC wearing course over AC binder course.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Mineral Filler = Stone dust & Silica
Asphalt source: Allied Mat'lss Corp-Sfroud OK
Geological Class = limestone
AC specific Gravity: 1.018, grade VAC-20
Thickness: 2.0 in., Crushed stones & gravel
Gradation: 100% passing 5/8" sieve
78% passing 3/8" sieve
64% passing no. 4
46% passing no. 8
28% passing no. 16
17% passing no. 30
11% passing no. 50
8% passing no. 100
7% passing no. 200

Base Layer: Dense Graded, hot laid, Central Plant Mix,
Thickness: 8.0 in., Crushed stones & gravel
AC specific Gravity: 1.018, grade VAC-20
Geological Class = limestone, Mineral filler = silica
Gradation: 100% passing 5/8" sieve
78% passing 3/8" sieve
64% passing no. 4
46% passing no. 8
28% passing no. 16
17% passing no. 30
11% passing no. 50
8% passing no. 100
7% passing no. 200

Subgrade Layer: Silty sand, AASHTO Soil class = A-7-6
99% passing no. 40 79% passing no. 200

Case 16 (Continued)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: April 26, 1996 at 10:03 am
Pavement surface temp 27 °C (81 °F); Air temp 16 °C (61 °F)

Table 6A-14: Case 16 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00211	0.00328	0.00425	0.00569
8	0.00199	0.00304	0.00394	0.00523
12	0.00187	0.00289	0.00378	0.00495
18	0.00172	0.00265	0.00347	0.00456
24	0.0016	0.00246	0.0032	0.00417
26	0.00133	0.00203	0.00265	0.00347
60	0.00086	0.0014	0.00191	0.00246

Case 17 (LTPP case no. 1029)
Location: Minnesoty, Isanti County, 5 mi S. of St-95, 0.7 mi N of Hwy-5, N of Isanti, MN, TH-65 NB, 2 miles N of Isanti, MN
Description: two 12 foot lanes, 500 ft section, constructed 1970
Pavement Type: AC wearing course over AC binder course.

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Thickness: 2.0 in
(no info on gradation)

Base Layer: Dense Graded, hot laid, Central Plant Mix,
Thickness: 6.0 in.,
(no info on gradation)

Subgrade Layer: Silty sand, AASHTO Soil class = A-3
82% passing no. 40
5.6% passing no. 200

Test Data: Plate radius = 150 mm (5.91 in) Test Date: September 9, 1994 at 10:02 am
Pavement surface temp 24 °C (75 °F); Air temp 23 °C (73 °F)

Table 6A-15: Case 17 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00722	0.00991	0.01342	0.01736
8	0.0064	0.00913	0.01217	0.01564
12	0.00585	0.00835	0.01112	0.01427
18	0.00495	0.00714	0.00952	0.01221
24	0.0041	0.00593	0.00796	0.01026
26	0.00226	0.00328	0.00445	0.00585
60	0.00117	0.00168	0.00226	0.00304

Case 18 (LTPP case no. 1004)
Location: Minnesota, Wright County, I-94 WB, 2 miles NW of Albertville, MN
Description: Rural Principal Arterial, two 12 foot lanes, 500 ft section, constructed 1994
Pavement Type: Asphalt over subgrade

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Koch Refining Co., Rosemount, MN
Crushed stone, crushed slag, gravel
AC specific Gravity: 1.032,
grade Asphalt cements 120-150 pen
Thickness: 8.8 in.
Gradation: 100% passing 3/4" sieve
92% passing ½" sieve
82% passing 3/8" sieve
67% passing no. 4
57% passing no. 10
27% passing no. 40
4% passing no. 200

Subgrade Layer: Silty clay
(no gradation info.)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: August 16, 1998 at 5:14 pm
Pavement surface temp 41 °C (106 °F); Air temp 28 °C (82 °F)

Table 6A-16: Case 18 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.01435	0.02266	0.03042	0.04212
8	0.00963	0.0156	0.02118	0.02984
12	0.00725	0.0119	0.01619	0.02297
18	0.00495	0.00815	0.01112	0.0158
24	0.00335	0.0055	0.00749	0.01065
26	0.00164	0.00269	0.00363	0.00507
60	0.00078	0.00129	0.00172	0.00242

Case 19 (LTPP case no. 1030)
Location: Nebraaska, Furnas County, D.5 mi E of US-2283, 2.7 mi W of US-136, US-6 WB lanes, ¾ mile E of Arapahoe, NE.
Description: Rural Principal Arterial, one 12 foot lane, 500 ft section, constructed 1982
Pavement Type: Asphalt over subgrade

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: Farmland Industries, Inc., Phillipsburg, KS
Conglomerate aggregate. Crushed stone, gravel
Mineral filler: soil
AC specific Gravity: 0.98
grade AC-10
Thickness: 7.0 in.
Gradation: 100% passing ½" sieve
94% passing 3/8" sieve
76% passing no. 4
43% passing no. 10
21% passing no. 50
9% passing no. 200

Subgrade Layer: Silty clay
AASHTO soil classification = A-4
98% passing no. 200

Test Data: Plate radius = 150 mm (5.91 in) Test Date: November 7, 1995 at 10:03 am
Pavement surface temp 7 °C (45 °F); Air temp 1 °C (34 °F)

Table 6A-17: Case 19 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.00745	0.01135	0.01525	0.02012
8	0.00675	0.0103	0.01388	0.01833
12	0.0062	0.0094	0.01264	0.01665
18	0.00534	0.00815	0.01092	0.01443
24	0.00449	0.00683	0.00917	0.01213
26	0.00215	0.00328	0.00449	0.00593
60	0.0016	0.00238	0.00316	0.00421

Case 20 (LTPP case no. 6420)
Location: Saskatchewan, RM # 153 of Willowdale, Hwy-9 SB lane, 33km S of Whitewood, SK
Description: Rural Principal Arterial, one 12 foot lane, 500 ft section, constructed 1971
Pavement Type: Asphalt over subgrade

Surface Layer: Hot mixed, Hot laid, Dense Graded Asphalt Concrete.
Asphalt source: GOLF
Crushed gravel
AC specific Gravity: 1.02
grade AC-5
Thickness: 7.0 in.
Gradation: 100% passing 5/8" sieve
85% passing 3/8" sieve
62% passing no. 4
43% passing no. 10
15% passing no. 40
7% passing no. 100
5% passing no. 200

Subgrade Layer: Silty clay
(no further info on subgrade)

Test Data: Plate radius = 150 mm (5.91 in) Test Date: September 28, 1995 at 10:02 am
Pavement surface temp 11 °C (52 °F); Air temp 19 °C (66 °F)

Table 6A-18: Case 20 Deflection Data (in)

Location (in)	Load(lb)			
	6007.5	9000	11992.5	15997.5
0	0.01182	0.01782	0.02457	0.03268
8	0.00959	0.01463	0.02028	0.02714
12	0.00819	0.01256	0.01743	0.0234
18	0.00632	0.00971	0.01357	0.01825
24	0.00488	0.00753	0.01053	0.01416
26	0.00308	0.00476	0.00659	0.00889
60	0.0016	0.00238	0.00332	0.00441

Case 21 (LTPP case no. 7613)
Location: Arizona, Butler County, US 60 WB, MP-179
Description: urban principal arterial, constructed in 1976, widened in 1985, 3 lanes, 12 ft wide, third lane added in 1985, 500 ft long
Pavement Type: JPCP placed directly on subgrade

Surface Layer: JPCP
Thickness: 13.0 in.
Contraction joints, avg 15 ft apart, joint load transfer is by aggregate interlock.
Mix: Cement type IIA, paver typee: slip form

Subgrade Layer: Silty clay, AASHTO soil classification A-6

Test Data: Plate radius = 150 mm (5.91 in) Test Date: December 8, 1998 at 4:00 am
FWD data were all taken 32 ft from start of section in outer wheel path.

Table 6A-19: Case 21 Deflection Data (in)

Location (in)	Load(lb)		
	9000	11992.5	15997.5
0	0.00382	0.00528	0.00705
8	0.00291	0.00406	0.00543
12	0.00276	0.00386	0.00528
18	0.00248	0.00343	0.00472
24	0.0022	0.00307	0.00417
26	0.00161	0.00228	0.00311
60	0.00087	0.00122	0.00169

**Note: Only 3 load levels were obtained for this case.

Appendix 6B
Program to Program Tables

KISS vs EVERCALC

Notes: **Case 8: EVERCALC only loads 1 and 3 gave printable results for AC modulus**
Case 15: EVERCALC gave no printable results for AC modulus
Case 18: EVERCALC ERROR on all four loads

Table 6B-1: KISS vs EVERCALC AC Layer

	K	E	100*(E-K)/K
	KISS	EVERCALC	% error
case 1	4479673	388500	-91.33
	4464929	365400	-91.82
	4463395	354000	-92.07
	4477174	376200	-91.60
case 2	647796	20252200	3026.32
	646228	20058500	3003.94
	645922	21208000	3183.37
	645372	22630000	3406.50
case 3	628970	18114600	2780.04
	610658	24131900	3851.79
	623164	28274800	4437.30
	622959	35446200	5589.97
case 4	3628155	64800	-98.21
	3813224	66600	-98.25
	3846028	64500	-98.32
	3853606	200000	-94.81
case 6	607716	361700	-40.48
	615778	516000	-16.20
	601795	477300	-20.69
	601035	462000	-23.13
case 7	636356	739000	16.13
	634542	780100	22.94
	629826	843600	33.94
	635424	812400	27.85
case 8	629269	91170200	14388.27
	624084	97353600	15499.44
case 9	661371	70255000	10522.63
	663657	77228800	11536.85
	657158	76089300	11478.54
	661953	76374900	11437.81
case 10	451887	3972800	779.16
	450198	4543600	909.24
	450470	5188700	1051.84
	449428	5840300	1199.50
case 11	151705	166800	9.95
	177996	173900	-2.30
	193802	188700	-2.63
	204747	190100	-7.15
case 12	248975	308400	23.87
	237555	270100	13.70
	252925	291300	15.17
	273540	306900	12.20
case 14	1275465	4615900	261.90
	1274238	4440200	248.46
	1495560	3809800	154.74
	1519209	2140000	40.86
case 16	978350	180000	-81.60
	978088	276300	-71.75
	977996	291700	-70.17
	977894	297000	-69.63
case 17	1030698	33675300	3167.23
	745065	39669000	5224.23
	1010654	39826800	3840.70
	997482	41239000	4034.31
case 19	1491876	616200	-58.70
	1434464	600000	-58.17
	1403522	603500	-57.00
	1444266	615900	-57.36
case 20	368372	263500	-28.47
	355285	263800	-25.75
	345032	258400	-25.11
	437508	261400	-40.25
		avg. =	**1731.04**

Table 6B-2: KISS vs EVERCALC Subgrade Layer

	K	E	100*(E-K)/K
	KISS	EVERCALC	% error
case 1	6368	28100	341.27
	6265	27500	338.92
	6248	27100	333.75
	6353	27900	339.16
case 2	7371	23400	217.46
	7266	21800	200.04
	7231	22400	209.79
	7196	23200	222.42
case 3	5195	22200	327.35
	8183	20900	155.40
	4335	21100	386.75
	4212	22300	429.44
case 4	3125	14800	373.63
	3243	15800	387.25
	3349	15000	347.93
	3374	15200	350.55
case 6	8061	41300	412.33
	7723	43400	461.97
	8339	44500	433.62
	8296	44900	441.21
case 7	2489	14400	478.44
	2457	14300	482.01
	2352	14400	512.32
	2387	14300	499.13
case 8	3422	23200	577.92
	3194	24000	651.39
case 9	7950	43200	443.39
	7564	40800	439.39
	7424	41200	454.98
	7582	41600	448.70
case 10	8951	27600	208.36
	8775	26300	199.72
	8793	26800	204.80
	8687	27000	210.80
case 11	9424	30800	226.83
	8986	28900	221.61
	8477	29400	246.82
	8442	28200	234.04
case 12	8664	37100	328.20
	9452	36100	281.92
	9519	36900	287.66
	9587	36200	277.61

	K	E	100*(E-K)/K
	KISS	EVERCALC	% error
case 14	9624	28500	196.14
	9584	28400	196.34
	6376	28000	339.14
	6717	29100	333.20
case 16	6950	33200	377.70
	6985	31400	349.53
	6985	31500	350.97
	7002	31100	344.16
case 17	4191	24400	482.14
	2754	26100	847.61
	4158	25800	520.47
	4235	25400	499.77
case 19	3774	19400	413.98
	3629	19100	426.27
	3551	18900	432.23
	3654	19000	419.97
case 20	4851	16000	229.83
	4679	15700	235.57
	4544	15000	230.13
	2817	14900	429.01
		avg. =	**236.15**

Table 6B-3: KISS vs EVERCALC
Stabilized Base

	K	E	100*(E-K)/K
	KISS	EVERCALC	% error
case 1	319304	830500	160.10
	312916	807700	158.12
	312263	781200	150.17
	318208	834500	162.25
case 2	91406	14200	-84.46
	89382	16000	-82.10
	88998	14600	-83.60
	88318	14200	-83.92
case 3	173939	46400	-73.32
	51585	34300	-33.51
	193987	29500	-84.79
	194684	20000	-89.73
case 4	108823	246600	126.61
	133869	291900	118.05
	138867	321800	131.73
	140048	345300	146.56
case 6	5649430	3012600	-46.67
	6055292	2949100	-51.30
	5347722	3013800	-43.64
	5308703	3054500	-42.46
case 7	333048	226000	-32.14
	324661	214500	-33.93
	304817	208800	-31.50
	328681	221500	-32.61
case 8	1256494	10000	-99.20
	1122204	3400	-99.70
case 9	2289563	17300	-99.24
	2337323	14500	-99.38
	2201441	11400	-99.48
	2301733	14100	-99.39
case 10	92435	79600	-13.89
	89866	55000	-38.80
	90271	44500	-50.70
	88738	32400	-63.49
		avg. =	**-15.86**

Table 6B-4: KISS vs EVERCALC
Granular Base

	K	E	100*(E-K)/K
	KISS	EVERCALC	% error
case 11	350	19300	5414.29
	411	19300	4595.86
	447	20800	4553.24
	473	21300	4403.17
case 12	575	21300	3604.87
	549	21900	3892.36
	584	22800	3803.85
	632	23500	3620.45
case 14	121551	25800	-78.77
	120865	37000	-69.39
	301653	62100	-79.41
	332945	97100	-70.84
		avg. =	**2799.14**

Table 6B-5: KISS vs EVERCALC
Asphalt Binder Course

	K	E	100*(E-K)/K
	KISS	EVERCALC	% error
case 16	919830	1382700	50.32
	926463	1245300	34.41
	928846	1201400	29.34
	931526	1174300	26.06
case 17	671365	6200	-99.08
	39417	5500	-86.05
	848044	5500	-99.35
	991940	6600	-99.33
		avg. =	**-30.46**

KISS vs BOUSDEF

Table 6B-6: KISS vs BOUSDEF AC Layer

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 1	4479673	3873370	-13.53
	4464929	2415190	-45.91
	4463395	2159302	-51.62
	4477174	2000802	-55.31
case 2	647796	2431060	275.28
	646228	390509	-39.57
	645922	2011700	211.45
	645372	1507342	133.56
case 3	628970	1096259	74.29
	610658	112044	-81.65
	623164	95807	-84.63
	622959	98488	-84.19
case 4	3628155	88485	-97.56
	3813224	90924	-97.62
	3846028	85662	-97.77
	3853606	84530	-97.81
case 6	607716	10000000	1545.51
	615778	9999989	1523.96
	601795	10000000	1561.70
	601035	10000000	1563.80
case 7	636356	855579	34.45
	634542	965008	52.08
	629826	931862	47.96
	635424	967157	52.21
case 8	629269	427456	-32.07
	627433	413858	-34.04
	624084	327262	-47.56
	627859	231213	-63.17
case 9	661371	156073	-76.40
	663657	143774	-78.34
	657158	162215	-75.32
	661953	118834	-82.05
case 10	451887	1196029	164.67
	450198	2257544	401.46
	450470	3678046	716.49
	449428	4615112	926.89
case 11	151705	168673	11.18
	177996	169633	-4.70
	193802	172160	-11.17
	204747	172219	-15.89

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 12	248975	260660	4.69
	237555	263441	10.90
	252925	272151	7.60
	273540	287383	5.06
case 14	1275465	228895	-82.05
	1274238	215655	-83.08
	1495560	208198	-86.08
	1519209	36791	-97.58
case 15	4169765	69667	-98.33
	4172697	643458	-84.58
	4183725	36444	-99.13
	4170920	65042	-98.44
case 16	978350	199018	-79.66
	978088	1914	-99.80
	977996	169826	-82.64
	977894	22210	-97.73
case 17	1030698	638	-99.94
	745065	10	-100.00
	1010654	781	-99.92
	997482	252	-99.97
case 18	66984	996017	1386.95
	69921	995824	1324.21
	70103	995797	1320.47
	67900	995643	1366.34
case 19	1491876	998779	-33.05
	1434464	998748	-30.37
	1403522	998736	-28.84
	1444266	998756	-30.85
case 20	368372	997364	170.75
	355285	997363	180.72
	345032	997280	189.04
	437508	996165	127.69
		avg. =	**110.13**

Table 6B-7: KISS vs BOUSDEF Subgrade Layer

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 1	6368	25825	305.54
	6265.35	25143	301.30
	6247.8	24793	296.83
	6353.1	25625	303.35
case 2	7371	20732	181.26
	7265.7	19682	170.89
	7230.6	19963	176.09
	7195.5	20774	188.71
case 3	5194.8	21354	311.06
	8183.214	19672	140.39
	4334.85	19803	356.83
	4212	20210	379.82
case 4	3124.778	14131	352.22
	3243	14221	338.55
	3349	14379	329.39
	3374	14629	333.63
case 6	8061.242	249593	2996.21
	7722.878	377470	4787.69
	8339	433825	5102.22
	8296	436832	5165.42
case 7	2489	14385	477.83
	2457	14310	482.42
	2352	14314	508.67
	2387	14227	496.07
case 8	3422	19446	468.22
	3352	17472	421.23
	3194	16656	421.46
	3229	18418	470.36
case 9	7950	36401	357.87
	7564	34301	353.47
	7424	33400	349.91
	7582	34734	358.14
case 10	8951	27295	204.96
	8775	25760	193.56
	8793	26017	195.90
	8687	26133	200.82
case 11	9424	1066	-88.69
	8986	1065	-88.15
	8477	1064	-87.45
	8442	1063	-87.41
case 12	8664	37930	337.78
	9452	39552	318.44
	9519	39278	312.64
	9587	38442	301.00

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 14	9624	26855	179.05
	9584	270099	2718.32
	6376	27219	326.89
	6717	28645	326.43
case 15	6305	27655	338.63
	6322	26922	325.83
	6388	28017	338.57
	6312	27570	336.81
case 16	6950	31395	351.73
	6985	12228	75.06
	6985	29710	325.34
	7002	29600	322.74
case 17	4191	18674	345.52
	2754	32450	1078.16
	4158	21407	414.82
	4235	21967	418.70
case 18	7409	484713	6442.61
	6959	483179	6843.30
	6800	483015	7002.88
	6587	482063	7218.94
case 19	3774.479	482553	12684.63
	3629.34	482365	13190.71
	3551.067	482365	13483.66
	3654.086	482326	13099.64
case 20	4851	479086	9776.03
	4679	478798	10133.67
	4544	477842	10416.59
	2817	486785	17182.72
		avg. =	**408.28**

Table 6B-8: KISS vs BOUSDEF
Stabilized Base

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 1	319304	967383	202.97
	312916	1001703	220.12
	312263	972825	211.54
	318208	1050188	230.03
case 2	91406	171000	87.08
	89382	264859	196.32
	88998	192252	116.02
	88318	193349	118.92
case 3	173939	193411	11.19
	51585	302384	486.19
	193987	324047	67.05
	194684	338073	73.65
case 4	108823	274103	151.88
	133869	316296	136.27
	138867	344373	147.99
	140048	366341	161.58
case 6	5649430	1510	-99.97
	6055292	1476	-99.98
	5347722	1484	-99.97
	5308703	1505	-99.97
case 7	333048	227659	-31.64
	324661	191121	-41.13
	304817	221137	-27.45
	328681	213783	-34.96
case 8	1256494	1343772	6.95
	1211469	1178315	-2.74
	1122204	1095116	-2.41
	1222109	1044406	-14.54
case 9	2289563	1578495	-31.06
	2337323	1638621	-29.89
	2201441	1474057	-33.04
	2301733	1557386	-32.34
case 10	92435	208040	125.07
	89866	84890	-5.54
	90271	57344	-36.48
	88738	41176	-53.60
		avg. =	**54.84**

Table 6B-9: KISS vs BOUSDEF
Granular Base

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 11	350	633	80.86
	411	628	52.80
	448	624	39.29
	473	623	31.71
case 12	575	21332	3610.43
	549	19736	3497.86
	584	21305	3547.87
	632	22081	3395.80
case 14	121551	389287	220.27
	120865	448803	271.33
	301653	498336	65.20
	332945	103823	-68.82
		avg. =	**1228.72**

Table 6B-10: KISS vs BOUSDEF
Asphalt Binder Course

	K	B	100*(B-K)/K
	KISS	BOUSDEF	% error
case 15	135906	581957	328.20
	136336	581203	326.30
	137967	644279	366.98
	136075	616452	353.02
case 16	919830	1442711	56.85
	926463	32571	-96.48
	928846	1570646	69.10
	931526	1768592	89.86
case 17	671365	533671	-20.51
	39417	8999994	22732.86
	848044	8999994	961.27
	991940	8999994	807.31
		avg. =	**2164.56**

KISS vs MODCOMP

Table 6B-11: KISS vs MODCOMP AC Layer

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 1	4479673	100000000	2132.31
	4464929	94700000	2020.97
	4463395	75100000	1582.58
	4477174	68700000	1434.45
case 2	647796	7160000	1005.29
	646228	6940000	973.92
	645922	7140000	1005.40
	645372	5180000	702.64
case 3	628970	1820000	189.36
	610658	128000	-79.04
	623164	183000	-70.63
	622959	172000	-72.39
case 4	3628155	59600	-98.36
	3813224	65400	-98.28
	3846028	62200	-98.38
	3853606	67400	-98.25
case 6	607716	292000	-51.95
	615778	352000	-42.84
	601795	350000	-41.84
	601035	335000	-44.26
case 7	636356	1460000	129.43
	634542	1760000	177.37
	629826	931862	47.96
	635424	1680000	164.39
case 8	629269	34500000	5382.55
	627433	22100000	3422.29
	624084	20700000	3216.86
	627859	8680000	1282.48
case 9	661371	5540000	737.65
	663657	1620000	144.10
	657158	1380000	110.00
	661953	2000000	202.14
case 10	451887	981000	117.09
	450198	1170000	159.89
	450470	1330000	195.25
	449428	1270000	182.58
case 11	151705	172000	13.38
	177996	183000	2.81
	193802	192000	-0.93
	204747	191000	-6.71

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 12	248975	313000	25.72
	237555	300000	26.29
	252925	310000	22.57
	273540	324000	18.45
case 14	1275465	2830000	121.88
	1274238	2050000	60.88
	1495560	1890000	26.37
	1519209	1760000	15.85
case 15	4169765	449000	-89.23
	4172697	96500	-97.69
	4183725	3730000	-10.85
	4170920	5290000	26.83
case 16	978350	192000	-80.38
	978088	304000	-68.92
	977996	434000	-55.62
	977894	395000	-59.61
case 17	1030698	4800000	365.70
	745065	46400000	6127.64
	1010654	30400000	2907.95
	997482	26700000	2576.74
case 18	66984	86700	29.43
	69921	88200	26.14
	70103	89900	28.24
	67900	91200	34.32
case 19	1491876	1380000	-7.50
	1434464	1300000	-9.37
	1403522	1290000	-8.09
	1444266	1300000	-9.99
case 20	368372	335000	-9.06
	355285	356000	0.20
	345032	354000	2.60
	437508	358000	-18.17
		avg. =	**360.16**

Table 6B-12: KISS vs MODCOMP Subgrade Layer

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 1	6368	23000	261.18
	6265	21700	246.35
	6248	20900	234.52
	6353	21400	236.84
case 2	7371	17800	141.49
	7266	16900	132.60
	7231	16700	130.96
	7196	12700	76.50
case 3	5195	20400	292.70
	8183	18900	130.96
	4335	19000	338.31
	4212	19200	355.84
case 4	3125	14600	367.23
	3243	14800	356.41
	3349	15000	347.93
	3374	15400	356.48
case 6	8061	38400	376.35
	7723	34100	341.55
	8339	37200	346.08
	8296	37700	354.42
case 7	2489	15700	530.66
	2457	16200	559.34
	2352	14314	508.67
	2387	15600	553.59
case 8	3422	16300	376.29
	3352	14300	326.60
	3194	13400	319.52
	3229	13400	314.96
case 9	7950	30300	281.12
	7564	30900	308.51
	7424	30200	306.81
	7582	31000	308.88
case 10	8951	25700	187.13
	8775	23800	171.23
	8793	23700	169.55
	8687	23200	167.06
case 11	9424	30200	220.46
	8986	28200	213.82
	8477	29000	242.10
	8442	27800	229.31
case 12	8664	38400	343.20
	9452	35900	279.80
	9519	36500	283.45
	9587	35800	273.44

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 14	9624	26400	174.32
	9584	26000	171.29
	6376	26300	312.48
	6717	27900	315.34
case 15	6305	25100	298.11
	6322	22700	259.05
	6388	22500	252.21
	6312	22200	251.73
case 16	6950	34200	392.09
	6985	32000	358.12
	6985	32500	365.28
	7002	31900	355.58
case 17	4191	14700	250.71
	2754	16500	499.06
	4158	16600	299.22
	4235	17300	308.50
case 18	7409	18300	147.01
	6959	16600	138.54
	6800	16100	136.76
	6587	15000	127.74
case 19	3774	14800	292.11
	3629	14800	307.79
	3551	14700	313.96
	3654	14900	307.76
case 20	4851	14800	205.09
	4679	14200	203.51
	4544	13500	197.12
	2817	13400	375.75
		avg. =	179.17

Table 6B-13: KISS vs MODCOMP
Stabilized Base

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 1	319304	311000	-2.60
	312916	356000	13.77
	312263	400000	28.10
	318208	458000	43.93
case 2	91406	112000	22.53
	89382	120000	34.26
	88998	131000	47.19
	88318	109000	23.42
case 3	173939	145000	-16.64
	51585	299000	479.63
	193987	287000	47.95
	194684	317000	62.83
case 4	108823	262000	140.76
	133869	297000	121.86
	138867	326000	134.76
	140048	336000	139.92
case 6	5649430	5030000	-10.96
	6055292	9880000	63.16
	5347722	7850000	46.79
	5308703	7930000	49.38
case 7	333048	72600	-78.20
	324661	37000	-88.60
	304817	221137	-27.45
	328681	60500	-81.59
case 8	1256494	434000	-65.46
	1211469	467000	-61.45
	1122204	458000	-59.19
	1222109	718000	-41.25
case 9	2289563	805000	-64.84
	2337323	1050000	-55.08
	2201441	996000	-54.76
	2301733	945000	-58.94
case 10	92435	194000	109.88
	89866	159000	76.93
	90271	153000	69.49
	88738	151000	70.16
		avg. =	**29.44**

Table 6B-14: KISS vs MODCOMP
Granular Base

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 11	350	19300	5414.29
	411	19300	4595.86
	448	20800	4542.86
	473	21500	4445.45
case 12	575	19900	3361.35
	549	20700	3673.60
	584	22200	3701.12
	632	23100	3557.13
case 14	121551	837000	588.60
	120865	147000	21.62
	301653	167000	-44.64
	332945	184000	-44.74
		avg. =	**2818**

Table 6B-15: KISS vs MODCOMP
Asphalt Binder Course

	K	M	100*(M-K)/K
	KISS	MODCOMP	% error
case 15	135906	578000	325.29
	136336	1130000	728.83
	137967	452000	227.61
	136075	441000	224.09
case 16	919830	1260000	36.98
	926463	1160000	25.21
	928846	973000	4.75
	931526	1020000	9.50
case 17	671365	547000	-18.52
	39417	82400	109.05
	848044	187000	-77.95
	991940	230000	-76.81
		avg. =	**126.50**

KISS vs MODULUS

Notes: **Case 2, no solutions obtained, stuck on E1 max**
Case 6, no solutions obtained, stuck on E1 max
Case 7, program error
Case 8, program error
Case 17, program error

Table 6B-16: KISS vs MODULUS AC Layer

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 1	4479673	637000	-85.78
	4464929	690000	-84.55
	4463395	857000	-80.80
	4477174	1047000	-76.61
case 3	628970	1200000	90.79
	610658	141000	-76.91
	623164	176000	-71.76
	622959	193000	-69.02
case 4	3628155	1244000	-65.71
	3813224	1418000	-62.81
	3846028	1472000	-61.73
	3853606	8000000	107.60
case 9	661371	873000	32.00
	663657	758000	14.22
	657158	713000	8.50
	661953	724000	9.37
case 10	451887	2873000	535.78
	450198	1778000	294.94
	450470	2014000	347.09
	449428	2374000	428.23
case 11	151705	197000	29.86
	177996	208000	16.86
	193802	216000	11.45
	204747	215000	5.01
case 12	248975	238000	-4.41
	237555	280000	17.87
	252925	301000	19.01
	273540	322000	17.72
case 14	1275465	2127000	66.76
	1274238	1673000	31.29
	1495560	1655000	10.66
	1519209	1241000	-18.31

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 15	4169765	551000	-86.79
	4172697	574000	-86.24
	4183725	551000	-86.83
	4170920	601000	-85.59
case 16	978350	3632000	271.24
	978088	2866000	193.02
	977996	2514000	157.06
	977894	2438000	149.31
case 18	66984	114000	70.19
	69921	113000	61.61
	70103	11300	-83.88
	67900	111000	63.48
case 19	1491876	1057000	-29.15
	1434464	1045000	-27.15
	1403522	1051000	-25.12
	1444266	1069000	-25.98
case 20	368372	392000	6.41
	355285	403000	13.43
	345032	396000	14.77
	437508	405000	-7.43
		avg. =	**25.70**

Table 6B-17: KISS vs MODULUS Subgrade Layer

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 1	6368	20400	220.35
	6265	19700	214.43
	6248	19400	210.51
	6353	20000	214.81
case 3	5195	17300	233.03
	8183	17400	112.63
	4335	17200	296.78
	4212	17400	313.11
case 4	3125	11500	268.03
	3243	11500	254.64
	3349	11700	249.39
	3374	13100	288.31
case 9	7950	29100	266.03
	7564	27800	267.53
	7424	26900	262.36
	7582	28200	271.95
case 10	8951	23900	167.02
	8775	22000	150.71
	8793	22000	150.21
	8687	21800	150.94
case 11	9424	13400	42.19
	8986	12600	40.22
	8477	12200	43.92
	8442	11500	36.22
case 12	8664	15100	74.28
	9452	19400	105.24
	9519	19600	105.91
	9587	19400	102.37
case 14	9624	23100	140.03
	9584	23400	144.16
	6376	23700	271.70
	6717	25000	272.17
case 15	6305	23400	271.14
	6322	23300	268.54
	6388	23500	267.87
	6312	23400	270.74
case 16	6950	26900	287.05
	6985	25300	262.20
	6985	25300	262.20
	7002	25000	257.04

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 18	7409	10700	44.43
	6959	9700	39.39
	6800	9500	39.70
	6587	8900	35.12
case 19	3774	9200	143.74
	3629	9000	147.98
	3551	8800	147.81
	3654	8900	143.56
case 20	4851	11600	139.13
	4679	11300	141.52
	4544	10800	137.69
	2817	10600	276.34
		avg. =	155.54

Table 6B-18: KISS vs MODULUS
Stabilized Base

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 1	319304	1244800	289.85
	312916	1193800	281.51
	312263	1075300	244.36
	318208	1093700	243.71
case 3	173939	173500	-0.25
	51585	292900	467.80
	193987	297800	53.52
	194684	313700	61.13
case 4	108823	163400	50.15
	133869	181400	35.51
	138867	193100	39.05
	140048	116300	-16.96
case 9	2289563	1288300	-43.73
	2337323	1317800	-43.62
	2201441	1230500	-44.10
	2301733	1298600	-43.58
case 10	92435	116200	25.71
	89866	131100	45.88
	90271	123200	36.48
	88738	106500	20.02
		avg. =	**85.12**

Table 6B-19: KISS vs MODULUS
Granular Base

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 11	350	21700	6100.00
	411	21700	5179.81
	448	24300	5324.11
	473	25200	5227.70
case 12	575	30500	5205.09
	549	23200	4129.35
	584	24300	4060.68
	632	24700	3810.43
case 14	121551	134500	10.65
	120865	180500	49.34
	301653	198600	-34.16
	332945	248100	-25.48
		avg. =	**3253.13**

Table 6B-20: KISS vs MODULUS
Asphalt Binder Course

	K	M	100*(M-K)/K
	KISS	MODULUS	% error
case 15	135906	551700	305.94
	136336	540300	296.30
	137967	537300	289.44
	136075	538700	295.88
case 16	919830	690100	-24.98
	926463	776900	-16.14
	928846	819400	-11.78
	931526	820400	-11.93
		avg. =	**140.34**

KISS vs ILLIBACK

DENSE LIQUID MODEL (k in pci)

Table 6B-21: KISS vs ILLIBACK
Dense Liquid model
AC Layer

	K	L	100*(L-K)/K
	KISS	ILLIBACK	% error
case 21	1085533	557781	-48.62
	1080398	559863	-48.18
	1098706	597429	-45.62
		avg =	-47.47

Table 6B-22: KISS vs ILLIBACK
Dense Liquid model
Subgrade

	K	L	100*(L-K)/K
	KISS	ILLIBACK	% error
case 21	508	684	34.65
	479	637	32.99
	451	601	33.26
		avg =	33.63

ELASTIC SOLID MODEL (E3 in psi)

Table 6B-23: KISS vs ILLIBACK
Elastic Solid Model
AC Layer

	K	L	100*(L-K)/K
	KISS	ILLIBACK	% error
case 21	1085533	327856	-69.80
	1080398	337239	-68.79
	1098706	374451	-65.92
		avg =	-68.17

Table 6B-24: KISS vs ILLIBACK
Elastic Solid Model
Subgrade

	K	L	100*(L-K)/K
	KISS	ILLIBACK	% error
case 21	8915	65356	633.07
	8406	61400	630.39
	7915	58877	643.86
		avg =	635.77

Appendix 6C
Case by Case Tables

NOTE: An empty box on any of the tables means that no result was obtained from the program.

Table 6C-1: Cases 1 to 10: AC Layer Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
Case 1	6007	4479673	388500	3873370	100000000	637000
	9000	4464929	365400	2415190	94700000	690000
	11993	4463395	354000	2159302	75100000	857000
	15998	4477174	376200	2000802	68700000	1047000
Case 2	6007	647796	20252200	2431060	7160000	
	9000	646228	20058500	390509	6940000	
	11993	645922	21208000	2011700	7140000	
	15998	645372	22630000	1507342	5180000	
Case 3	6007	628970	18114600	1096259	1820000	1200000
	9000	610658	24131900	112044	128000	141000
	11993	623164	28274800	95807	183000	176000
	15998	622959	35446200	98488	172000	193000
Case 4	6007	3628155	64800	88485	59600	1244000
	9000	3813224	66600	90924	65400	1418000
	11993	3846028	64500	85662	62200	1472000
	15998	3853606	64600	84530	67400	8000000
Case 6	6007	607716	361700	10000000	292000	
	9000	615778	516000	9999989	352000	
	11993	601795	477300	10000000	350000	
	15998	601035	462000	10000000	335000	3630000
Case 7	6007	636356	739000	855579	1460000	
	9000	634542	780100	965008	1760000	536000
	11993	629826	843600	931862	1740000	
	15998	635424	812400	967157	1680000	
Case 8	6007	629269	91170200	427456	34500000	
	9000	627433		413858	22100000	
	11993	624084	97353600	327262	20700000	
	15998	627859		231213	8680000	
Case 9	6007	661371	70255000	156073	5540000	873000
	9000	663657	77228800	143774	1620000	758000
	11993	657158	76089300	162215	1380000	713000
	15998	661953	76374900	118834	2000000	724000
Case 10	6007	451887	3972800	1196029	981000	2873000
	9000	450198	4543600	2257544	1170000	1778000
	11993	450470	5188700	3678046	1330000	2014000
	15998	449428	5840300	4615112	1270000	2374000
average		**1389291**	**20731474**	**2110791**	**13242156**	**1515818**

Table 6C-2: Cases 1 to 10: Stabilized Base Layer Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
Case 1	6007	319304	830500	967383	311000	1244800
	9000	312916	807700	1001703	356000	1193800
	11993	312263	781200	972825	400000	1075300
	15998	318208	834500	1050188	458000	1093700
Case 2	6007	91406	14200	171000	112000	
	9000	89382	16000	264859	120000	
	11993	88998	14600	192252	131000	
	15998	88318	14200	193349	109000	
Case 3	6007	173939	46400	193411	145000	173500
	9000	51585	34300	302384	299000	292900
	11993	193987	29500	324047	287000	297800
	15998	194684	20000	338073	317000	313700
Case 4	6007	108823	246600	274103	262000	163400
	9000	133869	291900	316296	297000	181400
	11993	138867	321800	344373	326000	193100
	15998	140048	345300	366341	336000	116300
Case 6	6007	5649430	3012600	1510	5030000	
	9000	6055292	2949100	1476	9880000	
	11993	5347722	3013800	1484	7850000	
	15998	5308703	3054500	1505	7930000	
Case 7	6007	333048	226000	227659	72600	
	9000	324661	214500	191121	37000	
	11993	304817	208800	221137	52200	
	15998	328681	221500	213783	60500	
Case 8	6007	1256494	10000	1343772	434000	
	9000	1211469	3800	1178315	467000	
	11993	1122204	3400	1095116	458000	
	15998	1222109	4600	1044406	718000	
Case 9	6007	2289563	17300	1578495	805000	1288300
	9000	2337323	14500	1638621	1050000	1317800
	11993	2201441	11400	1474057	996000	1230500
	15998	2301733	14100	1557386	945000	1298600
Case 10	6007	92435	79600	208040	194000	116200
	9000	89866	55000	84890	159000	131100
	11993	90271	44500	57344	153000	123200
	15998	88738	32400	41176	151000	106500
average		1130905	495558	539830	1158564	597595

Table 6C-3: Cases 1 to 10: Subgrade Layer Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
Case 1	6007	6368	28100	25825	23000	20400
	9000	6265	27500	25143	21700	19700
	11993	6248	27100	24793	20900	19400
	15998	6353	27900	25625	21400	20000
Case 2	6007	7371	23400	20732	17800	
	9000	7266	21800	19682	16900	
	11993	7231	22400	19963	16700	
	15998	7196	23200	20774	12700	
Case 3	6007	5195	22200	21354	20400	17300
	9000	8183	20900	19672	18900	17400
	11993	4335	21100	19803	19000	17200
	15998	4212	22300	20210	19200	17400
Case 4	6007	3125	14800	14131	14600	11500
	9000	3243	15800	14221	14800	11500
	11993	3349	15000	14379	15000	11700
	15998	3374	15200	14629	15400	13100
Case 6	6007	8061	41300	249593	38400	
	9000	7723	43400	377470	34100	
	11993	8339	44500	433825	37200	
	15998	8296	44900	436832	37700	
Case 7	6007	2489	14400	14385	15700	
	9000	2457	14300	14310	16200	
	11993	2352	14400	14314	15900	
	15998	2387	14300	14227	15600	
Case 8	6007	3422	23200	19446	16300	
	9000	3352	25100	17472	14300	
	11993	3194	24000	16656	13400	
	15998	3229	23600	18418	13400	
Case 9	6007	7950	43200	36401	30300	29100
	9000	7564	40800	34301	30900	27800
	11993	7424	41200	33400	30200	26900
	15998	7582	41600	34734	31000	28200
Case 10	6007	8951	27600	27295	25700	23900
	9000	8775	26300	25760	23800	22000
	11993	8793	26800	26017	23700	22000
	15998	8687	27000	26133	23200	21800
average		5843	26406	60887	21539	19915

Table 6C-4: Cases 11 to 14: AC Layer Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
CASE 11	6007	151705	166800	168673	172000	197000
	9000	177996	173900	169633	183000	208000
	11993	193803	188700	172160	192000	216000
	15998	204747	190100	172219	191000	215000
CASE 12	6007	248975	308400	260660	313000	238000
	9000	237555	270100	263441	300000	280000
	11993	252925	291300	272151	310000	301000
	15998	273540	306900	287383	324000	322000
CASE 14	6007	1275465	4615900	228895	2830000	2127000
	9000	1274238	4440200	215655	2050000	1673000
	11993	1495560	3809800	208198	1890000	1655000
	15998	1519209	2140000	36791	1760000	1241000
average		608810	1408508	204655	876250	722750

Table 6C-5: Cases 11 to 14 : Granular Base Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
CASE 11	6007	350	19300	633	19300	21700
	9000	411	19500	628	19300	21700
	11993	448	20800	624	20800	24300
	15998	473	21300	623	21500	25200
CASE 12	6007	575	21300	21332	19900	30500
	9000	549	21900	19736	20700	23200
	11993	584	22800	21305	22200	24300
	15998	632	23500	22081	23100	24700
CASE 14	6007	121551	25800	389287	837000	134500
	9000	120865	37000	448803	147000	180500
	11993	301653	62100	498336	167000	198600
	15998	332945	97100	103823	184000	248100
average		73420	32700	127268	125150	79775

Table 6C-6: Cases 11 to 14: Subgrade Layer Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
CASE 11	6007	9424	30800	1066	30200	13400
	9000	8986	28900	1065	28200	12600
	11993	8477	29400	1064	29000	12200
	15998	8442	28200	1063	27800	11500
CASE 12	6007	8664	37100	37930	38400	15100
	9000	9452	36100	39552	35900	19400
	11993	9519	36900	39278	36500	19600
	15998	9587	36200	38442	35800	19400
CASE 14	6007	9624	28500	26855	26400	23100
	9000	9584	28400	27099	26000	23400
	11993	6376	28000	27219	26300	23700
	15998	6717	29100	28645	27900	25000
average		8738	31467	22440	30700	18200

Table 6C-7: Cases 15 to 17: AC Wearing Course Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
CASE 15	6007	4169765		69667	449000	551000
	9000	4172697		643458	96500	574000
	11993	4183725		36444	3730000	551000
	15998	4170920		95042	5290000	601000
CASE 16	6007	978350	180000	199018	192000	3632000
	9000	978088	276300	1914	304000	2866000
	11993	977996	291700	169826	434000	2514000
	15998	977894	297000	22210	395000	2438000
CASE 17	6007	1030698	33675300	638	4800000	
	9000	745065	39669000	10	46400000	
	11993	1010654	39826800	781	30400000	
	15998	997482	41239000	252	26700000	
average		2032778	19431888	103272	9932542	1715875

Table 6C-8: Cases 15 to 17: AC Binder Course Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
CASE 15	6007	135906	48100	581957	578000	551700
	9000	136336	70900	581203	1130000	540300
	11993	137967	58100	644279	452000	537300
	15998	136075	54700	616452	441000	538700
CASE 16	6007	919830	1382700	1442711	1260000	690100
	9000	926463	1245300	32571	1160000	776900
	11993	928846	1201400	1570646	973000	819400
	15998	931526	1174300	1768592	1020000	820400
CASE 17	6007	671365	6200	533671	547000	
	9000	39417	5500	8999994	82400	
	11993	848044	5500	8999994	187000	
	15998	991940	6600	8999994	230000	
average		566976	438275	2897672	671700	659350

Table 6C-9: Cases 15 to 17: Subgrade Layer Data

	load (lb)	KISS moduli (psi)	EVERCALC moduli (psi)	BOUSDEF moduli (psi)	MODCOMP3 moduli (psi)	MODULUS moduli (psi)
CASE 15	6007	6305	29600	27655	25100	23400
	9000	6322	28300	26922	22700	23300
	11993	6388	29100	28017	22500	23500
	15998	6312	29500	27570	22200	23400
CASE 16	6007	6950	33200	31395	34200	26900
	9000	6985	31400	12228	32000	25300
	11993	6985	31500	29710	32500	25300
	15998	7002	31100	29600	31900	25000
CASE 17	6007	4191	24400	18674	14700	
	9000	2754	26100	32450	16500	
	11993	4158	25800	21407	16600	
	15998	4235	25400	21967	17300	
average		5716	28783	25633	24017	24513

Appendix 6D
Generated Deflections For 8 Theoretical Cases using SAP2000, Composite plate theory, and Elastic Layer theory

Note: For the first three cases, SAP is used both with (w/) and without (w/o) simply supported edges (s.s.)

Table 6D-1: Case 1: E1 = 500000 psi, E2 = 150000 psi, k = 100 pci (E3 = 1920 psi), h1=3, h2=6

Location	deflection (in)			
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	SAP w/o s.s.
0	0.0227	0.0608	0.0215	0.0219
6	0.0222	0.0584	0.0208	0.0211
12	0.0207	0.0543	0.0193	0.0197
24	0.0155	0.0449	0.0161	0.0165
36	0.0093	0.0365	0.0128	0.0131
54	0.0024	0.0266	0.0084	0.0087
72	0.0003	0.0198	0.005	0.0052

Table 6D-2: Case 2: E1 = 1000000 psi, E2 = 250000 psi, k = 150 pci (E3 = 2880 psi), h1=3, h2=6

Location	deflection (in)			
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	SAP w/o s.s.
0	0.0142	0.0382	0.0131	0.0134
6	0.0138	0.0369	0.0127	0.013
12	0.0129	0.0346	0.0119	0.0122
24	0.00985	0.0289	0.01	0.0103
36	0.00609	0.0238	0.0081	0.0084
54	0.0018	0.0176	0.0054	0.0057
72	0.0003	0.0133	0.0033	0.0035

Table 6D-3: Case 3: E1 = 1500000 psi, E2 = 500000 psi, k = 100 pci (E3 = 1920 psi), h1=3, h2=6

Location	deflection (in)			
(in)	Comp. Plate	Elsym5	Sap w/ s.s.	SAP w/o s.s.
0	0.0128	0.0418	0.0116	0.0125
6	0.0126	0.041	0.0113	0.0123
12	0.0122	0.0386	0.0108	0.0118
24	0.0104	0.0349	0.0096	0.0105
36	0.0079	0.0307	0.0082	0.0092
54	0.004	0.0249	0.0061	0.0071
72	0.0013	0.02	0.0043	0.0051

Table 6D-4: Case 4: E1 = 1500000 psi, E2 = 500000 psi, k = 100 pci (E3 = 1920 psi), h1=2, h2=6

Location	deflection (in)		
(in)	Comp. Plate	Elsym5	Sap w/ s.s.
0	0.0148	0.0465	0.014
6	0.0146	0.0454	0.0137
12	0.014	0.0432	0.0131
24	0.0117	0.0382	0.0114
36	0.0086	0.0329	0.0096
54	0.0039	0.0258	0.007
72	0.0008	0.0203	0.0047

Table 6D-5: Case 5: E1 = 1500000 psi, E2 = 500000 psi, k = 100 pci (E3 = 1920 psi), h1=4, h2=6

Location	deflection (in)		
(in)	Comp. Plate	Elsym5	Sap w/ s.s.
0	0.0112	0.0384	0.0096
6	0.0111	0.0377	0.0094
12	0.0107	0.0349	0.0091
24	0.0092	0.0321	0.0081
36	0.0072	0.0287	0.007
54	0.004	0.0239	0.0054
72	0.0017	0.0197	0.0038

Table 6D-6: Case 6: E1 = 1500000 psi, E2 = 500000 psi,
k = 100 pci (E3 = 1920 psi), h1=3, h2=10

Location	deflection (in)		
(in)	Comp. Plate	Elsym5	Sap w/ s.s.
0	0.00804	0.0322	0.0061
6	0.00797	0.0317	0.0059
12	0.00774	0.0276	0.0057
24	0.00691	0.0261	0.0051
36	0.00571	0.0241	0.0045
54	0.00375	0.021	0.0036
72	0.00217	0.0182	0.0026

Table 6D-7: Case 7: E1 = 1500000 psi, E2 = 500000 psi,
k = 100 pci (E3 = 1920 psi), h1=3, h2=18

Location	deflection (in)		
(in)	Comp. Plate	Elsym5	Sap w/ s.s.
0	0.00419	0.0215	0.00229
6	0.00418	0.02113	0.00219
12	0.00412	0.01928	0.00203
24	0.0039	0.01716	0.00183
36	0.00356	0.01664	0.00163
54	0.0029	0.01543	0.00132
72	0.00215	0.01419	0.001

Table 6D-8: Case 8: E1 = 1500000 psi, E2 = 500000 psi,
k = 100 pci (E3 = 1920 psi), h1=3, h2=6
(Same as case 3 but with a 50'x50' plate)

Location	deflection (in)		
(in)	Comp. Plate	Elsym5	Sap w/ s.s.
0	0.0115	0.0432	0.0118
6	0.0114	0.042	0.0116
12	0.0111	0.0386	0.0111
24	0.0098	0.0349	0.0099
36	0.0078	0.0307	0.0085
54	0.0045	0.0249	0.0065
72	0.0016	0.02	0.0047

Appendix 6E
Moduli and Observations for Generated Deflections

Table 6E-1: KISS Results for Generated Deflections

		backcalculated:						thoeretical:						Error:		
case	deflection source	Dbar	k	Q	E1	E2	F	E1	E2	k	(E3)	h1	h2	E1 %err	E2 %err	k %err
1	ELSYM	23595457	13.70	6.91E-05	446053	270039	9.17E-08	500000	150000	100	1920	3	6	10.79	-80.03	86.30
low	composite plate	16720789	100.00	9.35E-14	437792	162057	4.16E-05	500000	150000	100	1920	3	6	12.44	-8.04	0.00
	SAP w/ simple support	49700000	47.00	4.31E-06	402000	1210000	8.42E-06	500000	150000	100	1920	3	6	19.60	-706.67	53.00
	SAP w/o simple support	50000000	45.30	4.21E-06	402000	1220000	9.61E-06	500000	150000	100	1920	3	6	19.60	-713.33	54.70
2	ELSYM	40829996	20.18	2.31E-05	672357	524036	2.38E-05	1000000	250000	150	2880	3	6	32.76	-109.61	86.54
ave	composite plate	29726954	149.81	8.65E-11	688979	311870	2.19E-05	1000000	250000	150	2880	3	6	31.10	-24.75	0.13
	SAP w/ simple support	88300000	71.90	1.26E-06	584000	2470000	6.06E-08	1000000	250000	150	2880	3	6	41.60	-888.00	52.07
	SAP w/o simple support	90200000	67.90	1.23E-06	585000	2550000	1.10E-05	1000000	250000	150	2880	3	6	41.50	-920.00	54.73
3	ELSYM	53100000	13.00	1.84E-05	1170000	543000	1.16E-05	1500000	500000	100	1920	3	6	22.00	-8.60	87.00
high	composite plate	53610181	100.00	6.40E-11	1163386	589151	5.29E-05	1500000	500000	100	1920	3	6	22.44	-17.83	0.00
	SAP w/ simple support	140000000	60.10	5.87E-07	881000	4050000	5.31E-08	1500000	500000	100	1920	3	6	41.27	-710.00	39.90
	SAP w/o simple support	123000000	52.50	1.23E-06	885000	3250000	6.82E-06	1500000	500000	100	1920	3	6	41.00	-550.00	47.50
4	ELSYM	37700000	13.80	2.69E-05	1870000	424000	1.23E-04	1500000	500000	100	1920	2	6	-24.67	15.20	86.20
high	composite plate	37336868	100.00	5.48E-11	1870673	440870	1.97E-05	1500000	500000	100	1920	2	6	-24.71	11.83	0.00
	SAP w/ simple support	114000000	51.30	7.09E-07	1590000	3080000	3.48E-06	1500000	500000	100	1920	2	6	-6.00	-516.00	48.70
5	ELSYM	70000000	12.50	1.69E-05	1740000	402000	3.01E-04	1500000	500000	100	1920	4	6	-16.00	19.60	87.50
high	composite plate	73700000	100.00	4.47E-11	1760000	458000	1.69E-06	1500000	500000	100	1920	4	6	-17.33	8.40	0.00
	SAP w/ simple support	163000000	70.00	4.93E-07	1610000	1840000	6.62E-06	1500000	500000	100	1920	4	6	-7.33	-268.00	30.00
6	ELSYM	124000000	10.80	2.21E-05	959000	415000	4.43E-04	1500000	500000	100	1920	3	10	36.07	17.00	89.20
high	composite plate	159000000	100.00	2.65E-11	900000	720000	7.52E-06	1500000	500000	100	1920	3	10	40.00	-44.00	0.00
	SAP w/ simple support	302000000	101.00	2.18E-07	854000	2000000	2.00E-06	1500000	500000	100	1920	3	10	43.07	-300.00	-1.00
7	ELSYM	401544934	1.00	1.13E-05	1008252	287365	1.69E-02	1500000	500000	100	1920	3	18	32.78	42.53	99.00
high	composite plate	633000000	100.00	3.37E-11	958000	657000	8.46E-03	1500000	500000	100	1920	3	18	36.13	-31.40	0.00
	SAP w/ simple support	873000000	265.00	6.80E-08	961000	1010000	8.54E-04	1500000	500000	100	1920	3	18	35.93	-102.00	-165.00
8	ELSYM	43100000	12.80	2.90E-05	1180000	384000	9.91E-06	1500000	500000	100	1920	3	6	21.33	23.20	87.20
high	composite plate	53600000	100.00	2.52E-11	1160000	589000	3.53E-05	1500000	500000	100	1920	3	6	22.67	-17.80	0.00
	SAP w/ simple support	108000000	57.20	1.03E-06	903000	2540000	9.55E-06	1500000	500000	100	1920	3	6	39.80	-408.00	42.80

OBSERVATIONS:

I expected best results when using Composite plate theory, this was NOT always the case.

k estimates were very good with cmposite plate theory expect this is because k is unique and E's depend on my equations. Therefore E1, E2 errors from CPT deflections purely reflect my equation errors.

Did ok with ELSYM equations

Surprisingly high E2 errors with SAP deflections.

Table 6E-2: EVERCALC 5.0 Results for Generated Deflections

		backcalculated:				theoretical:						error:			notes:
case	deflection source	E1	E2	E3	RMS err	E1	E2	k	(E3)	h1	h2	E1 %err	E2 %err	E3 %err	
1	ELSYM	2000000	82500	1900	0.24	500000	150000	100	1920	3	6	-300.00	45.00	1.04	E1max
	composite plate	532100	5000	87600	63.19	500000	150000	100	1920	3	6	-6.42	96.67	-4462.50	E2min
	SAP w/ simple support	181400	788800	6500	7.19	500000	150000	100	1920	3	6	63.72	-425.87	-238.54	
	SAP w/o simple support	185400	800300	6300	6.99	500000	150000	100	1920	3	6	62.92	-433.53	-228.13	
2	ELSYM	309000	5000	21500	51.34	1000000	250000	150	2880	3	6	69.10	98.00	-646.53	
	composite plate	2000000	5000	80700	55.73	1000000	250000	150	2880	3	6	-100.00	98.00	-2702.08	E1max, E2min
	SAP w/ simple support	552900	1000000	10100	6.63	1000000	250000	150	2880	3	6	44.71	-300.00	-250.69	E2max
	SAP w/o simple support	601200	1000000	9600	6.37	1000000	250000	150	2880	3	6	39.88	-300.00	-233.33	E2max
3	ELSYM	1365600	493500	1900	0.46	1500000	500000	100	1920	3	6	8.96	1.30	1.04	
	composite plate	2000000	81200	17400	33.66	1500000	500000	100	1920	3	6	-33.33	83.76	-806.25	E1max
	SAP w/ simple support	2000000	1000000	8900	5.00	1500000	500000	100	1920	3	6	-33.33	-100.00	-363.54	E1max, E2min
	SAP w/o simple support	2000000	1000000	7800	5.89	1500000	500000	100	1920	3	6	-33.33	-100.00	-306.25	E1max, E2min
4	ELSYM	2000000	432200	1900	0.40	1500000	500000	100	1920	2	6	-33.33	13.56	1.04	E1max
	composite plate	2000000	26300	27700	50.33	1500000	500000	100	1920	2	6	-33.33	94.74	-1342.71	E1max
	SAP w/ simple support	2000000	1000000	7900	5.49	1500000	500000	100	1920	2	6	-33.33	-100.00	-311.46	E1max, E2min
5	ELSYM	1302300	482800	1900	1.19	1500000	500000	100	1920	4	6	13.18	3.44	1.04	
	composite plate	2000000	108300	15100	22.42	1500000	500000	100	1920	4	6	-33.33	78.34	-686.46	E1max
	SAP w/ simple support	2000000	1000000	10100	5.17	1500000	500000	100	1920	4	6	-33.33	-100.00	-426.04	E1max, E2min
6	ELSYM	2000000	323100	2000	3.17	1500000	500000	100	1920	3	10	-33.33	35.38	-4.17	E1max
	composite plate	336900	694500	14800	11.76	1500000	500000	100	1920	3	10	77.54	-38.90	-670.83	
	SAP w/ simple support	2000000	1000000	14600	4.90	1500000	500000	100	1920	3	10	-33.33	-100.00	-660.42	E1max, E2min
7	ELSYM	2000000	228100	2200	2.85	1500000	500000	100	1920	3	18	-33.33	54.38	-14.58	E1max
	composite plate	1233800	1000000	15100	7.08	1500000	500000	100	1920	3	18	17.75	-100.00	-686.46	E2max
	SAP w/ simple support	2000000	10000000	37200	6.45	1500000	500000	100	1920	3	18	-33.33	-1900.00	-1837.50	E1max, E2min
8	ELSYM	1978600	380200	1900	1.30	1500000	500000	100	1920	3	6	-31.91	23.96	1.04	
	composite plate	173500	808300	15400	26.54	1500000	500000	100	1920	3	6	88.43	-61.66	-702.08	
	SAP w/ simple support	2000000	1000000	8500	5.83	1500000	500000	100	1920	3	6	-33.33	-100.00	-342.71	E1max, E2min

OBSERVATIONS:

Program often got stuck on limits.

Usually good estimates on E3 (subgrade moduli) with ELSYM data.

Expected best results from ELSYM data, this was not always the case, especially with E1 often good results with E2 and E3.

Extreme E3 errors with Composite Plate Theory and SAP deflections.

Table 6E-3: BOUSDEF Results for Generated Deflections

		backcalculated:				theoretical:						error:			notes:
case	deflection source	E1	E2	E3	ABS SUM % DIFF	E1	E2	k	(E3)	h1	h2	E1 %err	E2 %err	E3 %err	
1	ELSYM	100000	72389	1810	185.75	500000	150000	100	1920	3	6	80.00	51.74	5.73	E1min
	composite plate	100000	5000	17888	826.82	500000	150000	100	1920	3	6	80.00	96.67	-831.67	E1min, E2min
	SAP w/ simple support	100000	1000000	6469	62.99	500000	150000	100	1920	3	6	80.00	-566.67	-236.93	E1min, E2max
	SAP w/o simple support	100000	1000000	6301	61.78	500000	150000	100	1920	3	6	80.00	-566.67	-228.18	E1min, E2max
2	ELSYM	1999999	261079	2705	3.92	1000000	250000	150	2880	3	6	-100.00	-4.43	6.08	E1max
	composite plate	734459	7480	27271	540.03	1000000	250000	150	2880	3	6	26.55	97.01	-846.91	
	SAP w/ simple support	100000	1000000	10992	77.65	1000000	250000	150	2880	3	6	90.00	-300.00	-281.67	E1min, E2max
	SAP w/o simple support	100000	1000000	10617	82.12	1000000	250000	150	2880	3	6	90.00	-300.00	-268.65	E1min, E2max
3	ELSYM	1999999	732462	1792	11.13	1500000	500000	100	1920	3	6	-33.33	-46.49	6.67	E1max
	composite plate	486345	604417	12310	194.00	1500000	500000	100	1920	3	6	67.58	-20.88	-541.15	
	SAP w/ simple support	1999999	1000000	9360	68.48	1500000	500000	100	1920	3	6	-33.33	-100.00	-387.50	E1max, E2max
	SAP w/o simple support	1999999	1000000	8223	74.45	1500000	500000	100	1920	3	6	-33.33	-100.00	-328.28	E1max, E2max
4	ELSYM	1999999	868428	1775	7.37	1500000	500000	100	1920	2	6	-33.33	-73.69	7.55	E1max
	composite plate	100000	186217	13888	372.46	1500000	500000	100	1920	2	6	93.33	62.76	-623.33	E1min
	SAP w/ simple support	1999999	1000000	8378	74.29	1500000	500000	100	1920	2	6	-33.33	-100.00	-336.35	E1max, E2max
5	ELSYM	100000	584571	1816	190.39	1500000	500000	100	1920	4	6	93.33	-16.91	5.42	E1min
	composite plate	550245	947368	12165	127.58	1500000	500000	100	1920	4	6	63.32	-89.47	-533.59	
	SAP w/ simple support	1999999	1000000	10502	62.77	1500000	500000	100	1920	4	6	-33.33	-100.00	-446.98	E1max, E2max
6	ELSYM	1999999	523185	1950	30.49	1500000	500000	100	1920	3	10	-33.33	-4.64	-1.56	E1max
	composite plate	100000	656018	12828	98.53	1500000	500000	100	1920	3	10	93.33	-31.20	-568.13	E1min
	SAP w/ simple support	1999999	1000000	15647	60.88	1500000	500000	100	1920	3	10	-33.33	-100.00	-714.95	
7	ELSYM	1999999	331759	2310	27.75	1500000	500000	100	1920	3	18	-33.33	33.65	-20.31	E1max
	composite plate	1999999	1000000	15317	33.69	1500000	500000	100	1920	3	18	-33.33	-100.00	-697.76	E1max, E2max
	SAP w/ simple support	1999999	1000000	39899	55.50	1500000	500000	100	1920	3	18	-33.33	-100.00	-1978.07	E1max, E2max
8	ELSYM	1999999	632263	1819	17.59	1500000	500000	100	1920	3	6	-33.33	-26.45	5.26	E1max
	composite plate	100000	1000000	13618	172.28	1500000	500000	100	1920	3	6	93.33	-100.00	-609.27	E1min, E2max
	SAP w/ simple support	1999999	1000000	8952	72.38	1500000	500000	100	1920	3	6	-33.33	-100.00	-366.25	E1min, E2max

OBSERVATIONS:

Program often stuck on limits.
Pretty good E3 results with ELSYM data.
E2 and E3 not good with composite plate theory or SAP deflections.
E1 often stuck on limits for all three sets of deflections.

Table 6E-4: MODCOMP3 Results for Generated Deflections

		backcalculated:				theoretical:						error:			notes:
	deflection source	E1	E2	E3	RMS err	E1	E2	k	(E3)	h1	h2	E1 %err	E2 %err	E3 %err	
1	ELSYM	11400	5280000	1000	33.31	500000	150000	100	1920	3	6	97.72	-3420.00	47.92	no convergence in 20 iterations, soln. Not satisfactory
	composite plate	13200000	87500	4770	971.90	500000	150000	100	1920	3	6	-2540.00	41.67	-148.44	no convergence in 20 iterations, soln. Not satisfactory
	cpt with stiff layer	12700000	123000	3720	884.94	500000	150000	100	1920	3	6	-2440.00	18.00	-93.75	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/ simple support	2780000	407000	5000	21.55	500000	150000	100	1920	3	6	-456.00	-171.33	-160.42	defl w/I 0.15%, mod w/I 1%, soln not acceptable, RMS too large
	SAP ss w/ stiff layer	2990000	394000	4930	20.83	500000	150000	100	1920	3	6	-498.00	-162.67	-156.77	defl w/I 0.15%, mod w/I 1%, soln not acceptable, RMS too large
	SAP w/o simple support	314000	1380000	5120	18.15	500000	150000	100	1920	3	6	37.20	-820.00	-166.67	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/ stiff layer	361000	1270000	5080	17.94	500000	150000	100	1920	3	6	27.80	-746.67	-164.58	no convergence in 20 iterations, soln. Not satisfactory
2	ELSYM	391000	568000	2750	3.59	1000000	250000	150	2880	3	6	60.90	-127.20	4.51	no convergence in 20 iterations, soln. Not satisfactory
	composite plate	4620000	721000	7050	659.44	1000000	250000	150	2880	3	6	-362.00	-188.40	-144.79	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	cpt with stiff layer	5110000	730000	4310	592.49	1000000	250000	150	2880	3	6	-411.00	-192.00	-49.65	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	SAP w/ simple support	3090000	969000	7660	21.68	1000000	250000	150	2880	3	6	-209.00	-287.60	-165.97	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	SAP ss w/ stiff layer	3460000	923000	7520	21.13	1000000	250000	150	2880	3	6	-246.00	-269.20	-161.11	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	SAP w/o simple support	3070000	979000	7390	20.17	1000000	250000	150	2880	3	6	-207.00	-291.60	-156.60	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	SAP w/ stiff layer	3360000	952000	7250	20.09	1000000	250000	150	2880	3	6	-209.00	-287.60	-165.97	defl w/I 0.15%, soln. Not acceptable, RMS too large.
3	ELSYM	8110000	74000	2340	9.44	1500000	500000	100	1920	3	6	-440.67	85.20	-21.88	no convergence in 20 iterations, soln. Not satisfactory
	composite plate	4140000	2910000	5330	172.85	1500000	500000	100	1920	3	6	-176.00	-482.00	-177.60	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	cpt with stiff layer	8540000	2470000	4930	174.00	1500000	500000	100	1920	3	6	-469.33	-394.00	-156.77	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	SAP w/ simple support	697000	5100000	7580	8.16	1500000	500000	100	1920	3	6	53.53	-920.00	-294.79	no convergence in 20 iterations, soln. Not satisfactory
	SAP ss w/ stiff layer	698000	5090000	7580	8.15	1500000	500000	100	1920	3	6	53.47	-918.00	-294.79	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/o simple support	11500000	1410000	5840	11.08	1500000	500000	100	1920	3	6	-666.67	-182.00	-204.17	defl w/I 0.15%, soln. Not acceptable, RMS too large.
4	ELSYM	4170000	385000	1820	2.01	1500000	500000	100	1920	2	6	-178.00	23.00	5.21	defl w/I 0.15%, soln. questionable, RMS >2%
	composite plate	65700000	1010000	4960	326.27	1500000	500000	100	1920	2	6	-4280.00	-102.00	-158.33	no convergence in 20 iterations, soln. Not satisfactory
	cpt with stiff layer	65000000	1090000	3350	304.81	1500000	500000	100	1520	2	6	-4233.33	-118.00	-74.48	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/ simple support	7060000	1830000	5810	17.21	1500000	500000	100	1920	2	6	-370.67	-266.00	-202.60	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	SAP ss w/ stiff layer	8020000	1750000	5710	17.20	1500000	500000	100	1920	2	6	-434.67	-250.00	-197.40	defl w/I 0.15%, soln. Not acceptable, RMS too large.
5	ELSYM	3540000	30300	2840	17.00	1500000	500000	100	1920	4	6	-136.00	93.94	-47.92	terminated, sensor 2 (at r = 6 in.) not sensitiv to mod of layer 2. Soln Not satisfactory
	composite plate	22200000	787000	5900	107.11	1500000	500000	100	1920	4	6	-1380.00	-57.40	-207.29	no convergence in 20 iterations, soln. Not satisfactory
	cpt with stiff layer	19400000	1010000	4230	102.19	1500000	500000	100	1920	4	6	-1193.33	-102.00	-120.31	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/ simple support	610000	13700000	6800	18.19	1500000	500000	100	1920	4	6	59.33	-2640.00	-254.17	no convergence in 20 iterations, soln. Not satisfactory
6	ELSYM	14800000	17000	2830	12.83	1500000	500000	100	1920	3	10	-886.67	96.60	-47.40	terminated, sensor 2 (at r = 6 in.) not sensitiv to mod of layer 2. Soln Not satisfactory
	composite plate	79800000	230000	8780	38.19	1500000	500000	100	1920	3	10	-5220.00	54.00	-357.29	no convergence in 20 iterations, soln. Not satisfactory
	cpt with stiff layer	76400000	283000	7910	37.65	1500000	500000	100	1920	3	10	-4993.33	43.40	-311.98	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/ simple support	136000	100000000	3280	35.10	1500000	500000	100	1920	3	10	90.93	-19900.00	-70.83	E2max
7	ELSYM	14800000	87300	2620	6.14	1500000	500000	100	1920	3	18	-886.67	82.54	-36.46	defl w/I 0.15%, soln. Not acceptable, RMS too large.
	composite plate	100000000	768000	9930	14.02	1500000	500000	100	1920	3	18	-6566.67	-53.60	-417.19	E1max
	SAP w/ simple support	11900000	1290000	27900	8.33	1500000	500000	100	1920	3	18	-693.33	-158.00	-1353.13	no convergence in 20 iterations, soln. Not satisfactory
8	ELSYM	5570000	25500	2880	19.23	1500000	500000	100	1920	3	6	-271.33	94.90	-50.00	terminated, sensor 2 (at r = 6 in.) not sensitiv to mod of layer 2. Soln Not satisfactory
	composite plate	29900000	2340000	4800	138.28	1500000	500000	100	1920	3	6	-1893.33	-368.00	-150.00	no convergence in 20 iterations, soln. Not satisfactory
	cpt with stiff layer	26300000	2560000	3010	131.12	1500000	500000	100	1920	3	6	-1653.33	-412.00	-56.77	no convergence in 20 iterations, soln. Not satisfactory
	SAP w/ simple support	12300000	1350000	6410	10.94	1500000	500000	100	1920	3	6	-720.00	-170.00	-233.85	defl w/I 0.15%, soln. Not acceptable, RMS too large.

OBSERVATIONS:

The program does not minimize RMS error, it stops after max iterations, delfection convergence, or modulus convergence.
usually this is not the solution with minimum RMS.

In looking at results, solutions with minimum RMS are usually more reasonable than the final solutions.

Including the stiff layer as estimated bythe program seems to have little effect on the backcalculated moduli values.

Table 6E-5: MODULUS Results for Generated Deflections

		backcalculated:				thoeretical:						error:			notes:
case	deflection source	E1	E2	E3	abs sum % err	E1	E2	k	(E3)	h1	h2	E1 %err	E2 %err	E3 %err	
1	ELSYM	148100	448100	1600	6.21	500000	150000	100	1920	3	6	70.38	-198.73	16.67	
	composite plate	117200	450500	4700	91.70	500000	150000	100	1920	3	6	76.56	-200.33	-144.79	failed convexity test
	SAP w/ simple support	427000	1000000	4300	6.11	500000	150000	100	1920	3	6	14.60	-566.67	-123.96	E2 max
	SAP w/o simple support	435600	1000000	4200	6.04	500000	150000	100	1920	3	6	12.88	-566.67	-118.75	E2 max
2	ELSYM	866000	344400	2300	3.55	1000000	250000	150	2880	3	6	13.40	-37.76	20.14	
	composite plate	220900	736300	7400	78.10	1000000	250000	150	2880	3	6	77.91	-194.52	-156.94	
	SAP w/ simple support	1710300	1000000	6800	9.00	1000000	250000	150	2880	3	6	-71.03	-300.00	-136.11	E2max
	SAP w/o simple support	1843500	1000000	6500	7.89	1000000	250000	150	2880	3	6	-84.35	-300.00	-125.69	E2max
3	ELSYM	1459400	526600	1500	4.64	1500000	500000	100	1920	3	6	2.71	-5.32	21.88	near E1max
	composite plate	1785000	595000	6000	43.70	1500000	500000	100	1920	3	6	-19.00	-19.00	-212.50	near E1max
	SAP w/ simple support	2000000	1000000	7600	41.80	1500000	500000	100	1920	3	6	-33.33	-100.00	-295.83	E1max, E2max
	SAP w/o simple support	2000000	1000000	6700	44.30	1500000	500000	100	1920	3	6	-33.33	-100.00	-248.96	E1max, E2max
4	ELSYM	1634600	380100	1600	20.20	1500000	500000	100	1920	2	6	-8.97	23.98	16.67	near E1max
	composite plate	1126900	1000000	4900	46.00	1500000	500000	100	1920	2	6	24.87	-100.00	-155.21	E2max
	SAP w/ simple support	2000000	1000000	6700	43.70	1500000	500000	100	1920	2	6	-33.33	-100.00	-248.96	E1max, E2max
5	ELSYM	1555600	366400	1600	12.00	1500000	500000	100	1920	4	6	-3.71	26.72	16.67	near E1max
	composite plate	1074200	1000000	6200	34.90	1500000	500000	100	1920	4	6	28.39	-100.00	-222.92	E2 max
	SAP w/ simple support	2000000	1000000	8300	40.50	1500000	500000	100	1920	4	6	-33.33	-100.00	-332.29	E1max, E2max
6	ELSYM	1447600	352400	1400	18.60	1500000	500000	100	1920	3	10	3.49	29.52	27.08	near E1 max
	composite plate	1110700	1000000	8000	28.90	1500000	500000	100	1920	3	10	25.95	-100.00	-316.67	E2max
	SAP w/ simple support	2000000	1000000	11900	37.70	1500000	500000	100	1920	3	10	-33.33	-100.00	-519.79	E1max, E2max
7	ELSYM	100000	553400	1500	12.50	1500000	500000	100	1920	3	18	93.33	-10.68	21.88	E1min
	composite plate	1616400	1000000	10000	33.70	1500000	500000	100	1920	3	18	-7.76	-100.00	-420.83	E2max
	SAP w/ simple support	2000000	1000000	28800	44.10	1500000	500000	100	1920	3	18	-33.33	-100.00	-1400.00	E1max,E2max, failed conexity test
8	ELSYM	1453300	505300	1500	9.66	1500000	500000	100	1920	3	6	3.11	-1.06	21.88	near E1max
	composite plate	1900800	745200	6300	50.10	1500000	500000	100	1920	3	6	-26.72	-49.04	-228.13	near E1max
	SAP w/ simple support	2000000	1000000	7300	unknown	1500000	500000	100	1920	3	6	-33.33	-100.00	-280.21	E1max, E2max, could not print error table

OBSERVATIONS:

A lot of trouble with computer lock up in printing results
Most often least error with ELSYM deflections.
Often stuck on maximums with SAP deflections.